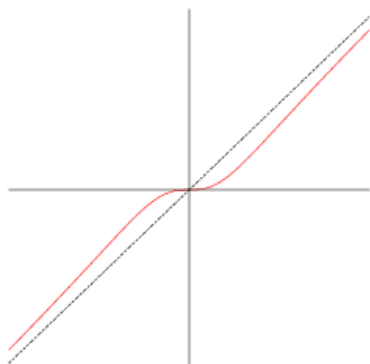


Week 10 Friday Homework (1328515)

Question 12345678910111213141516

1. Question DetailsSCalcET6 4.5.AE.06. [1290372]

[Video Example](#)[Online Textbook](#)**EXAMPLE 6** Sketch the graph of the function below.

$$f(x) = \frac{x^3}{x^2 + 2}$$

(A) The domain is $\mathbb{R} = (-\infty, \infty)$.(B) The x - and y -intercepts are both .(C) Since $f(-x) = -f(x)$, f is and its graph is symmetric about the origin.(D) Since $x^2 + 2$ is never 0, there is no vertical asymptote. Since $f(x) \rightarrow \infty$ as $x \rightarrow \infty$ and $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$, there is no horizontal asymptote. But long division gives

$$f(x) = \frac{x^3}{x^2 + 2} = x - \frac{\text{input}}{x^2 + 2}$$

$$f(x) - x = \frac{\text{input}}{x^2 + 2} = \frac{-2/x}{1 + 2/x^2} \rightarrow \text{input} \text{ as } x \rightarrow \pm\infty$$

So the line $y = \text{input}$ is a slant asymptote.

$$(E) f'(x) = \frac{3x^2(x^2 + 2) - x^3 \cdot 2x}{(x^2 + 2)^2} = \frac{x^2(x^2 + 6)}{(x^2 + 2)^2}$$

Since $f'(x) > 0$ for all x (except 0), f is increasing on $(-\infty, \infty)$.(F) Although $f'(0) = 0$, $f'(x)$ does not change sign at , so there is no local maximum or minimum.

$$(G) f''(x) = \frac{(4x^3 + 12x)(x^2 + 2)^2 - (x^4 + 6x^2) \cdot 2(x^2 + 2)2x}{(x^2 + 2)^4} = \frac{4x(6 - x^2)}{(x^2 + 2)^3}$$

Since $f''(x) = 0$ when $x = 0$ or $x = \pm\sqrt{6}$, we set up the following chart.

Interval	$2x$	$6 - x^2$	$(x^2 + 2)^3$	$f''(x)$	f
$x < -\sqrt{6}$	-	-	+	+	CU on $(-\infty, -\sqrt{6})$
$-\sqrt{6} < x < 0$	-	+	+	-	CD on $(-\sqrt{6}, 0)$
$0 < x < \sqrt{6}$	+	+	+	+	CU on $(0, \sqrt{6})$
$\sqrt{6} < x$	+	-	+	-	CD on $(\sqrt{6}, \infty)$

The points of inflection are $(-\sqrt{6}, \text{input})$, $(0, 0)$, and $(\sqrt{6}, \text{input})$.

(H) The graph is sketched in the figure.

2. Question DetailsSCalcET6 4.5.Tut.01. [708890]

3. Question DetailsSCalcET6 4.5.Tut.03. [708893]

4. Question DetailsSCalcET6 4.6.009.MI. [1387771]

Consider the function below. (Give your answers correct to two decimal places. If you need to use $-\infty$ or ∞ , enter -INFINITY or INFINITY. Round your answers to two decimal places.)

$$f(x) = 1 + \frac{1}{x} + \frac{7}{x^2} + \frac{1}{x^3}$$

(a) Produce graphs of f that reveal all the important aspects of the curve. (Do this on paper. Your instructor may ask you to turn in this graph.)

(b) Use calculus to find the interval of increase.

(,)

(c) Use calculus to find the intervals of decrease. (Enter the interval that contains smaller numbers first.)

(,) \cup (,) \cup (,)

(d) Use calculus to find the intervals where the function is concave up. (Enter the interval that contains smaller numbers first.)

(,) \cup (,)

(e) Use calculus to find the intervals where the function is concave down. (Enter the interval that contains smaller numbers first.)

(,) \cup (,)

[Tutorial](#)

5. Question DetailsSCalcET6 4.6.011. [703747]

Consider the following function. (If you need to use $-\infty$ or ∞ , enter -INFINITY or INFINITY. Round the answers to four decimal places.)

$$f(x) = x^9 \ln(x)$$

$$x > 0$$

(a) Graph the function. (Do this on paper. Your instructor may ask you to turn in this graph.)

(b) Use l'Hospital's Rule to determine the limit as $x \rightarrow 0^+$.

(c) Use calculus to find the minimum value.

(d) Find the interval where the function is concave up.

(,)

(e) Find the interval where the function is concave down.

(,)

6. Question DetailsSCalcET6 4.6.012. [703709]

Consider the following function. (If you need to use $-\infty$ or ∞ , enter -INFINITY or INFINITY. Round the answers to two decimal places.)

$$f(x) = xe^{8/x}$$

(a) Graph the function. (Do this on paper. Your instructor may ask you to turn in this graph.)

(b) Use l'Hospital's Rule to determine the limit as $x \rightarrow 0^+$.

(c) Use calculus to find the local minimum value.

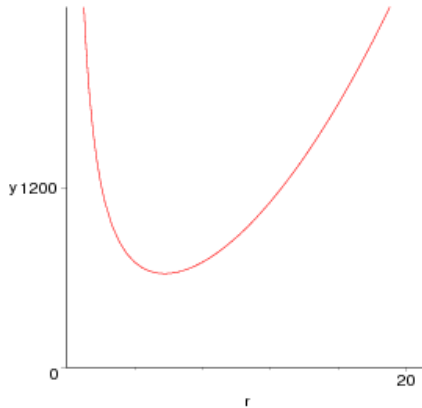
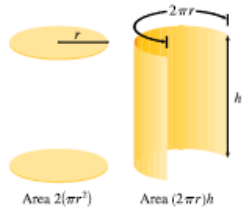
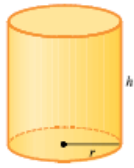
(d) Find the interval where the function is concave up.

(,)

(e) Find the interval where the function is concave down.

(,)

7. Question DetailsSCalcET6 4.7.AE.02. [1291255]



[Video Example](#)

[Online Textbook](#)

EXAMPLE 2 A cylindrical can is to be made to hold 1200 cm^3 of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

SOLUTION Draw the diagram as in the figure, where r is the radius and h the height (both in centimeters). In order to minimize the cost of the metal, we minimize the total surface area of the cylinder (top, bottom, and sides). From the figure, we see that the sides are made from a rectangular sheet with dimensions $2\pi r$ and h . So the surface area is

$$A = 2\pi r^2 + \boxed{}$$

To eliminate h we use the fact that the volume is given as 1200 cm^3 . Thus $\pi r^2 h = 1200$ which gives $h =$

$$\boxed{}$$

. Substitution of this into the expression for A gives

$$\begin{aligned} A &= 2\pi r^2 + 2\pi r \left(\boxed{} \right) \\ &= 2\pi r^2 + \boxed{} \end{aligned}$$

Therefore the function we want to minimize is

$$A(r) = 2\pi r^2 + \frac{2400}{r}, r > 0$$

To find the critical numbers, we differentiate:

$$\begin{aligned} A'(r) &= 4\pi r - \boxed{} \\ &= 4 \left(\boxed{} \right) \\ &\quad \underline{\hspace{2cm}} \\ &\quad r^2 \end{aligned}$$

Then $A'(r) = 0$ when

$\boxed{} = 600$, so the only critical number is $r = \boxed{}$. Since the domain of A is $(0, \infty)$, we can't use the endpoint arguments to determine if the critical point is a maximum

or a minimum. But we can observe that $A'(r) < 0$ for $r < \boxed{}$ and $A'(r) > 0$ for $r >$

$\boxed{}$, so A is decreasing for all r to the left of the critical number and increasing for

all r to the right. Thus $r = \boxed{}$ must give give the absolute minimum.

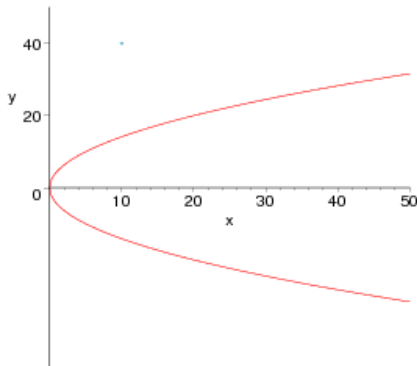
[Alternatively, we could argue that $A(r) \rightarrow \infty$ as $r \rightarrow 0^+$ and $A(r) \rightarrow \infty$ as $r \rightarrow \infty$, so there must be a minimum value of $A(r)$, which must occur at the critical number. See the graph.]

The value of h corresponding to $r = \sqrt[3]{600/\pi}$ is

$$\begin{aligned} h &= \frac{1200}{\pi r^2} = \frac{1200}{\pi \left(\boxed{} \right)^2} \\ &= \boxed{} = 2r \end{aligned}$$

Thus, to minimize the cost of the can, the radius should be $\sqrt[3]{600/\pi}$ cm and the height should be equal to twice the radius, namely, the diameter.

8. Question DetailsSCalcET6 4.7.AE.03. [1290792]



[Video Example](#)

[Online Textbook](#)

EXAMPLE 3 Find the point on the parabola $y^2 = 20x$ that is closest to the point $(10, 40)$.

SOLUTION The distance between the point $(10, 40)$ and the point (x, y) is

$$d = \sqrt{(x - 10)^2 + (y - 40)^2}$$

(See the graph.) But if (x, y) lies on the parabola, then $x = y^2/20$, so the expression for d becomes

$$d = \sqrt{\left(\frac{y^2}{20} - 10\right)^2 + (y - 40)^2}$$

(Alternatively, we could have substituted $y = \sqrt{20x}$ to get d in terms of x alone.) Instead of minimizing d , we minimize its square:

$$d^2 = f(y) = \left(\boxed{} - 10 \right)^2 + (y - 40)^2$$

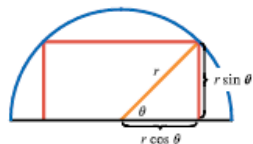
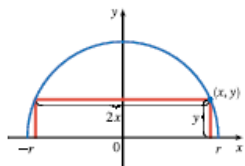
(You should convince yourself that the minimum of d occurs at the same point as the minimum of d^2 , but d^2 is easier to work with.) Differentiating, we obtain

$$f'(y) = 2\left(\frac{y^2}{20} - 10\right)\left(\boxed{}\right) + 2(y - 40)$$

$$= \boxed{}$$

so $f'(y) = 0$ when $y = \boxed{}$. Observe that $f'(y) < 0$ when $y < 20$ and $f'(y) > 0$ when $y > 20$, so by the First Derivative Test for Absolute Extreme Values, the absolute minimum occurs when $y = 20$ (or we could simply say that because of the geometric nature of the problem, it's obvious that there is a closest point but not a furthest point). The corresponding value of x is $x = y^2/20 = \boxed{}$. Thus the point on $y^2 = 20x$ closest to $(10, 40)$ is $(\boxed{}, \boxed{})$.

9. Question DetailsSCalcET6 4.7.AE.05. [1290748]



[Video Example](#)

[Online Textbook](#)

EXAMPLE 5 Find the area of the largest rectangle that can be inscribed in a semicircle of radius r .

SOLUTION 1 Let's take the semicircle to be the upper half of the circle $x^2 + y^2 = r^2$ with center the origin. Then the word inscribed means that the rectangle has two vertices on the semicircle and two vertices on the x -axis as shown in the top figure.

Let (x, y) be the vertex that lies in the first quadrant. Then the rectangle has sides of lengths $2x$ and y , so its area is $A =$

$$\boxed{}$$

To eliminate y we use the fact that (x, y) lies on the circle $x^2 + y^2 = r^2$ and so $y =$

$$\boxed{}. \text{ Thus } A = \boxed{}.$$

The domain of this function is $0 \leq x \leq r$. Its derivative is

$$A' = 2\sqrt{r^2 - x^2} - \frac{2x^2}{\sqrt{r^2 - x^2}} = \frac{2(r^2 - 2x^2)}{\sqrt{r^2 - x^2}}$$

which is 0 when $2x^2 = r^2$, that is, $x =$

$$\boxed{}$$

(since $x \geq 0$). This value of x gives a maximum value of A since $A(0) = 0$ and $A(r) = 0$. Therefore the area of the largest inscribed rectangle is

$$A\left(\frac{r}{\sqrt{2}}\right) = 2 \frac{r}{\sqrt{2}} \boxed{} = \boxed{}$$

SOLUTION 2 A simpler solution is possible if we think of using an angle as a variable. Let θ be the angle shown in the bottom figure. Then the area of the rectangle is

$$A(\theta) = (2r \cos(\theta))(r \sin(\theta)) = r^2(2\sin(\theta)\cos(\theta)) = r^2\sin(2\theta)$$

We know that $\sin(2\theta)$ has a maximum value of 1 and it occurs when $2\theta = \pi/2$. So $A(\theta)$ has a maximum value of r^2 and it occurs when $\theta = \pi/4$. Notice that this trigonometric solution doesn't involve differentiation. In fact, we didn't need to use calculus at all.

10. Question DetailsSCalcET6 4.7.AE.06. [1291765]

[Video Example](#) 

[Online Textbook](#)

EXAMPLE 6 A store has been selling 200 DVD burners a week at \$350 each. A market survey indicates that for each \$10 rebate offered to buyers, the number of units sold will increase by 20 a week. Find the demand function and the revenue function. How large a rebate should the store offer to maximize its revenue?

SOLUTION If x is the number of DVD burners sold per week, then the weekly increase in sales is

. For each increase of 20 units sold, the price is decreased by \$10. So for each additional unit sold, the decrease in price will be $10/20$ and the demand function is

$$p(x) = 350 - \text{[]}(x - 200) = 450 - \text{[]}$$

The revenue function is

$$R(x) = xp(x) = \text{[]}$$

Since $R'(x) =$

, we see that $R'(x) = 0$ when $x = \text{[]}$. This value of x gives an absolute maximum by the First Derivative Test (or simply by observing that the graph of R is a parabola that opens downward). The corresponding price is

$$p(450) = 450 - 450/2 = \text{[]}$$

and the rebate is $350 - 225 = \text{[]}$. Therefore, to maximize revenue, the store should offer a rebate of \$125.

11. Question DetailsSCalcET6 4.7.Tut.03. [708881]

12. Question DetailsSCalcET6 4.7.005.MI. [1387248]

Find the dimensions of a rectangle with perimeter 60 m whose area is as large as possible.

m (smaller value)

m (larger value) [Tutorial](#)

13. Question DetailsSCalcET6 4.7.010. [1291745]

Consider the following problem: A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have. Let x denote the length of the side of the square being cut out. Let y denote the length of the base.

(a) Draw several diagrams to illustrate the situation, some short boxes with large bases and some tall boxes with small bases. Find the volumes of several such boxes. (Do this on paper. Your teacher may ask you to turn in this work.)

(b) Draw a diagram illustrating the general situation. Introduce notation and label the diagram with your symbols. (Do this on paper. Your teacher may ask you to turn in this work.)

(c) Write an expression for the volume V .

$$V = \boxed{}$$

(d) Use the given information to write an equation that relates the variables. (Do this on paper. Your teacher may ask you to turn in this work.)

(e) Use part (d) to write the volume as a function of x .

$$V(x) = \boxed{}$$

(f) Finish solving the problem by finding the largest volume that such a box can have.

$$V = \boxed{} \text{ ft}^3$$

14. Question DetailsSCalcET6 4.7.011. [703810]

A farmer wants to fence an area of 24 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence? (Give the dimensions in increasing order.)

feet

feet

15. Question DetailsSCalcET6 4.7.014. [656911]

A rectangular storage container with an open top is to have a volume of 10 m^3 . The length of this base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.

\$

16. Question DetailsSCalcET6 4.7.017. [703949]

Find the point on the line $y = 4x + 6$ that is closest to the origin. (Give your answers correct to three decimal places.)

(,)

Assignment Details

Name (AD): **Week 10 Friday Homework (1328515)**

Submissions Allowed: **5**

Category: **Homework**

Code:

Locked: **No**

Author: **Jernigan, John** (jjernigan@ccp.edu)

Last Saved: **Jul 30, 2010 11:19 AM EDT**

Permission: **Protected**

Randomization: **Person**

Which graded: **Last**

Feedback Settings

Before due date

Question Score

Assignment Score

Publish Essay Scores

Question Part Score

Mark

Add Practice Button

Help/Hints

Response

Save Work

After due date

Question Score

Assignment Score

Publish Essay Scores

Key

Question Part Score

Solution

Mark

Add Practice Button

Help/Hints

Response