Community College of Philadelphia Spring 2011
Math 172-Calculus II Dr. Yun Yoo

<u>This solution is generated by Ms. Louesa Huf who is taking math 172 currently.</u>

After reading page 464-465 of the textbook I found the information needed to solve the problem from class:

$\int tan^4x \ \ sec^3x \ \ dx$

Involves using this formula: $\int \sec x \, dx = ln|\sec x + \tan x| + c$

I may have gotten a few signs mixed up but here goes....

1. express tan^4x in terms of secant

   $\int (sec^2x - 1)^2 \, sec^3x \, dx$

   $\int (sec^4x - 2sec^2x + 1) sec^3x \, dx$

   $\int sec^7x - 2sec^5x + sec^3x \, dx$

2. Then integrate each secant individually

   $\int sec^7x \, dx - 2\int sec^5x \, dx + \int sec^3x \, dx$

3. I integrated using integration by parts from the smallest to the largest because it turns out that I needed the solution from the smaller power of secant to solve the others

   $\int sec^3 \, x \, dx = \ ?$ $u = sec \ x$ $dv = sec^2x \, dx$

   $du = \sec x \tan x$ $v = \tan x$

   $\int sec^3x \, dx = \sec x \tan x - \int \sec x \, tan^2x \, dx$

   $\int sec^3x \, dx = \sec x \tan x - \int \sec x \, (sec^2x - 1) \, dx$

   $\int sec^3x \, dx = \sec x \tan x - \int sec^3x - sec \ x \ dx$

   $\int sec^3x \, dx = \sec x \tan x - \int sec^3x \, dx - \int sec \ x \ dx$

   $2\int sec^3x \, dx = \sec x \tan x - \int \sec x \, dx$

   $\int sec^3x \, dx = \frac{1}{2}(\sec x \tan x - ln|\sec x + \tan x|)$ using the identity $\int \sec x \, dx = ln|\sec x + \tan x| + c$

4. Next I used integration by parts for the next lowest power of secant

   $-2\int sec^5x \, dx = \ ?$ $u = (\sec x)^3$ $dv = sec^2x \, dx$

   $du = 3sec^3x \ tanx \ dx$ $v = \tan x$

$-2\int sec^5x\,dx = \tan x\,sec^3x - 3\int tan^2x\,sec^3x\,dx$

$-2\int sec^5x\,dx = \tan x\ \ sec^3x - 3\int (sec^2x - 1)sec^3x\,dx$

$-2\int sec^5x\,dx = \tan x\,sec^3x - 3\int sec^5x - sec^3x\,dx$

$-2\int sec^5x\,dx = \tan x\,sec^3x - 3\int sec^5x\,dx - \int sec^3x\,dx$

$\int sec^5x\,dx = \tan x\,sec^3x - \int sec^3x\,dx$ since I already solved $\int sec^3x\,dx$ I'll substitute it when I write out the whole answer

5. Next I used integration by parts for the highest power of sign.

$\int sec^7x\,dx = \ ?$ $u = (\sec x)^5$ $dv = sec^2x\,dx$
$du = 5sec^4x\tan x\sec x\,dx$ $v = \tan x$

$\int sec^7x\,dx = \tan x\,sec^5x - 5\int sec^5x\,tan^2x\,dx$

$\int sec^7x\,dx = \tan x\,sec^5x - 5\int sec^5x(sec^2x - 1)dx$

$\int sec^7x\,dx = \tan x\,sec^5x - 5\int sec^7x - sec^5x\,dx$

$\int sec^7x\,dx = \tan x\,sec^5x - 5\int sec^7x\,dx - \int sec^5x\,dx$

$6\int sec^7x\,dx = \tan x\,sec^5x - \int sec^5x\,dx$

$\int sec^7x\,dx = \frac{1}{6}\left(\tan x\,sec^5x - \int sec^5\,x\,dx\right)$ since I already solved $\int sec^5x\,dx$ I'll substitute it when I write out the answer

6. Here it is

$$\int tan^4x\ \ sec^3x\ \ dx = \frac{1}{6}\left\{\tan x\,sec^5x + \left[\tan x\,sec^3x - \frac{1}{2}(\sec x\tan x - ln|\sec x + \tan x|)\right]\right\} + \left[\tan x\,sec^3x - \frac{1}{2}(\sec x\tan x - ln|\sec x + \tan x|)\right] + \frac{1}{2}(sec\ x\tan x - ln|\sec x + \tan x|) + C$$

If it's wrong then I got a lot of practice with trigonometric identities and such! :-)