Community College of Philadelphia **Spring 2011**
Math 172-Calculus II **Dr. Yun Yoo**

<u>Infinite Sequences and Series</u>

Definition: A sequence $\{a_n\}$ has the limit L and we write

$$\lim_{n\to\infty} a_n = L \qquad \text{or} \qquad a_n \to L \text{ as } n \to \infty$$

If we can make the terms a_n as close to L as we like by taking n sufficiently large. If $\lim_{n->\infty} a_n$ exists, we say the sequence **converges** (or is **convergent**). Otherwise, we say the sequence **diverges** (or is **divergent**).

Theorem: The sequence $\{r_n\}$ is convergent if $-1 < r \le 1$ and divergent for all other values of r.
In other words, $\lim_{n\to\infty} r_n = 0$ if $-1 < r < 1$ and $\lim_{n\to\infty} r_n = 1$ if $r = 1$.

<u>1. The Test for Geometric Series:</u>

The geometric series $\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + ...$ is convergent if $-1 < r < 1$ and its sum is $\dfrac{a}{1-r}$. The Geometric Series is divergent if $r < -1$ or $r > 1$

<u>2. The P-test:</u>

The series $\sum_{n=1}^{\infty} \dfrac{1}{n^p}$ is convergent if $p > 1$ and the series $\sum_{n=1}^{\infty} \dfrac{1}{n^p}$ is divergent if $p \le 1$.

<u>3. The Test for Divergence:</u>

If $\lim_{n->\infty} a_n$ does not exist or if $\lim_{n->\infty} a_n \ne 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

(Notice that if $\lim_{n->\infty} a_n = 0$, we can not conclude that $\sum_{n=1}^{\infty} a_n$ is convergent. For example, in the harmonic series $\sum_{n=1}^{\infty} \dfrac{1}{n}$, $\lim_{n->\infty} \dfrac{1}{n} = 0$, but $\sum_{n=1}^{\infty} \dfrac{1}{n}$ is divergent by p-test.)

4. The integral Test:

For $\sum_{n=1}^{\infty} a_n$, let $f(n) = a_n$. suppose $f(n)$ is a continuous, positive, decreasing function on $[1,\infty)$.

If $\int_1^{\infty} f(n)dn$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent.

If $\int_1^{\infty} f(n)dn$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is convergent.

5. The Comparison Test:

Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms.

If $\sum_{n=1}^{\infty} b_n$ is convergent and $a_n \le b_n$ for all n , then $\sum_{n=1}^{\infty} a_n$ is also convergent.

If $\sum_{n=1}^{\infty} b_n$ is divergent and $a_n \ge b_n$ for all n , then $\sum_{n=1}^{\infty} a_n$ is also convergent.

6. The Limit Comparison Test:

Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$, then either both series converge or both diverge.

7. The Alternating Test:

The alternating series $\sum_{n=0}^{\infty} (-1)^n b_n = b_0 - b_1 + b_2 - b_3 + b_4 - \cdots$ satisfies two conditions:

$b_n \le b_{n+1}$ for all n and $\lim_{n\to\infty} b_n = 0$, then the series $\sum_{n=0}^{\infty} (-1)^n b_n$ is convergent.

Definition: A series $\sum_{n=1}^{\infty} a_n$ is called absolutely convergent (AC) if $\sum_{n=1}^{\infty} |a_n|$ is convergent.

Definition: A series $\sum_{n=1}^{\infty} a_n$ is called conditional convergent (CC) if it is convergent but not absolutely convergent.

Theorem: If a series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then it is convergent.

<u>8. The Ratio Test:</u>

If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

<u>9. The Root Test:</u>

If $\lim_{n\to\infty}\sqrt[n]{a_n} = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

If $\lim_{n\to\infty}\sqrt[n]{a_n} = L > 1$ or $\lim_{n\to\infty}\sqrt[n]{a_n} = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.