1. On which intervals is the function \( f(x) = x^3 - 3x^2 + 1 \) increasing? Where is it decreasing? Here is a picture from maple to help.

\[ f(x) = x^3 - 3x^2 + 1 \]

2. Where is the above function concave up (-leaning to the left) and concave down?

3. Locate the relative extrema of the function \( f(x) = 3\sqrt{x^3} - 15\sqrt{x^2} \) and determine whether they are maxima or minima.

4. Let \( C = 7L + \frac{48}{L} \) with domain \( 0 < L \). Find the least possible value of \( C \). Make sure to explain why the value you chose was the least, not perhaps the greatest.

5. (Do you understand Newton’s method?) Here is a simple test. Fill in the question mark in each table with the number Newton would give for his guess at the right answer.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( y' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>-27</td>
</tr>
<tr>
<td>?</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( y' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-1</td>
<td>10</td>
</tr>
<tr>
<td>?</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

6. Find a good approximation for the positive root of \( x^2 - x - 4 \) using Newton’s method.

7. What does the mean value theorem say about the function \( f(x) = x^2 - x - 4 \) on the interval \([0, 4]\)?

8. For question above, find the number in the interval \([0, 4]\) guaranteed by the mean value theorem to exist.

9. Extra credit. Show that it is no accident that it lies at the midpoint of the interval.

10. If \( f \) and \( g \) are function for which \( f'(x) = g(x) \) and \( g'(x) = f(x) \) for all \( x \), show that \( f^2(x) - g^2(x) \) is a constant. (Hint: take the derivative using the chain rule, show that it is zero, and then quote a theorem.)

11. What does the mean value theorem say about the function \( f(x) = |x - 1| \) on the interval \([-2, 2]\)?

12. Use L’Hopital’s rule if applicable to find the following limits:
a) \[ \lim_{x \to 1} \frac{\ln x}{x - 1} \]

b) \[ \lim_{x \to 0} \frac{\sin(ax)}{x} \]

c) \[ \lim_{x \to 0} x^x \]

d) \[ \lim_{x \to 0} \frac{\cos x}{x} \]

13. Definition: If \( f \) is continuous on \([a, b]\), the **definite integral of** \( f \) **from** \( a \) **to** \( b \) **is**

\[ \int_{a}^{b} f(x) \, dx = \]

14. Define each symbol on the right hand side of the equal sign above.

15. Given that

\[ \sum_{k=1}^{n} k^3 = \left[ \frac{n(n+1)}{2} \right]^2 \]

find \[ \int_{0}^{2} x^3 \, dx \]

16. Suppose \( \int_{3}^{7} f(x) \, dx = 12, \int_{3}^{10} f(x) \, dx = 20 \). What is \( \int_{7}^{10} f(x) \, dx \)?

17. For the function above, what is \( \int_{7}^{10} f(x) \, dx \)?

18. What is the biggest \( \int_{0}^{\pi} x \sin x \, dx \) can possibly be?

19. At this point, why is the fundamental theorem of calculus useless to you for evaluating the above integral?

20. What is the derivative of \( \int_{a}^{x} f(t) \, dt \)?

21. What is the derivative of \( \int_{1}^{x} \frac{1}{2t} \, dt \)?

22. Find another expression for all such functions whose derivative is the same as above.

23. Evaluate \( \int_{1}^{2} \frac{1}{2t} \, dt \)

24. Evaluate \( \int_{1}^{2} \frac{x^2 + 1}{\sqrt{x}} \, dx \)

25. Show that the derivative of \(-x\cos x + \sin x\) is \( x\sin x \).

26. Evaluate the integral in problem 18.