

## Week 3 Tuesday Homework (1319287)

Question 123456789101112131415161718

## 1. Question DetailsSCalcET6 2.1.002. [679759]

A cardiac monitor is used to measure the heart rate of a patient after surgery. It compiles the number of heartbeats after  $t$  minutes. When the data in the table are graphed, the slope of the tangent line represents the heart rate in beats per minute. The monitor estimates this value by calculating the slope of a secant line. Use the data to estimate the patient's heart rate after 42 minutes using the secant line between the points with the given values of  $t$ . (Give your answers correct to 1 decimal place.)

$t$ (min)	36	38	40	42	44
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Heartbeats	2529	2668	2805	2940	3077
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(a)  $t = 36$  and  $t = 42$ (b)  $t = 38$  and  $t = 42$ (c)  $t = 40$  and  $t = 42$ (d)  $t = 42$  and  $t = 44$ 

## 2. Question DetailsSCalcET6 2.1.003. [679930]

The point  $P(1, 1/6)$  lies on the curve  $y = x/(5 + x)$ . If  $Q$  is the point  $(x, x/(5 + x))$ , use a scientific calculator to find the slope of the secant line  $PQ$  (correct to six decimal places) for the following values of  $x$ .

(a) 0.5

(b) 0.9

(c) 0.99

(d) 0.999

(e) 1.5


(f) 1.1

(g) 1.01

(h) 1.001

## 3. Question DetailsSCalcET6 2.1.AE.03. [679805]

Time interval	Average velocity (m/s)
$6 \leq t \leq 7$	63.7
$6 \leq t \leq 6.1$	59.29
$6 \leq t \leq 6.05$	59.045
$6 \leq t \leq 6.01$	58.849
$6 \leq t \leq 6.001$	58.8049

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**EXAMPLE 3** Suppose that a ball is dropped from the upper observations deck of the CN Tower in Toronto, 450 m above the ground. Find the velocity of the ball after 6 seconds.

**SOLUTION** Through experiments carried out four centuries ago, Galileo discovered that the distance fallen by any freely falling body is proportional to the square of the time it has been falling. (This model for free fall neglects air resistance.) If the distance fallen after  $t$  seconds is denoted by  $s(t)$  and measured in meters, then Galileo's law is expressed by the equation

$$s(t) = 4.9t^2$$

The difficulty in finding the velocity after 6 s is that we are dealing with a single instant of time ( $t = 6$ ), so no time interval is involved. However, we can approximate the desired quantity by computing the average velocity over the brief interval of a tenth of a second from  $t = 6$  to  $t = 6.1$ :

$$\begin{aligned} \text{average velocity} &= \frac{\text{change in position}}{\text{time elapsed}} \\ &= \frac{s(6.1) - s(6)}{0.1} \\ &= \frac{4.9(\boxed{\phantom{00}})^2 - 4.9(\boxed{\phantom{00}})^2}{0.1} = \boxed{\phantom{00}} \end{aligned}$$

The table shows the results of similar calculations of the average velocity over successively smaller time periods.

It appears that as we shorten the time period, the average velocity is becoming closer to  $\boxed{\phantom{00}}$  m/s. The instantaneous velocity when  $t = 6$  is defined to be the limiting value of these average velocities over shorter and shorter time periods that start at  $t = 6$ . Thus the (instantaneous) velocity after 6 s is

$$v = \boxed{\phantom{00}} \text{ m/s}$$

## 4. Question DetailsSCalcET6 2.1.005. [679833]

If a ball is thrown in the air with a velocity 34 ft/s, its height in feet  $t$  seconds later is given by  $y = 34t - 16t^2$ .

(a) Find the average velocity for the time period beginning when  $t = 2$  and lasting 0.5 second.

$$\boxed{\phantom{00}} \text{ ft/s}$$

(b) Find the average velocity for the time period beginning when  $t = 2$  and lasting 0.1 second.

$$\boxed{\phantom{00}} \text{ ft/s}$$

(c) Find the average velocity for the time period beginning when  $t = 2$  and lasting 0.05 second.

$$\boxed{\phantom{00}} \text{ ft/s}$$

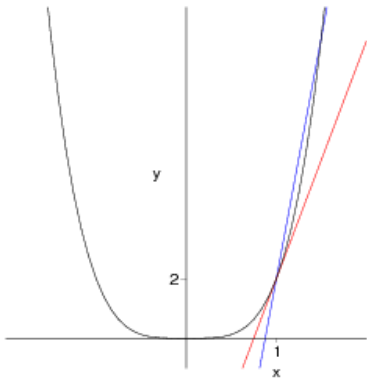
(d) Find the average velocity for the time period beginning when  $t = 2$  and lasting 0.01 second.

$$\boxed{\phantom{00}} \text{ ft/s}$$

(e) Estimate the instantaneous velocity when  $t = 2$ .

$$\boxed{\phantom{00}} \text{ ft/s}$$

5. Question DetailsSCalcET6 2.1.AE.01. [679714]



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**EXAMPLE 1** Find an equation of the tangent line to the function  $y = 2x^4$  at the point  $P(1, 2)$ .

**SOLUTION** We will be able to find an equation of the tangent line  $t$  as soon as we know its slope  $m$ . The difficulty is that we know only one point,  $P$ , on  $t$ , whereas we need two points to compute the slope. But observe that we can compute an approximation to  $m$  by choosing a nearby point  $Q(x, 2x^4)$  on the graph and computing the slope  $m_{PQ}$  of the secant line  $PQ$ . We choose  $x \neq 1$  so that  $P \neq Q$ . Then

$$m_{PQ} = \frac{2x^4 - 2}{x - 1}$$

For instance, for the point  $Q(1.5, 10.125)$  we have

$$m_{PQ} = \frac{\boxed{\phantom{00}} - 2}{\boxed{\phantom{00}} - 1} = \frac{\boxed{\phantom{00}}}{.5} = \boxed{\phantom{00}}$$

The tables below show the values of  $m_{PQ}$  for several values of  $x$  close to 1. The closer  $Q$  is to  $P$ , the closer  $x$  is to 1 and, it appears from the tables, the closer  $m_{PQ}$  is to  $\boxed{\phantom{00}}$ . This suggests that the slope of the tangent line  $t$  should be  $m = \boxed{\phantom{00}}$ .

$x$	$m_{PQ}$	$x$	$m_{PQ}$
2	30	0	2
1.5	16.25	0.5	3.75
1.1	9.282	0.9	6.878
1.01	8.121	0.99	7.881
1.001	8.012	0.999	7.988

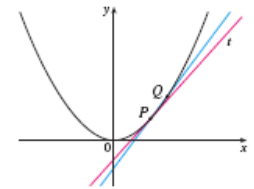
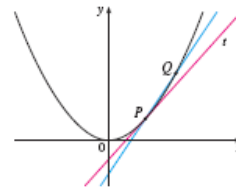
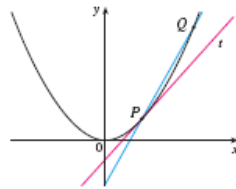
We say that the slope of the tangent line is the *limit* of the slopes of secant lines, and we express this symbolically by writing

$$\lim_{Q \rightarrow P} m_{PQ} = 8 \quad \text{and} \quad \lim_{x \rightarrow 1} \frac{2x^4 - 2}{x - 1} = 8$$

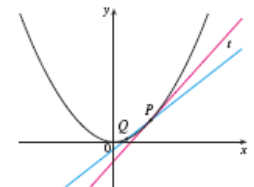
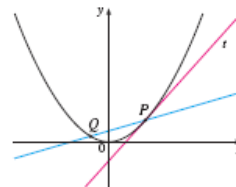
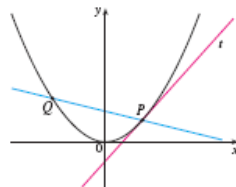
Assuming that the slope of the tangent line is indeed **8**, we use the point-slope form of the equation of a line (see Appendix B) to write the equation of the tangent line through  $(1, 2)$  as

$$y - \boxed{\phantom{00}} = \boxed{\phantom{00}}(x - 1) \quad \text{or} \quad y = \boxed{\phantom{00}}x - \boxed{\phantom{00}}$$

The graphs below illustrate the limiting process that occurs in this example. As  $Q$  approaches  $P$  along the graph, the corresponding secant lines rotate about  $P$  and approach the tangent line  $t$ .



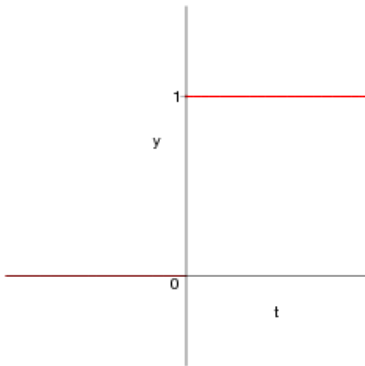
$Q$  approaches  $P$  from the right




$Q$  approaches  $P$  from the left

6. Question DetailsSCalcET6 2.1.Tut.01. [697543]

7. Question DetailsSCalcET6 2.2.AE.06. [679743]



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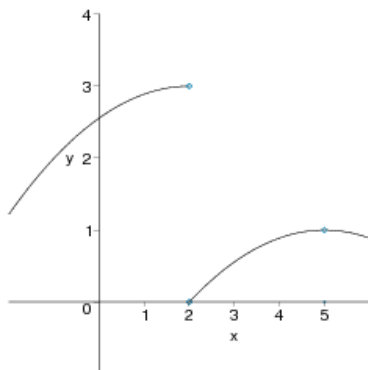
**EXAMPLE 6** The Heaviside function  $H$  is defined by

$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$

[This function is named after the electrical engineer Oliver Heaviside (1850-1925) and can be used to describe an electric current that is switched on at time  $t = 0$ .] Its graph is shown in the figure.

As  $t$  approaches 0 from the left,  $H(t)$  approaches . As  $t$  approaches 0 from the right,  $H(t)$  approaches . Therefore the limit as  $t$  approaches 0 of  $H(t)$  does not exist.

## 8. Question DetailsSCalcET6 2.2.AE.07. [679845]



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**EXAMPLE 7** The graph of a function  $g$  is shown in the figure. Use it to state the values (if they exist) of the following:

(a)  $\lim_{x \rightarrow 2^-} g(x)$    (b)  $\lim_{x \rightarrow 2^+} g(x)$    (c)  $\lim_{x \rightarrow 2} g(x)$

(d)  $\lim_{x \rightarrow 5^-} g(x)$    (e)  $\lim_{x \rightarrow 5^+} g(x)$    (f)  $\lim_{x \rightarrow 5} g(x)$

**SOLUTION** From the graph we see that the values of  $g(x)$  approach  as  $x$  approaches 2 from the left, but they approach  as  $x$  approaches 2 from the right. Therefore

(a)  $\lim_{x \rightarrow 2^-} g(x) =$   and (b)  $\lim_{x \rightarrow 2^+} g(x) =$

(c) Since the left and right limits are different, we conclude that the limit as  $x$  approaches 2 of  $g(x)$  does not exist.

The graph also shows that

(d)  $\lim_{x \rightarrow 5^-} g(x) =$   and (e)  $\lim_{x \rightarrow 5^+} g(x) =$

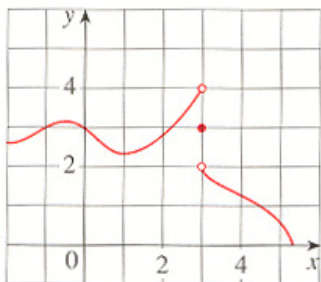
(f) This time, the left and right limits are the same, and so we have

$\lim_{x \rightarrow 5} g(x) =$

Despite this fact, notice that  $g(5) \neq 1$

## 9. Question DetailsSCalcET6 2.2.004. [657127]

For the function  $f$  whose graph is given, state the value of the given quantity, if it exists. (If it does not exist, enter NONE.)



(a)  $\lim_{x \rightarrow 0} f(x)$

(b)  $\lim_{x \rightarrow 3^-} f(x)$

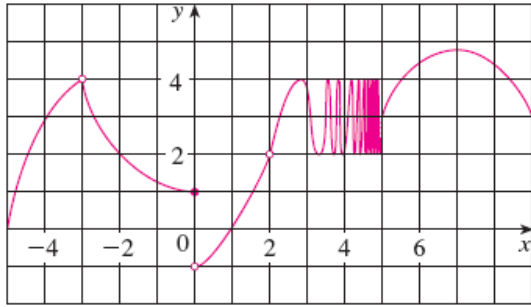
(c)  $\lim_{x \rightarrow 3^+} f(x)$

(d)  $\lim_{x \rightarrow 3} f(x)$

(e)  $f(3)$

10. Question DetailsSCalcET6 2.2.006. [679738]

For the function  $h$  whose graph is given, state the value of each quantity, if it exists. (If it does not exist, enter NONE.)



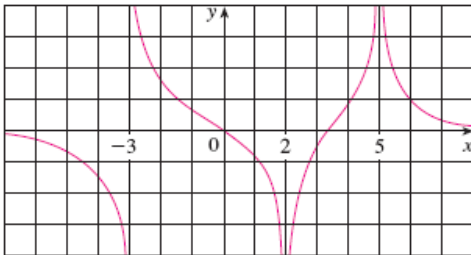
(a)  $\lim_{x \rightarrow -3^-} h(x)$   (b)  $\lim_{x \rightarrow -3^+} h(x)$   (c)  $\lim_{x \rightarrow -3} h(x)$   (d)  $h(-3)$

(e)  $\lim_{x \rightarrow 0^-} h(x)$   (f)  $\lim_{x \rightarrow 0^+} h(x)$   (g)  $\lim_{x \rightarrow 0} h(x)$   (h)  $h(0)$

(i)  $\lim_{x \rightarrow 2} h(x)$   (j)  $h(2)$   (k)  $\lim_{x \rightarrow 5^+} h(x)$   (l)  $\lim_{x \rightarrow 5^-} h(x)$

11. Question DetailsSCalcET6 2.2.008. [679769]

For the function  $R$  whose graph is shown, state the following. (If you need to use  $-\infty$  or  $\infty$ , enter -INFINITY or INFINITY.)



(a)  $\lim_{x \rightarrow 2} R(x)$

(b)  $\lim_{x \rightarrow 5} R(x)$

(c)  $\lim_{x \rightarrow -3^-} R(x)$

(d)  $\lim_{x \rightarrow -3^+} R(x)$

(e) The equations of the vertical asymptotes.

$x =$   (smallest value)

$x =$

$x =$   (largest value)

12. Question DetailsSCalcET6 2.2.AE.03. [679852]

$x$	$\frac{1 - \cos(x)}{x}$
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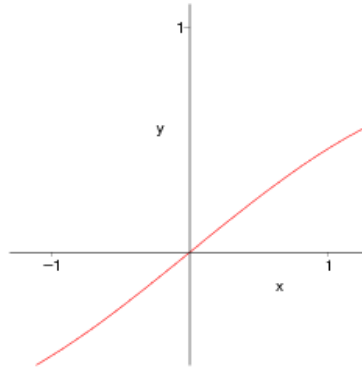
[Video Example](#)[Online Textbook](#)**EXAMPLE 3** Guess the value of the limit below.

$$\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x}$$

**SOLUTION** The function  $f(x) = (1 - \cos x) / x$  is not defined when  $x = \boxed{\phantom{0}}$ . Using a calculator (and remembering that, if  $x \in \mathbb{R}$ ,  $\cos(x)$  means the **cosine** of the angle whose *radian* measure is  $x$ ), we construct a table of values correct to eight decimal places. From the table and the graph in the figure we guess that

$$\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x} = \boxed{\phantom{0}}$$

This guess is in fact correct, as will be proved in Chapter 3 using a geometric argument.



13. Question DetailsSCalcET6 2.2.019. [679786]

Consider the following function.

$$f(x) = \frac{e^x - 1 - x}{x^2}$$

(a) Evaluate the function at the given numbers (correct to six decimal places).

$x$	$f(x)$	$x$	$f(x)$
1	<input type="text"/>	-1	<input type="text"/>
0.5	<input type="text"/>	-0.5	<input type="text"/>
0.1	<input type="text"/>	-0.1	<input type="text"/>
0.05	<input type="text"/>	-0.05	<input type="text"/>
0.01	<input type="text"/>	-0.01	<input type="text"/>

(b) Guess the value of the limit of  $f(x)$  as  $x$  approaches 0. (If it does not exist, enter NONE.)

14. Question DetailsSCalcET6 2.2.023. [780384]

Use a table of values to estimate the value of the limit. If you have a graphing device, use it to confirm your result graphically.

15. Question DetailsSCalcET6 2.2.025. [679767]

Determine the infinite limit.

  $\infty$   $-\infty$

16. Question DetailsSCalcET6 2.2.026.MI. [1387171]  
Determine the infinite limit.

$$\lim_{x \rightarrow -6^-} \frac{x+5}{x+6}$$

$\infty$

$-\infty$

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17. Question DetailsSCalcET6 2.2.034. [780407]  
Find the vertical asymptotes of the function below.

$x =$   (smaller value)

$x =$   (larger value)

18. Question DetailsSCalcET6 2.2.035. [679808]  
Estimate the value of the limit to five decimal places.

Assignment Details

Name (AD): **Week 3 Tuesday Homework (1319287)**

Submissions Allowed: **5**

Category: **Homework**

Code:

Locked: **No**

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