

171 test 3 Solutions

1. On which intervals is the function $f(x) = 2x^3 - 3x^2 - 36x$ increasing? Where is it decreasing? Here is a picture from maple to help, but you must show your work.

$$f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x-3)(x+2)$$

Since the derivative is a quadratic with positive leading coefficient, it is facing up. The zeros are -2 and 3 , so it is positive outside the zeros and negative between them.

Answer: Increasing over $(-\infty, -2), (3, \infty)$, decreasing over $(-2, 3)$

2. Over what interval(s) is the above function concave up (leaning to the left) and concave down?

$$f''(x) = 12x - 6, \text{ a line, positive for } x > \frac{1}{2}, \text{ negative for } x < \frac{1}{2}.$$

Answer: concave up over $(\frac{1}{2}, \infty)$, concave down over $(-\infty, \frac{1}{2})$.

3. Locate the maximum and minimum values of the function $f(x) = 4\sqrt{x} - x^2$ on the interval $[0, 2]$. Don't forget to check the endpoints as well as the critical points.

$$f'(x) = \frac{2}{\sqrt{x}} - 2x \quad \text{Set } \frac{2}{\sqrt{x}} - 2x = 0 \Rightarrow \frac{1}{\sqrt{x}} = x \Rightarrow 1 = x^{\frac{3}{2}} \Rightarrow x = 1$$

Critical point are $x = 0$ where the derivative is undefined, and $x = 1$.

$$f(0) = 0, \quad f(1) = 3, \quad f(2) = 4\sqrt{2} - 4$$

Answer: Minimum is 0, maximum is 3.

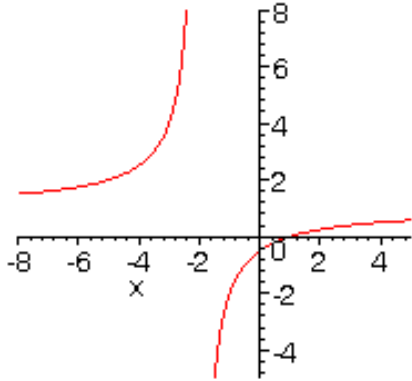
$$\text{Let } f(x) = \frac{x-1}{x+2}$$

4. f has a vertical asymptote at $x = -2$ and a horizontal asymptote at $y = 1$.

5. Find $f'(x) = \frac{(x+2)-(x-1)}{(x+2)^2} = \frac{3}{(x+2)^2}$ and therefore always positive, hence f is

increasing.

6. Using the information from 4 and 5 above, draw a rough sketch of f .



7. Approximate the root of $f(x) = x^3 - 2x - 2$ by using Newton's method. Start with the first guess of 2 and fill in the table.

x	y	y'
2	2	10
1.8		ignore

$$f(2) = 2^3 - 2 \times 2 - 2 = 2, \quad f'(2) = 3 \times 2^2 - 2 = 10 \Rightarrow x_2 = 2 - \frac{2}{10} = 1.8$$

8. What does the mean value theorem say about the function $f(x) = x^3 - 2x - 2$ on the interval $[0, 2]$?

There is a number c between 0 and 2 with

$$f'(c) = 3c^2 - 2 = \frac{f(2) - f(0)}{2 - 0} = \frac{2 - (-2)}{2} = 2$$

9. For question above, find the number in the interval $(0, 2)$ guaranteed by the mean value theorem to exist.

$$3c^2 - 2 = 2 \Rightarrow 3c^2 = 4 \Rightarrow c = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$$

Use L'Hôpital's rule if applicable to find the following limits:

$$10. \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2}$$

$$\frac{\sin^2 0}{0^2} = \frac{0}{0} \text{ so L'Hôpital's rule is applicable.}$$

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} = \lim_{x \rightarrow 0} \frac{2 \sin x \cos x}{2x} = \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} = \lim_{x \rightarrow 0} \frac{2 \cos x}{2} = \frac{2 \cos 0}{2} = 1$$

or

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} = \lim_{x \rightarrow 0} \frac{2 \sin x \cos x}{2x} = \lim_{x \rightarrow 0} \frac{\sin x \cos x}{x} = \lim_{x \rightarrow 0} \frac{\cos x \cos x - \sin x \sin x}{1} = \cos^2 0 - \sin^2 0 = 1$$

$$11. \lim_{x \rightarrow 3} \frac{\sqrt{x} - 9}{x - 3}$$

$$\frac{\sqrt{3} - 9}{3 - 3} = \frac{\sqrt{3} - 9}{0} \neq \frac{0}{0} \text{ so L'Hôpital's rule is not applicable. The limit does not exist.}$$

$$12. \lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x}$$

$$\frac{e^{3 \times 0} - 1}{0} = \frac{1 - 1}{0} = \frac{0}{0} \text{ so L'Hôpital's rule is applicable.}$$

$$\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x} = \lim_{x \rightarrow 0} \frac{3e^{3x}}{1} = 3e^{3 \times 0} = 3e^0 = 3$$

13. Definition: If f is continuous on $[a, b]$, the **definite integral of f from a to b is**

$$\int_a^b f(x) dx =$$

Look it up.

14. Define each symbol on the right hand side of the equal sign above.

$$15. \text{ Suppose } \int_a^b f(x) dx = 4, \int_a^c f(x) dx = -3. \text{ What is } \int_c^b f(x) dx ?$$

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \Rightarrow \int_c^b f(x) dx = 7$$

$$16. \text{ For the function above, what is } \int_b^a f(x) dx ?$$

$$\int_b^a f(x) dx = -\int_a^b f(x) dx \Rightarrow \int_b^a f(x) dx = -4$$

17. Let $F(x) = \int_a^x f(t)dt$. F is a function of what variable? x

Which is why it is written F of x !!

18. For the function defined above, what is $F'(x)$?

$$f(x)$$

19. More specifically, if $F(x) = \int_1^x \sqrt{t^2 - 1}dt$ then $F'(x) =$

$$f(x) = \sqrt{x^2 - 1}$$

20. Evaluate $\int_1^4 \frac{1}{t^2} dt$

$$f(t) = \frac{1}{t^2} = t^{-2} \Rightarrow F(t) = \frac{t^{-1}}{-1} = -\frac{1}{t}$$

$$F(4) - F(1) = -\frac{1}{4} - \left(-\frac{1}{1}\right) = 1 - \frac{1}{4} = \frac{3}{4}$$

21. What are $\int_1^4 \frac{1}{z^2} dz$ and $\int_1^4 \frac{1}{x^2} dx$? Answer: $\frac{3}{4}$ and $\frac{3}{4}$

22. Let $F(x) = xe^x - e^x$ show that $F'(x) = xe^x$ (Don't forget the product rule!)

23. Evaluate $\int_0^1 xe^x dx$

$$\int_0^1 xe^x dx = F(1) - F(0) = 1e^1 - e^1 - (0e^0 - e^0) = e - e + 1 = 1$$

Note that this works since the derivative of F is xe^x