A ball thrown in the air with initial velocity 64 feet per second has height given by the equation: \( h = -16t^2 + 64t \) where \( t \) is the time in seconds after it is released.

1. How high is the ball after 1 second?
2. How high is the ball after 2 seconds?
3. What is the average velocity of the ball between one and two seconds?
4. Is the ball going up or down in that time?
5. What is the instantaneous velocity at 2 seconds?

6. State, as precisely as you can either in clear English or using mathematics, what it means to say \( \lim_{x \to a} f(x) = L \)

7. Fill in the blank: a function \( f \) is continuous at a point \( a \) if ________________

8. Give an example of a function continuous at 3.

Let the function \( f \) be given by the formula \( f(x) = \frac{x^2 - 25}{x^2 - 5x} \)

9. What kind of function is this?
10. What is the domain of this function?
11. Where is the function continuous?

For the function above, find, if they exist,

12. \( \lim_{x \to 2} f(x) \)
13. \( \lim_{x \to 5} f(x) \)
14. \( \lim_{x \to 0} f(x) \)

Suppose \( \lim_{x \to a} f(x) = 6 \) and \( \lim_{x \to a} g(x) = -1 \)

15. Find \( \lim_{x \to a} \frac{f^2 + g}{g^2 + f} (x) \)
16. Find \( \lim_{x \to -2} \frac{x^3 + 8}{x + 2} \).

The function \( f(x) = \frac{x^3 + 8}{x + 2} \) is not continuous at \(-2\).

17. What kind of discontinuity is this?
18. How would you define the function above so that it would be continuous at \(-2\)?
19. The greatest integer function is not continuous at 3. State precisely what this means and why.

Below is a picture from maple of the function defined piecewise as

\[
 f(x) = \begin{cases} 
 x^2 & x < 2 \\
 x + 3 & x \geq 2 
\end{cases}
\]
20. Find \( \lim_{x \to 2^-} f(x) \) and \( \lim_{x \to 2^+} f(x) \).

21. State a precisely as you can why \( \lim_{x \to 2} f(x) \) does not exist.

Here is another nice picture, this time of \( \sin\left(\frac{1}{x}\right) \)

Clearly there is no limit as \( x \) approaches 0, although if asked maple will return the answer \(-1\ 1\).

22. Why is this incorrect?

23. What epsilon could you pick to show that this limit does not exist?

24. Suppose the numbers 2.40 and 3.30 are known to be in the range of a certain function (the price you pay for a cab ride, for example). Must the range necessarily contain all numbers between 2.40 and 3.30?

25. What condition on the function in problem 24 would assure you that the range contains all numbers between 2.40 and 3.30?
26. Definition: The **tangent line** to the curve $y = f(x)$ at the point $(a, f(a))$ is the line with slope $m: = _____________________$

27. Definition: the **instantaneous rate of change** of the function $f(x)$ at $(a, f(a))$ is

28. Definition: The **derivative of a function $f$ at a number $a$**, denoted by $f'(a)$, is _____________________

29. Find the formula for $f'(a)$ if $f(x) = 2x^2 - 5x$ (Answer: $4a - 5$)

30. Use the formula in problem 29 to find the equation for the tangent line at (2, -2)

31. Use the formula in problem 29 to guess the vertex of the parabola given by the equation $y = 2x^2 - 5x$

Another nice picture, this time of a polynomial. Guess the degree. (This can only be a guess, but there are some possibilities and some impossibilities.)

32. If this is the graph of $y = g(x)$ arrange in order from smallest to greatest the following:
   $g'(-1), g'(-.5), g'(0), g'(1.5), g'(1), g'(2), g'(2.5)$