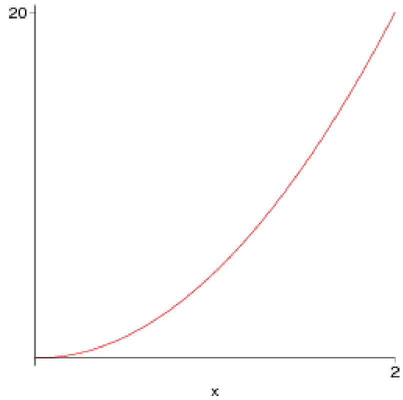
Week 12 Tuesday Homework (1329184)

Question **1234567891011121314151617181920**

1. Question DetailsSCalcET6 5.1.AE.01. [708910]



Video Example

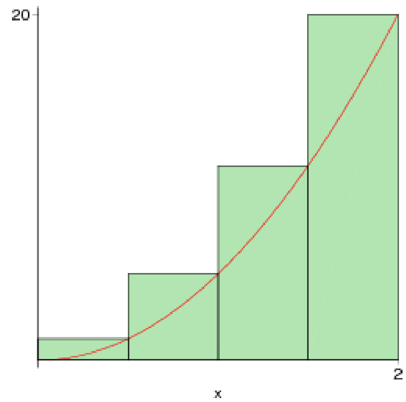
Online Textbook

EXAMPLE 1 Use rectangles to estimate the area under the parabola $y = 5x^2$ from 0 to 2 (the parabolic region illustrated to the left).

SOLUTION We first notice that the area must be somewhere between 0 and 40 because it is contained in a rectangle of side lengths 2 and 20, but we can certainly do better than that.

Suppose we divide the region into 4 strips by drawing vertical lines $x = 1/2$, $x = 1$, and $x = 3/2$ as in the figure.

We can approximate each strip by a rectangle whose base is the same as the strip and whose height is the same as the right edge of the strip (as in the figure). In other words, the heights of these rectangles are the values of the function $f(x) = 5x^2$ at the right endpoints of the subintervals [0, 1/2], [1/2, 1], [1, 3/2], and [3/2, 2].



Each rectangle has width 1/2 and the heights are 1.25, 5, 11.25, and 20. If we let R_4 be the sum of the areas of these approximating rectangles, we get

$$R_4 = \frac{1}{2} \cdot 1.25 + \frac{1}{2} \cdot 5 + \frac{1}{2} \cdot 11.25 + \frac{1}{2} \cdot 20 = \boxed{}$$

We see that the area A is less than R_4, so $A < 18.75$.

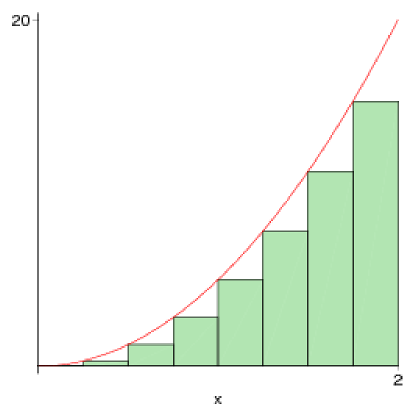
Instead of using the rectangles above we could use the smaller rectangles whose heights are the values of f at the left endpoints of the sub intervals. (The leftmost rectangle has collapsed because its height is 0.) The sum of the areas of these approximating rectangles is

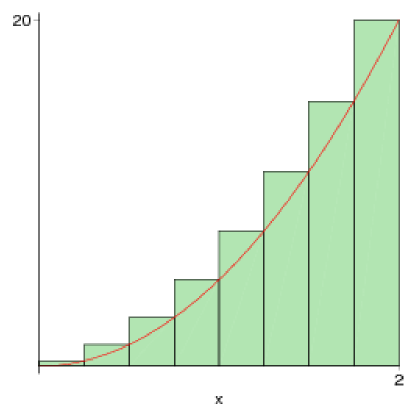
$$L_4 = \frac{1}{2} \cdot 0^2 + \frac{1}{2} \cdot 1.25 + \frac{1}{2} \cdot 5 + \frac{1}{2} \cdot 11.25 = \boxed{}$$

We see that the area is larger than L_4, so we have lower and upper estimates for A:

$\boxed{} < A < 18.75$.

We can repeat this procedure with a larger number of strips. The figure shows what happens when we divide the region into eight strips of equal width.





By computing the sum of these areas of the smaller rectangles (L_8) and the sum of the areas of the larger rectangles (R_8), we obtain better lower and upper estimates for A: $10.9375 < A < 15.9375$. So one possible answer to the question is to say that the true area lies somewhere between 10.9375 and 15.9375.

We could obtain better estimates by increasing the number of strips. The table below shows the results of similar calculations (with a computer) using n rectangles whose heights are found with left endpoints (L_n) or right endpoints (R_n). In particular, we see by using 50 strips that the area lies between 12.9360 and 13.7360. With 1000 strips, we narrow it down even more: A lies between 13.3133400 and 13.3533400. A good estimate is obtained by averaging these numbers:

$A \approx \boxed{}$.

n	L_n	R_n
10	11.4000000	15.4000000
20	12.3500000	14.3500000
30	12.6740741	14.0074074
50	12.9360000	13.7360000
100	13.1340000	13.5340000
1000	13.3133400	13.3533400

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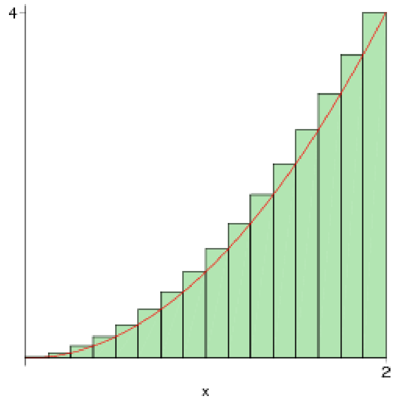
2. Question DetailsSCalcET6 5.1.AE.02. [1291704]



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EXAMPLE 2 For the region under $f(x) = x^2$ on [0, 2], show that the sum of the areas of the upper approximating rectangle approaches 8/3, that is

$$\lim_{n\to\infty} R_n = \frac{8}{3}$$

SOLUTION R_n is the sum of the areas of the n rectangles in the figure. Each rectangle has width $2/n$ and the heights are the values of the function $f(x) = x^2$ at the points $2/n$, $4/n$, $6/n$,…,$2n/n$; that is, the heights are $(2/n)^2$,

$\boxed{}$, $(6/n)^2$,…, $(2n/n)^2$. Thus,

$$R_n = \frac{2}{n}\left(\frac{2}{n}\right)^2 + \frac{2}{n}\left(\frac{4}{n}\right)^2 + \frac{2}{n}\left(\frac{6}{n}\right)^2 + \dots + \frac{2}{n}\left(\frac{2n}{n}\right)^2$$

$$= \frac{2}{n}\cdot \boxed{} (1^2 + 2^2 + 3^2 + \dots + n^2)$$

$$= \boxed{} (1^2 + 2^2 + 3^2 + \dots + n^2)$$

Here we need the formula for the sum of the squares of the first n positive integers:

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Perhaps you have seen this formula before. Putting this formula into our expression for R_n, we get

$$R_n = \frac{8}{n^3}\cdot \boxed{}$$

$$= \frac{8(n+1)(2n+1)}{6n^2}$$

Thus we have

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{8(n+1)(2n+1)}{6n^2} = \lim_{n\to\infty} \frac{8}{6}\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)$$

$$= \lim_{n\to\infty} \frac{8}{6}\left(1 + \boxed{}\right)\left(2 + \boxed{}\right)$$

$$= \frac{8}{6}\cdot 1 \cdot 2 = \frac{8}{3}$$

3. Question DetailsSCalcET6 5.1.AE.04. [708918]

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EXAMPLE 4 Suppose the odometer on our car is broken and we want to estimate the distance driven over a 30 second time interval. We take the speedometer readings every five seconds and record them in the following table:

Time (s)	0	5	10	15	20	25	30
Velocity (mi/h)	18	19	25	28	31	30	28

In order to have the time and the velocity in consistent units, let's convert the velocity readings to feet per second (1 mi/h = 5280/3600 ft/s)

Time (s)	0	5	10	15	20	25	30
Velocity (ft/s)	26		37		45	44	41

During the first five seconds the velocity doesn't change very much, so we can estimate the distance traveled during that time by assuming that the velocity is constant. If we take the velocity during that time interval to be the initial velocity (26 ft/s), then we obtain the approximate distance traveled during the first five seconds:

26 ft/s × 5 s = ____ ft

Similarly, during the second time interval the velocity is approximately constant and we take it to be the velocity when t = 5 s. So our estimate for the distance traveled from t = 5 s to t = 10 s is

28 ft/s × 5 s = ____ ft

If we add similar estimates for the other time intervals, we obtain an estimate for the total distance traveled:

(26 × 5) + (28 × 5) + (37 × 5) + (41 × 5) + (45 × 5) + (44 × 5) = ____ ft

We could just as well have used the velocity at the end of each time period instead of the velocity at the beginning as our assumed constant velocity. Then our estimate becomes

(28 × 5) + (37 × 5) + (41 × 5) + (45 × 5) + (44 × 5) + (41 × 5) = ____ ft

If we had wanted a more accurate estimate, we could have taken velocity readings every two seconds, or even every second.

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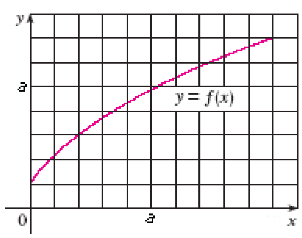
4. Question DetailsSCalcET6 5.1.001. [698780]

Do the following.

a = 20



(a) By reading values from the given graph of f, use five rectangles to find a lower estimate and an upper estimate for the area under the given graph of f from x = 0 to x = 40.

$A \approx$ ____ (lower estimate)

$A \approx$ ____ (upper estimate)

(b) Find new estimates using ten rectangles in each case.

$A \approx$ ____ (lower estimate)

$A \approx$ ____ (upper estimate)

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5. Question DetailsSCalcET6 5.1.011.MI. [1387813]

The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find lower and upper estimates for the distance, d that she traveled during these three seconds. (Round your answers to one decimal place.)

***t*(s)**	0	0.5	1.0	1.5	2.0	2.5	3.0
***v*(ft/s)**	0	5.7	11.2	15.5	17.5	19.4	20

d = ____ ft (lower estimate)

d = ____ ft (upper estimate) Tutorial

6. Question DetailsSCalcET6 5.1.017. [698774]
The area *A* of the region *S* that lies under the graph of the continuous function is the limit of the sum of the areas of approximating rectangles.

$$A = \lim_{n \to \infty} R_n = \lim_{n \to \infty} \left(f(x_1)\Delta x + f(x_2)\Delta x + \ldots + f(x_n)\Delta x \right)$$

Use this definition to find an expression for the area under the graph of *f* as a limit. Do not evaluate the limit.

$f(x) = \sqrt[5]{x}, 1 \le x \le 17$

- ○ $\displaystyle\lim_{n \to \infty} \sum_{i=1}^{n} \sqrt[5]{1 + \frac{16i}{n}} \cdot \frac{16}{n}$
- ○ $\displaystyle\lim_{n \to \infty} \sum_{i=1}^{n} \sqrt[5]{\frac{16i}{n}} \cdot \frac{16}{n}$
- ○ $\displaystyle\lim_{n \to \infty} \sum_{i=0}^{n} \sqrt[5]{1 + \frac{16i}{n}} \cdot \frac{16}{n}$
- ○ $\displaystyle\lim_{n \to \infty} \sum_{i=1}^{n} \sqrt[5]{1 + \frac{17i}{n}} \cdot \frac{16}{n}$
- ○

7. Question DetailsSCalcET6 5.1.020. [698783]
Determine a region whose area is equal to the given limit. Do not evaluate the limit.

- ○
- ○
- ○
- ○
- ○

8. Question DetailsSCalcET6 5.1.022. [806503]
The area *A* of the region *S* that lies under the graph of the continuous function is the limit of the sum of the areas of approximating rectangles:

(a) Use this definition to find an expression for the area under the curve $y = x^3$ from 0 to 1 as a limit.

- ○
- ○
- ○
- ○
- ○

(b) Use the following formula for the sum of the cubes of the first integers to evaluate the limit in part (a).

9. Question DetailsSCalcET6 5.2.AE.04. [1291715]

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EXAMPLE 4 Evaluate the following integrals by interpreting each in terms of areas.

SOLUTION (a) Since $f(x) = \sqrt{25 - x^2} \geq 0$, we can interpret this integral as the area under the curve $y = \sqrt{25 - x^2}$ from 0 to ____. But since $y^2 =$

____, we get $x^2 + y^2 = 25$, which shows that the graph of f is a quarter-circle with radius ____ in the top figure. Therefore,

____

(b) The graph of $y = x - 1$ is the line with slope ____ shown in the bottom figure. We compute the integral as the difference of the areas of the two triangles:

$= A_1 - A_2 =$ ____ $- 0.5 =$ ____

10. Question DetailsSCalcET6 5.2.AE.05. [708907]

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EXAMPLE 5 Use the Midpoint Rule with $n = 5$ to approximate the following integral.

SOLUTION The endpoints of the subintervals are 1, 1.8, 2.6, 3.4, 4.2, and 5 so the midpoints are ____, 2.2, ____, 3.8, and ____. the width of the subintervals is $\Delta x = (5 - 1)/5 =$ ____, so the Midpoint Rule gives

$$\approx \Delta x[f(1.4) + f(2.2) + f(3) + f(3.8) + f(4.6)]$$

$$= \frac{4}{5}\left(\frac{2}{1.4} + \frac{2}{2.2} + \frac{2}{3} + \frac{2}{3.8} + \frac{2}{4.6}\right)$$

$\approx$ ____

Since $f(x) = 2/x > 0$ for $1 \leq x \leq 5$, the integral represents an area, and the approximation given by the Midpoint Rule is the sum of the areas of the rectangles shown in the figure.

11. Question DetailsSCalcET6 5.2.AE.07. [708895]

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EXAMPLE 7 If the first two integrals are known, find the third integral.

SOLUTION By Property 5, we have

$= 16 -$ ____ $=$ ____

12. Question DetailsSCalcET6 5.2.Tut.04. [700167]

13. Question DetailsSCalcET6 5.2.Tut.07. [700164]

14. Question DetailsSCalcET6 5.2.001.MI. [1386355]
Consider the given function.

Evaluate the Riemann sum for $2 \le x \le 14$, with six subintervals, taking the sample points to be left endpoints. (Give an exact answer.)

L_6 = ______ Tutorial

15. Question DetailsSCalcET6 5.2.017.MI. [1386642]
Express the limit as a definite integral on the given interval.

______

Tutorial

16. Question DetailsSCalcET6 5.2.018. [1290502]
Express the limit as a definite integral on the given interval.

______

17. Question DetailsSCalcET6 5.2.019. [1289909]
Express the limit as a definite integral on the given interval.

______

18. Question DetailsSCalcET6 5.2.022. [657060]
If *f* is integrable on [*a*, *b*], the following equation is correct.

Use the given form of the definition to evaluate the integral.

______

19. Question DetailsSCalcET6 5.2.034.MI. [1386539]
The graph of *g* consists of two straight lines and a semicircle. Use it to evaluate each integral.

a = 12
b = 21
c = 6
d = 12

______ Tutorial

______ Tutorial

______ Tutorial

20. Question DetailsSCalcET6 5.2.036. [1291444]
Evaluate the integral by interpreting it in terms of areas.

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Assignment Details

Name (AID): **Week 12 Tuesday Homework (1329184)**

Submissions Allowed: **5**

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Author: **Jernigan, John (jjernigan@ccp.edu)**

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