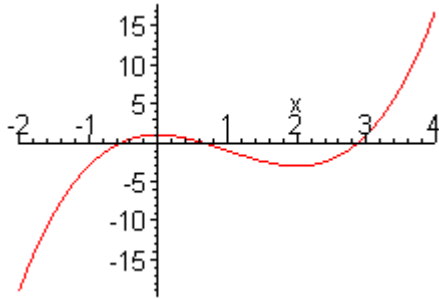


171 Sample Test 3

1. On which intervals is the function  $f(x) = x^3 - 3x^2 + 1$  increasing? Where is it decreasing? Here is a picture from maple to help.



2. Where is the above function concave up (leaning to the left) and concave down?
3. Locate the relative extrema of the function  $f(x) = 3\sqrt[3]{x^5} - 15\sqrt[3]{x^2}$  and determine whether they are maxima or minima.
4. Let  $C = 7L + \frac{48}{L}$  with domain  $0 < L$ . Find the least possible value of  $C$ . Make sure to explain why the value you chose was the least, not perhaps the greatest.
5. (Do you understand Newton's method?) Here is a simple test. Fill in the question mark in each table with the number Newton would give for his guess at the right answer.

x	y	$y'$
1	1	-27
?	0	

x	y	$y'$
-2	-1	10
?	0	

6. Find a good approximation for the positive root of  $x^2 - x - 4$  using Newton's method.
7. What does the mean value theorem say about the function  $f(x) = x^2 - x - 4$  on the interval  $[0, 4]$ ?
8. For question above, find the number in the interval  $[0, 4]$  guaranteed by the mean value theorem to exist.
9. Extra credit. Show that it is no accident that it lies at the midpoint of the interval.
10. If  $f$  and  $g$  are function for which  $f'(x) = g(x)$  and  $g'(x) = f(x)$  for all  $x$ , show that  $f^2(x) - g^2(x)$  is a constant. (Hint: take the derivative using the chain rule, show that it is zero, and then quote a theorem.)
11. What does the mean value theorem say about the function  $f(x) = |x - 1|$  on the interval  $[-2, 2]$ ?
12. Use L'Hopital's rule if applicable to find the following limits:

a)  $\lim_{x \rightarrow 1} \frac{\ln x}{x - 1}$

b)  $\lim_{x \rightarrow 0} \frac{\sin(ax)}{x}$

c)  $\lim_{x \rightarrow 0^+} x^x$

d)  $\lim_{x \rightarrow 0} \frac{\cos x}{x}$

13. Definition: If  $f$  is continuous on  $[a, b]$ , the **definite integral of  $f$  from  $a$  to  $b$**  is

$$\int_a^b f(x) dx =$$

14. Define each symbol on the right hand side of the equal sign above.

15. Given that  $\sum_{k=1}^n k^3 = \left[ \frac{n(n+1)}{2} \right]^2$  find  $\int_0^2 x^3 dx$

16. Suppose  $\int_3^7 f(x) dx = 12$ ,  $\int_3^{10} f(x) dx = 20$ . What is  $\int_7^{10} f(x) dx$ ?

17. For the function above, what is  $\int_{10}^7 f(x) dx$

18. What is the biggest  $\int_0^\pi x \sin x dx$  can possibly be?

19. At this point, why is the fundamental theorem of calculus useless to you for evaluating the above integral?

20. What is the derivative of  $\int_a^x f(t) dt$ ?

21. What is the derivative of  $\int_1^x \frac{1}{2t} dt$

22. Find another expression for all such functions whose derivative is the same as above.

23. Evaluate  $\int_1^9 \frac{1}{2t} dt$

24. Evaluate  $\int_1^2 \frac{x^2 + 1}{\sqrt{x}} dx$

25. Show that the derivative of  $-x \cos x + \sin x$  is  $x \sin x$ .

26. Evaluate the integral in problem 18.