171 Final Exam. Name_____

1. Definition: The **derivative of a function *f* at a number** *a*, denoted by $f'(a)$, is

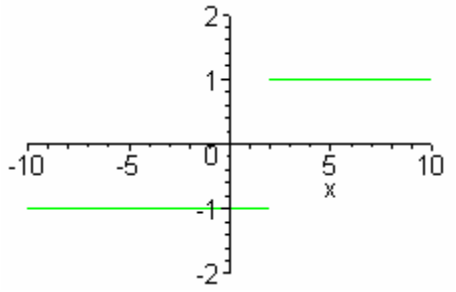
2. Use the ***definition*** above (not the power rule) to find the derivative of the function $f(x) = x^2 - x$

3. Find $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$ by any method you choose.

4. State, as precisely as you can either in clear English or using mathematics, what it means to say $\lim_{x \to a} f(x) = L$

5. Definition: a function is **continuous** at a point *a* in its domain if

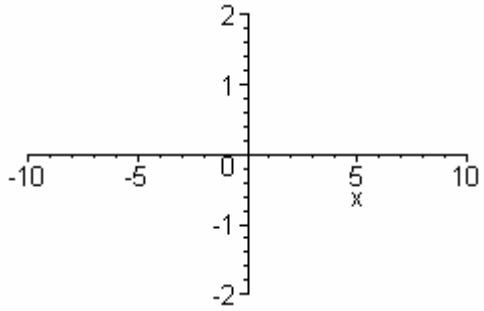
Here is a picture of the graph of a function. Let's call it the graph of $y = f(x)$




6. For which value of *x* is this function discontinuous? _____

7. What kind of discontinuity does this function have? _____

8. Assuming that this function above is the derivative of another function *F*, i.e. $F'(x) = f(x)$, and that $F(2) = 0$ draw a picture of *F*.




Find the following derivatives. Simplify if possible, but don't do anything silly.

9. $\frac{d}{dx}\left[\sqrt{x^2 - 2x}\right]$

10. $\frac{d}{dx}\left[\frac{x+1}{x-2}\right]$

11. $\frac{d}{dx}\left[e^{\cos x}\right]$

12. $\frac{d}{dx}\left[x \sin x\right]$

13. $\frac{d}{dx}\left[x^{x}\right]$

14. What is the derivative of $f(x) = 2^{x}$?

15. What is the derivative of $\log_2 x$?

16. Find the slope of the line tangent to the curve $x^2 + y^2 = 25$ at the point (3,4).

17. On which intervals is the function $f(x) = 2x^3 - 3x^2$ increasing? Where is it decreasing? (And this time, no picture.)

18. Where is the above function concave up and concave down?

19. Locate the maximum and minimum values of the function $f(x) = x - \ln x$ on the interval $[\frac{1}{2}, 2]$. Don't forget to check the endpoints as well as the critical points.

20. What does the mean value theorem say about the function $f(x) = x^2 + 2x - 3$ on the interval [1 , 3]?

21. For question above, find the number in the interval (1 , 3) guaranteed by the mean value theorem to exist.

Suppose $f(2) = 4, f'(2) = -1$ $f''(2) = 3$

22. Let $y = \sqrt{f(x)}$. Use the rule for square roots to find $y'$ and $y''$ at $x = 2$

23. Describe the behavior of $\sqrt{f}$ at $(2,2)$ (Increasing or decreasing, leaning left or right)

24. Find the equation for the line tangent to the graph of $y = \sqrt{f}$ at $(2,2)$

25. Does the line lie above the graph, or below?

Use L'Hopital's rule if applicable to find the following limits:

26. $\lim_{x \to 0} \frac{x^2}{e^x - 1}$

27. $\lim_{x \to 1} \frac{\ln x}{x - 1}$

28. $\lim_{x \to 0} \frac{\sin 3x}{x}$

29. Definition: If *f* is continuous on [*a* , *b*], the **definite integral of *f* from *a* to *b* is**

$\int_a^b f(x)dx =$

30. Let $F(x) = \int_0^x \sin t dt$ . $F$ is a function of what variable? _____

31. For the function defined above, what is $F'(x)$? _____

32. For the function defined above, what is $F(0)$? _____

33. Evaluate $\int_1^2 \frac{1}{t^4} dt$

34. Find the general antiderivative of $f(x) = 2\sqrt{x} + 2x + \frac{1}{\sqrt{1-x^2}}$

35. Evaluate $\int_0^1 \left( 2\sqrt{x} + 2x + \frac{1}{\sqrt{1-x^2}} \right) dx$

36. Evaluate $\int_0^{\frac{\pi}{2}} (\sin x + x\cos x) dx$ Hint: look at your answer to problem 12.