Math 152 (Probability) Course Outline and References

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Probability has its origins in the study of "games of chance" (that is, gambling). The related field of statistics has its origins in the study of mortality rates and the value of life insurance contracts. At the time (1600s), "life insurance" contracts were offered for wealthy and/or famous individuals, and were a subject of financial speculation, so in a sense statistics also is rooted in gambling. It wasn't until the early 1900s that the usefulness of probability and statistics in science, technology, manufacturing (quality control, process control), medicine (the first application of statistics to a clinical trial of a drug was around 1920), engineering, demography, economics, finance, communications, and many other areas began to be seriously realized.

Probability and intuition Probability is distinguished in part by its role in serving as a corrective to often bad intuition. A large number of magic tricks and scams (not to mention casinos) rely on people's misunderstanding of basic ideas of probability. Persi Diaconis, a magician-turned-mathematician/statistician, has stated something to the effect that people's brains are just not wired to do probability problems well (quoted in Bennett, Randomness). What he means is that without conscious application of the mathematical theory of probability it is often very easy to be misled. This is even true for people who are experts in the theory. A stunning case in point is the famous "Monty Hall Problem", in which many mathematicians publicly embarassed themselves getting a simple question wrong. Reportedly even Paul Erdos (the subject of the book "The Man Who Loved Only Numbers" and a major figure in the development of probability) was initially misled by this question.

The Monty Hall is loosely based on a TV game show called "Let's Make a Deal" and named after the show's host. It goes like this: there are three doors, and behind one of the doors there is a prize. You (the contestant) choose a door. The host, Monty Hall, does not open the door but instead opens another door behind which there is no prize. He asks you if you'd like to change your choice (there's only one other door left) or keep your chosen door. What should you do?

It is often argued that there is no point in switching. There are only two doors, so it's 50-50, right? On the other hand, Monty does give some real information by showing you a door that isn't a winner. So maybe you should switch? Huge quantities of ink were wsasted in discussing this question. It was rare to hear someone say "let's just try the two strategies out and see which is better".

Course In this course we'll start by discussing the Monty Hall problem. We will actually try out the two strategies. We'll discuss what the important points of the problem are, and whether we faithfully reproduced them in our tests (if we didn't, then our test results wouldn't help settle the question.) We'll learn how to do experiments like these on a computer (so we don't have to spend years rolling dice or flipping coins, totaling long columns of numbers, etc.), using the statistical software R (www.r-project.org). We'll have about half of our class meetings in a classroom equipped with computers.

Here's a brief outline (tentative) of the course.

1. Introduction; Monty Hall problem; flipping coins
2. Computing (using R)
3. Basic ideas of probability: how to put our findings into mathematical form
4. Statements, sets and functions. Our translation of probability ideas will be into the language of sets and functions so we take time out to describe this.
5. Experiments and probability models. Repetition, sampling, independence, conditional probability. Markov chains. Randomization and randomized algorithms.
6. Random variables. Expectation, variance. Joint distribution, conditional distribution, conditional expectation. Martingales.
7. Statistical inference. Sampling revisited. Randomization.
8. Law of large numbers (what it means and what it doesn't mean). Other laws.
9. Central Limit Theorem and the normal distribution.
10. Applications. One possibility is "image restoration".

Textbook and references There are several useful references. I will use my own notes to lecture from. I will likely assign homework (exercises and reading) from the Kemeny, Knapp and Snell book listed below. As it's free there's no reason not to have a copy handy.

References

[1] Feller, W. *Introduction to Probability Theory and its Applications*, Volume 1. (This is a classic, and too hard for a textbook at this level, but an excellent reference.)

[2] Bennett, D. *Randomness.* (A recent popular book, worth a read.)

[3] Hodges, J. and Lehmann, E. *Elements of Finite Probability.* (An excellent text by reknowned statisticians, with very good discussions of sample space, sampling, and issues close to statistics.)

[4] Kemeny, Knapp and Snell. *Finite Mathematics.* (This is currently under an "open-source" license, which means you can download it free and even edit it. The current version includes only the chapters needed for probability; older out-of-print versions are also freely downloadable. See http://www.math.dartmouth.edu/~doyle/)