Exercises on generating Pythagorean triples.

Recall that all Pythagorean triples (integers *a, b, c* such that $a^2 + b^2 = c^2$) are generated by integers *m, n* with $a = m^2 - n^2, b = 2mn, c = m^2 + n^2$

Ex. 1. A primitive Pythagorean triple is one where the numbers have no common factors. For example, (3,4,5) is primitive, while (6,8,10) is not. To generate a primitive Pythagorean triple, *m* and *n* should be of *opposite parity,* i.e. one is even and the other is odd. In addition, they should be *relatively prime*, that is, they should have no common factors. Find the Pythagorean triple with *m* = 6 and *n* = 3. We can predict that it is not primitive. What is the common factor for the resulting triple? Generalize.

Ex. 2. Verify that $(m-n)(m+n) = m^2 - n^2$ by multiplying.

Ex. 3. Suppose we have two equations such as the following:

$m - n = 1$

$m + n = 7$

Add the two equations together. What must *m* be? Now find *n*.

Ex. 4. Find the Pythagorean triple that has 7 as one side.

Ex. 5. Find a Pythagorean triple that has 11 as one side.

Ex. 6. Factor the number 15 two different ways. Find two different Pythagorean triples with one number (side) 15.

Ex. 7. Write down a procedure for finding a Pythagorean triple that includes a specific odd integer as one of the sides.

Ex. 8. Find a Pythagorean triple that has one side 10. Is it Primitive? What conditions must an even number have for it to be a leg of a Pythagorean triple?

Ex. 9 (A hard one.) Notice that for all primitive Pythagorean triples, one leg is even and one is odd. Why is that? Also notice that one of the numbers (not necessarily the legs) is always divisible by 5. Why?