Calculus Final Exam/Homework.
Please complete 8 of the following 10 problems. You may do any 8 that you choose, but you must answer all parts of each question. Put at most one question on each page.

1. Find the following derivatives:
   (a) $\frac{d}{dx}\left[5x\sqrt{1-3x}\right]$
   (b) $\frac{d}{dx}\left[\frac{x}{x-6}\right]$
   (c) $\frac{d}{dx}\left[\left(\frac{x}{x-6}\right)^2\right]$
   (d) $\frac{d}{dx}\left[x^3-\frac{1}{x}\right]$
   (e) $\frac{d}{dx}\left[\left(x^3-\frac{1}{x}\right)^3\right]$

2. Show that the "rule for squares" $(g(x)^2)' = 2g(x)g'(x)$, the "rule for reciprocals" $\left(\frac{1}{g(x)}\right)' = \frac{-g'(x)}{g^2(x)}$ and the "rule for square roots" $\left(\sqrt{g(x)}\right)' = \frac{g'(x)}{2\sqrt{g(x)}}$ are simple consequences of the chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
   Hint for first one: let $u = g(x), y = u^2$

3. Consider the function $f(x) = 5x\sqrt{1-3x}$
   (a) Find $f'(x)$
   (b) Find $f''(x)$
   (c) Using the above, describe the local behaviour of f at the point $(0,0)$. That is, is it increasing or decreasing, leaning left or right?

4. Consider the curve $C = 7L + \frac{48}{L}$ for $0 < L$
   (a) Find $\frac{dC}{dL}$
   (b) Find $\frac{d^2C}{dL^2}$ (This is the unfortunate notation for the second derivative.)
   (c) Find the coordinate for the point where the tangent line is horizontal. I.e. set the first derivative equal 0 and solve.
   (d) Is the curve leaning left or right at that point?

(e) From your answer above, is the point you found the highest or lowest point on the curve?

5. Find the point on the graph of $y^2 = 4x$ nearest the point $(2, 1)$ using the following steps:

   (a) What is the square of the distance between any point (x, y) and $(2, 1)$. Use the distance formula without the square root sign.

   (b) Replace x by $\dfrac{y^2}{4}$ in your answer above.

   (c) Multiply out carefully, remembering that $(a - b)^2 = a^2 - 2ab + b^2$

   (d) Now you should have an expression in one variable, y. Take the derivative.

   (e) Set the derivative equal 0 and solve.

   (f) Answer the question.

6. Let $f(x) = 3x^3 - 12x$

   (a) Find $f'(x)$

   (b) Find $f''(x)$

   (c) Carefully factor your answer in part a and make a quick sketch of the parabola.

   (d) Using your answer above, over what intervals is f increasing, and over what interval is f decreasing?

   (e) Make a quick sketch of the second derivative.

   (f) Using the second derivative, over what interval is f leaning left, and over what interval is f leaning right?

   (g) Using all the information together, draw the graph of $y = x^3 - 12x$

7. A rock is thrown upward from an initial height of 160 feet and its height in feet after t seconds is given by $h = -16t^2 + 256t + 160$.

   (a) What is the average speed for the first two seconds of flight?

   (b) What is the instantaneous speed when $t = 2$?

   (c) After 10 seconds, is the rock going up or down?

   (d) What is the maximum height attained by the rock?

   (e) What is the rocks initial speed?

   (f) When does it hit the ground?

8. A man 6 feet tall walks directly away from the base of a 10 foot street light at a rate of 4 feet per second. How fast does the length of his shadow increase?

9. Evaluate the following integrals using the Fundamental Theorem of Calculus:

   (a) $\int_1^4 2x\,dx$

   (b) $\int_1^4 1 - x^2\,dx$

   (c) $\int_1^4 x^3\,dx$

   (d) $\int_1^4 1 - x^2 + x^3\,dx$

   (e) $\int_0^1 \frac{-x}{\sqrt{1-x^2}}\,dx$ Hint: We have seen $\frac{-x}{\sqrt{1-x^2}}$ many times before.

10. Compute $\int_2^6 x\,dx$ directly from the definition

$$\int_a^b f(x)\,dx = \lim_{\Delta x \to\ 0} \sum_{k=1}^{n} f(x_k)\Delta x$$

This is not as hard as it seems, especially since there is a worked out example in the book: Example 9 page 257.