

# Appendix D

## Topics and sample problems for Math 017 Final Exam (short version)

Below there are 59 categories:

9 “arithmetic” categories

13 “easy” categories

24 “basic” categories

9 “difficult” categories

4 “very difficult” categories

Each version of the exam will be created as follows:

First, 45 categories will be selected: All 9 arithmetic categories will be included. From the algebraic categories, 8 categories will be selected at random from 13 “easy” categories, all “basic” categories will be represented, 3 out of the “difficult” categories, and 1 “very difficult” category. After this process we will have 45 categories. One question out of each such category will be selected at random. Thus, the test will consist of 45 questions.

More examples for each category, can be found in “Topics and sample problems for Math 017 Final Exam (long version)” at <http://faculty.ccp.edu/dept/math/devmath.html>.

### **ARITHMETIC PART: 9 QUESTIONS**

*Select randomly one question from each category.*

#### **1A**

**Addition, subtraction, multiplication, or division of integers (using both:  $\div$  and fraction bar as a division sign) - up to 3 integers.**

Compute, if possible. If not possible, write “not possible”:  $-3 - (-2) + (-1) =$

Compute, if possible. If not possible, write “not possible”:  $-2 \times (-1) \times (-1) =$

#### **2A**

**Order of operations, integers only, two operations (only integers as an answer, no exponents).**

Compute, if possible. If not possible, write “not possible”:  $4 - (7 - 8) =$

Evaluate, if possible. If not possible, write “not possible”:  $-1 - 1(-1) =$

#### **3A**

**Addition and subtraction of fractions or of fractions and integers (positive and negative).**

Compute, if possible. If not possible, write “not possible”:  $-\frac{2}{3} + \frac{4}{5} =$

Compute, if possible. If not possible, write “not possible”:  $\frac{3}{5} - \left(-\frac{5}{6}\right) =$

## 4A

**Multiplication of fractions and of fractions and integers (positive and negative).**

Compute, if possible. If not possible, write “not possible”:  $\frac{3}{5} \times (-10) =$

Evaluate, if possible. If not possible, write “not possible”:  $-\frac{345}{5} \times \frac{1}{2} \times \frac{10}{345} =$

## 5A

**Division (using both  $\div$  and fraction bar) of fractions and of fractions and integers (positive and negative).**

Evaluate, if possible. If not possible, write “not possible”:  $\frac{\frac{3}{4}}{\frac{1}{2}}$

Compute, if possible. If not possible, write “not possible”:  $-\frac{3}{5} \div \frac{2}{7}$

## 6A

**“Zero” category(number/0; 0/number; addition of opposite numbers; multiplication by 0). “Ten” category: Multiplication and division of decimals and integers by powers of ten.**

Compute, if possible. If not possible, write “not possible”:  $234 \times 1000 =$

Compute, if possible. If not possible, write “not possible”:  $0 \div 23$

## 7A

**Addition, subtraction, multiplication and division of decimals (and integers).**

Evaluate, if possible. If not possible, write “not possible”:  $0.1 - 2.1 =$

Evaluate, if possible. If not possible, write “not possible”:  $0.33 \div (-0.011) =$

## 8A

**Exponential notation (fractions, integers, decimals)**

Evaluate, if possible. If not possible, write “not possible”:  $-1^6 =$

Evaluate, if possible. If not possible, write “not possible”:  $-\left(-\frac{2}{3}\right)^2 =$

## 9A

**Addition, subtraction, multiplication and division including mixed number representations. Order of operation (up to 3 operations).**

Compute, if possible. If not possible, write “not possible”:  $2\frac{1}{3} - 1\frac{5}{6} =$

Evaluate, if possible. If not possible, write “not possible”:  $(2-3)\left(-\frac{1}{3} - \frac{2}{3}\right) =$

## ALGEBRA PART: 36 QUESTIONS

*EASY CATEGORY: select randomly 8 categories and then 1 question from each category selected:*

### 1E

**Writing phrases (involving only one operation) as algebraic expressions.**

Write the following phrase as an algebraic expression: The sum of  $a$  and  $-b$

Write the following phrase as an algebraic expression:  $-x$  raised to  $m$ -th power

### 2E

**Ability to distinguish between an algebraic expression and equation.**

Determine whether the following examples represent an equation or an algebraic expression.

Any time you find an equation, circle its left hand side.

- a)  $4x - 2$       b)  $4x - 2 = 7$

Determine whether the following examples represent an equation or an algebraic expression. In the case of an equation, circle the right-hand side of the equation:

- a)  $3x - 7y = 5$       b)  $x = 0$

### 3E

**Solving linear equations (all variables on one side, no parentheses involved, no need for collecting terms, no fractions or decimals unless they appear as a solution).**

Solve the following equation:  $0 = 1 - x$

Solve the following equation:  $-3 = -6w + 2$

## 4E

**Solving a linear inequality (all variables on one side, no parentheses involved, no need for collecting terms, no fractions or decimals unless they appear as a solution).**

Solve the following inequality:  $10 > -2x$

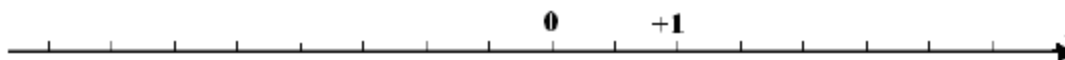
Solve the following inequality:  $-x \leq 0$

## 5E

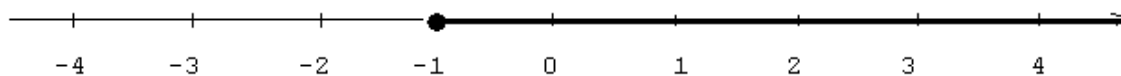
**Graphing sets on a number line.**

Graph the following number set on a number line. Assume that the distance between all marks is the same.

$$x \leq -1$$



Using inequality symbols, describe the set that is graphed below:



## 6E

**Checking if a given integer is a solution of a linear equation (inequality) in one variable.**

Determine if the number  $-5$  is a solution of the following inequality.  $x + 2 < -4$ .

For each of the following equations determine if the number  $0$  is its solution. If  $0$  is a solution, circle the equation.

a)  $3x = 0$

b)  $\frac{1}{x} = 0$

## 7E

**Understanding “<”, “≤” and “>”, “≥” notation. Using it to describe sets of numbers (phrases like “at least”, “at most”, “not less”, “not more”).**

Determine which of the following numbers satisfies the condition that  $x \leq -2$ .

$-4$ ,  $-2.1$ ,  $-2$ ,  $-0.9$ ,  $0$ ,  $1$

Describe the following set of numbers using inequality signs: All numbers  $x$  that are at most equal to  $9$ .

## 8E

**Rewriting the expression by replacing the variable with its value (only integers) and evaluating it, if possible. One operation needed for the evaluation.**

Let  $x = -1$ . Rewrite the expression replacing the variable with its value and evaluate it, if possible. If evaluation is not possible, write “not possible”:  $x - 2$

Let  $x = 0$ . Rewrite the expression replacing the variable with its value and evaluate it, if possible. If evaluation is not possible, write “not possible”:  $\frac{4}{x}$

## 9E

**Solving literal equations (variable one is seeking will not appear in the denominator). One operation needed. No simplification needed at the end.**

Solve for  $x$ :  $\frac{x}{a} = b$

Solve for  $x$ :  $x + a = b$

## 10E

**If possible, rewriting algebraic expressions with the use of exponential laws (non-negative exponents; only one law used) .**

Simplify, using one of the exponential laws, if possible, If not possible, write “not possible” :  
 $a^3 a^{15}$

Simplify, using one of the exponential laws, if possible, If not possible, write “not possible” :  
 $a^3 + a^{15}$

## 11E

**Removing parentheses in multiplication of a monomial by a polynomial (up to 3 terms). Performing numerical operations might be required but no use of exponential laws needed. No fractions, no decimals.**

Eliminate parentheses in the following expression:  $-4(x + 3y - z)$

Eliminate parentheses in the following expression:  $(3x - 10)2$

## 12E

**Collecting like terms (2 terms only), if possible. No fractions, no decimals.**

If possible, collect like terms. If not possible, write “not possible”:  $-3x + 4x$

If possible, collect like terms. If not possible, write “not possible”:  $xy - yx$

## 13E

**Factoring an indicated expression (a variable or an integer). No use of exponential laws needed. No instances when “an entire term is factored” and thus “1” becomes a term after factorization.**

Factor  $x$  from the following expression:  $3x - 2xy$

Factor 4 from the following expression:  $4y^2 + 8x$

*BASIC CATEGORY: 1 question from each category of 24 categories*

## 1B

**Simplifying an expression by performing numerical operations when possible. (may need to change the order of terms or factors). Performing an operation on fractions or on decimals might be needed.**

Simplify if possible. If no numerical operation can be performed write “not possible”:

$$\frac{2}{3} + \frac{1}{6} - Y$$

Simplify if possible. If no numerical operation can be performed write “not possible”:

$$12 + 3x$$

## 2B

**Simplifying algebraic expressions by applying exponential laws (only non-negative exponents; one variable, up to 2 types of operations). Performing an operation on fractions or on decimals might be needed.**

Simplify  $2(x^2)^3 x$

Simplify:  $\frac{a^3}{aa^2}$

## 3B

**The Distributive Law: removing parentheses in multiplication of a monomial by a polynomial with up to 3 terms. No collecting terms needed at the end but**

**numerical operations or application of exponential laws needed in the process. Performing an operation on fractions or on decimals might be needed.**

Use the Distributive Law to eliminate parentheses in the following expression. Simplify, if

possible.  $\left(3x - \frac{1}{2}\right)2x$

Use the Distributive Law to eliminate parentheses in the following expression. Simplify, if possible.  $-2(x + 3y - 4z)$

## 4B

**The Distributive Law: removing parentheses in multiplication of a binomial by a binomial. No collecting terms needed at the end but numerical operations or application of exponential laws needed in the process (no operations on fractions or decimals).**

Eliminate parentheses in the following expression. Simplify:  $(a - b)(2a + 3)$

Eliminate parentheses in the following expression. Simplify:  $(2b + c)(-c + c^5)$

## 5B

**Factorization of an indicated monomial (factorization of “an entire term” is and thus getting “1” as a term after factorization possible). No operations on fractions or decimals.**

Factor  $x^2$  from the following expression:  $3x^5 - 2x^3 + x^2$

Factor  $xy^2$  from the following expression:  $3x^2y^2 + 5xy^4 - 3x^3y^3$

## 6B

**Simplification of rational expressions. Factorization of a numerator OR a denominator (but not both) needed (unless simplification is not possible)**

Simplify, if possible. If not possible, write “not possible”:  $\frac{a + 2ab}{a}$

Simplify, if possible. If not possible, write “not possible”:  $\frac{a + 4d}{-4d - a}$

## 7B

**Understanding what it means that a number(s) is a solution of a given equation (“relatively” complicated evaluation needed). Performing an operation on fractions or on decimals might be needed.**

**or**

**Ability to rewrite the expression and evaluating it, if possible (one instance of a variable). If evaluation is not possible, recognizing that it is not possible to**

**evaluate. Performing an operation on fractions or on decimals might be needed.**

Is  $x = -1$  a solution of  $(-x)(-x) = 2$

Let  $x = 4$ . Rewrite the expression replacing the variable with its value and evaluate, if possible. If evaluation is not possible, write “not possible”.  $-2^x$

## 8B

**Recognizing equivalent expressions by applying the following**

- a) the order of terms of an expression can be change
- b) the order of factors of an expression can be changed

c)  $\frac{b}{a} \pm \frac{c}{a} = \frac{b \pm c}{a}$       d)  $\frac{ab}{c} = a \cdot \frac{b}{c} = ab \cdot \frac{1}{c}$       e)  $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$

Circle all expressions that are equal to  $2x - 5y + 4z$ :

$2x + 4z - 5y$        $-5y + 4z + 2x$        $-5y + 2x + 4z$

Is  $-3AB$  equivalent to  $BA - 3$ ? How about  $-3AB$  and  $-3BA$ ?

## 9B

**Expanding exponential expression; Rewriting expressions using exponential notation whenever possible. Understanding that the exponent pertains only to the closest expression.**

or

**Evaluation of exponential expressions raised to the zero-th power.**

or

**Identifying bases, exponents and numerical coefficients of exponential expressions.**

Expand (i.e. write without using exponential notation). Do not simplify.  $(5A)^3$

Rewrite using exponential notation whenever it is possible:  $\frac{-z - z - z}{zzzz}$

Evaluate  $(2x)^0$

## 10B

**Determining if rational expressions can be simplified and simplifying them. Factorization not needed.**

Simplify, if possible. If not possible, write “not possible”:

$$\frac{-a^2}{b^2a^2}$$

Simplify, if possible. If not possible, write “not possible”:

$$\frac{v-4z}{4z}$$

## 11B

**Collecting like terms: more than one type of like terms in the expression (fractions and decimals used but not as coefficients of variables).**

Simplify by collecting like terms:  $3x + a - 4x - a$

Simplify by collecting like terms:  $\frac{2}{3} - xy - y + y + \frac{1}{3} - yx$

## 12B

**Solving linear equations with a variable on both sides. No fractions or parentheses involved. “No solution” or “all real numbers” as a solution possible. No operations on decimals needed.**

Solve the following equation:  $3x = 5 - 4x$

Solve the following equation:  $x - 8 = x - 9$

## 13B

**Solving linear inequalities (no fractions, no decimals involved; if a variable is only on one side its coefficient will be negative). “No solution” or “all real numbers” as a solution possible. No operations on decimals needed.**

Solve the following inequality:  $x \leq x$

Solve the following inequality:  $2 - 3x \leq 14$

## 14B

**“The difference of squares” factorization (formula not given).**

Factor the following expression:  $36x^2 - 1$

Factor the following expression:  $m^4 - 0.01n^2$

## 15B

**Rewriting algebraic expression in terms of another variable. Only a direct substitution needed; “relatively easy” simplification.**

Rewrite the expression  $\frac{3a}{2}$  in terms of  $x$ , given that  $a = 2x$  (i.e. substitute  $2x$  for  $a$ ) Simplify.

Express  $-2P$  in terms of  $m$ , when  $P = m^2 - m$  (i. e. substitute  $m^2 - m$  for  $P$ ). Write your final answer without parentheses.

**16B Simplifying algebraic expressions by removing parentheses first, and then collecting like terms. Performing an operation on fractions or on decimals might be needed (no expressions of the form  $(a \pm b)^2$ )**

Eliminate parentheses and simplify by collecting like terms:  $3(x^5 - 2) - (4 - x^5)$

Eliminate parentheses and simplify by collecting like terms:  $(a - 3)(a + 4)$

**17B**

**Collecting like terms (if possible) involving operations on fractions, or decimals.**

If possible, collect like terms. If not possible, write “not possible”  $-3x + 0.4x^2$

If possible, collect like terms. If not possible, write “not possible”  $\frac{ab}{3} - \frac{1}{6}ba$

**18B**

**Solving linear equations (inequalities) when the removal of parentheses is needed; no fractions involved. “No solution” or “all real numbers” as a solution is a possibility. Operations on decimals might be needed.**

Solve the following equation:  $3x = 2 - 4(x - 1)$

Solve the following inequality:  $3x + 1 > -11 + 3(x + 4)$

**19B**

**Solving equations for a given variable. Factorization of this variable might be needed in the process. Variable might be on both sides of the equation.**

**Variable could be in the denominator. No simplification needed at the end.**

**More than one operation might be needed in the process.**

Solve for  $x$ :  $ax - bx - c = 0$

Solve for  $x$ :  $\frac{a}{x} = b$

**20B**

**Writing phrases as algebraic expressions (involving 2 operations).**

Use the letter  $x$  to represent a number and write the following as algebraic expression:

Seven more than one third of a number  
Use the letter  $x$  to represent a number and write the following as algebraic expression:  
A quantity decreased by 9, and then multiplied by A

## 21B

**Identifying linear equations and writing them in a standard form.  
Recognizing the value of parameters in the representation.**

or

**Rewriting algebraic expressions to match a prescribed format and identifying the values of given variables.**

Determine if the following equation is linear in one variable. If so, express it in the form  $ax + b = 0$ ,  $a \neq 0$ . Determine the values of  $a$  and  $b$  in your representation:  $\frac{x}{4} - 1 = 0$

Rewrite the following expression in the form  $ax^3 + b$ , where  $a$  and  $b$  are any numbers. Identify  $a$  and  $b$  in your representation:  $(2x)^3 + 4$

## 22B

**Applying laws of exponents in order to evaluate an expression (up to two operations). Performing an operation on fractions or on decimals might be needed.**

Or

**Factoring -1**

Evaluate:  $\frac{4^7 \cdot 4^6}{4^{11}}$

Factor  $-1$  from the following expression:  $\frac{-X}{3} + y$ .

## 23B

**Removing parenthesis form expressions of the form  $(a \pm b)^2$  and simplifying them by collecting like terms.**

Eliminate parentheses and simplify by collecting like terms:  $(2 - x)^2$

Eliminate parentheses and simplify by collecting like terms:  $(3ab + 1)^2$

## 24B

**Evaluation of an algebraic expression when the value of a part of the algebraic expression is given (rather than the value of the variables). No rewriting the expression in its equivalent form is needed.**

If  $A^5 = -3$ , evaluate:  $-2A^5$

Evaluate the expression  $-\frac{a}{b}$ , if  $\frac{a}{b} = -3$

**DIFFICULT CATEGORY:** randomly select 3 categories and then 1 question from each category

## 1D

**Ability to rewrite the expression by substituting (up to 2 substitutions) value of variable(s) and then evaluating the expression, if possible. If evaluation is not possible, recognizing that it is not possible to evaluate. “More challenging” examples.**

The expression  $\frac{3-x}{y-5}$  cannot be evaluated for which of the following values of  $x$  and  $y$ ? Circle all of them.

- a)  $x = 3, y = -5$       b)  $x = 3, y = 5$       c)  $x = 3, y = 0$       d)  $x = 0, y = 5$

Rewrite the expression by substituting value of a variable and then evaluate it, if possible. If not possible, write “not possible”:  $2a^2 - (2a)^2$  if  $a = -1$

## 2D

**Recognizing equivalent expressions. Applying more than one of the following rules at a time.**

- a) the order of terms of an expression can be changed
- b) the order of factors of an expression can be changed
- c)  $\frac{b}{a} \pm \frac{c}{a} = \frac{b \pm c}{a}$
- d)  $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$
- e)  $\frac{ab}{c} = a \cdot \frac{b}{c} = ab \cdot \frac{1}{c}$

Circle all of the following expressions that are equivalent to  $m - n$ :

$$n - m, \quad m(-n), \quad m - (n), \quad -nm, \quad -n + m, \quad (-1)n - m$$

Rewrite each of the following expression in their equivalent form as a single fraction:

$$-\frac{a}{4s} - \frac{1}{4s}a^2$$

## 3D

**Simplifying expressions involving exponential expressions (“relatively difficult examples”). Operations on fractions or decimals might be needed.**

Circle all expressions that are equivalent to  $\left(\frac{a}{3}\right)^{20}$  :

$$\frac{a^{20}}{3}, \quad \frac{a^{20}}{3^{20}}, \quad \frac{(a^{12})^8}{3^{20}}, \quad \frac{a^{12}a^8}{3^{20}}, \quad \frac{(a^4)^5}{3^{20}}$$

Perform the indicated operations and simplify:  $\frac{-y^5}{y^8} \cdot y^3$

## 4D

**Applying the Distributive Law when removing parentheses in multiplication of a binomial by a trinomial. Collecting terms is needed.**

Eliminate parentheses:  $(b + c - 2)(b^2 - b^3)$

Eliminate parentheses:  $(x - x^2)(2 - 6x^3 + 2x^2)$

## 5D

**Factoring an indicated expression involving parentheses, fractions or decimals.**

Factor  $(a + 2b)$  from the following expression:  $4x(a + 2b) - (a + 2b)$ .

Factor  $\frac{2}{3}$  from the following expression:  $\frac{2}{3}x^2y - \frac{4}{3}z$

Factor 0.01 from the following expression:  $0.2x^2 + 0.03z$

## 6D

**Rewriting an algebraic expression in terms of a designated variable when a rewriting of the expression in an equivalent form is needed.**

Rewrite the expression  $a - 2b + 3c + 4d$  in terms of  $x$ , if it is given that  $a + 3c = 5x$  and  $4d - 2b = -x$ . Simplify.

Rewrite the expression  $\frac{x}{5} - \frac{y}{5}$  in the terms of  $a$  if  $x - y = a$ .

## 7D

**Solving linear equations (inequalities) involving fractions. “No solution” or “all real numbers” as a solution is a possibility.**

Solve the following equation:  $\frac{4x}{5} + 1 = \frac{2}{3}$

Solve the following inequality:  $\frac{4-x}{2} - \frac{x+2}{4} > 5$

## 8D

**Solving equations for a given variable. Factorization of this variable might be needed in the process. Variable could be in the denominator. Simplification at the end required.**

Solve for the indicated variable. Simplify your answer whenever possible:  $\frac{b}{a} = ac$  for  $b$

Solve for the indicated variable. Simplify your answer whenever possible:  $AX - A = 1 - X$  for  $A$

## 9D

**Evaluating algebraic expressions when the value of a part of it is given (rather than the value of variables). Rewriting the original expression in some equivalent form is required to make the needed substitution.**

Evaluate the expression  $a - c - b$ , if  $a - b = -1$  and  $c = 2$ .

Evaluate the expression  $2c - (b - a)$ , if  $a - b = -1$  and  $c = 2$ .

*VERY DIFFICULT CATEGORY: randomly select 1 category and then 1 question from the category selected*

## 1VD

**Writing phrases as algebraic expressions and then simplifying them:**

a) using the laws of exponents

b) removing parentheses and collecting like terms.

Write as a single algebraic expression using parentheses where appropriate, then eliminate the parentheses and simplify: The product of  $-x$  and  $x^2$ , then raised to the sixth power.

Write the following expressions using algebraic symbols, eliminate parentheses, if necessary, and collect like terms: multiply  $3x - 2$  and  $-4x + 2$

## 2VD

**Rewriting algebraic expressions in terms of a designated variable requiring “challenging” simplifications.**

Express  $(m + n)(m^2 - mn + n^2)$  in terms of  $x$ , if  $m = 2$ , and  $n = 3x$ . Simplify.

Express  $(m + n)(m^2 - mn + n^2)$  in terms of  $m$ , if  $n = m$ . Simplify.

### 3VD

**“Non standard” factoring of the difference of squares, difference (sum) of cubes (formula for the difference/sum of cubes given but students need to know the formula for the difference of squares should).**

Factor and then simplify the following expression  $m^2 - (1 - 3m)^2$

The following formula is true:  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ . Factor the following expression using the above formula:  $y^3x^3 - 64$  .

### 4VD

**Replacing up to 2 variables with an algebraic expression and solving the resulting equation.**

Let  $a = 3x - 2$  and  $b = -x + 2$ . Find the value of  $x$  so that the following is true:  $a = b$

Let  $a = 3x - 2$  and  $b = -x + 2$ . Find the value of  $x$  so that the following is true:  $a = 2b$