1. Definition: The **derivative of a function** $f$ at a number $a$, denoted by $f'(a)$, is

2. Use the **definition** above (not the power rule) to find the derivative of the function $f(x) = x^2 - x$

3. Find $\lim_{x\to 3} \frac{x^2 - 9}{x - 3}$ by any method you choose.

4. State, as precisely as you can either in clear English or using mathematics, what it means to say $\lim_{x\to a} f(x) = L$

5. Definition: a function is **continuous** at a point $a$ in its domain if

Here is a picture of the graph of a function. Let’s call it the graph of $y = f(x)$

6. For which value of $x$ is this function discontinuous? ___________
7. What kind of discontinuity does this function have?  ______________

8. Assuming that this function above is the derivative of another function $F$, i.e. $F'(x) = f(x)$, and that $F'(2) = 0$ draw a picture of $F$.

Find the following derivatives. Simplify if possible, but don’t do anything silly.

9. $\frac{d}{dx} \left[ \sqrt{x^2 - 2x} \right]$

10. $\frac{d}{dx} \left[ \frac{x+1}{x-2} \right]$

11. $\frac{d}{dx} \left[ e^{\cos x} \right]$
12. \( \frac{d}{dx} [x \sin x] \)

13. \( \frac{d}{dx} [x^{-x}] \)

14. What is the derivative of \( f(x) = 2^x \)?

15. What is the derivative of \( \log_2 x \)?

16. Find the slope of the line tangent to the curve \( x^2 + y^2 = 25 \) at the point (3,4).

17. On which intervals is the function \( f(x) = 2x^3 - 3x^2 \) increasing? Where is it decreasing? (And this time, no picture.)

18. Where is the above function concave up and concave down?
19. Locate the maximum and minimum values of the function \( f(x) = x - \ln x \) on the interval \([\frac{1}{2}, 2]\). Don’t forget to check the endpoints as well as the critical points.

20. What does the mean value theorem say about the function on the interval \([1, 3]\)?

21. For question above, find the number in the interval \((1, 3)\) guaranteed by the mean value theorem to exist.

Suppose \( f(2) = 4, f'(2) = -1, f''(2) = 3 \)

22. Let \( y = \sqrt{f(x)} \). Use the rule for square roots to find \( y' \) and \( y'' \) at \( x = 2 \)

23. Describe the behavior of \( \sqrt{f} \) at \((2, 2)\) (Increasing or decreasing, leaning left or right)
24. Find the equation for the line tangent to the graph of $y = \sqrt{f}$ at $(2,2)$

25. Does the line lie above the graph, or below?

Use L’Hopital’s rule if applicable to find the following limits:

26. $\lim_{x \to 0} \frac{x^2}{e^x - 1}$

27. $\lim_{x \to 1} \frac{\ln x}{x - 1}$

28. $\lim_{x \to 0} \frac{\sin 3x}{x}$

29. Definition: If $f$ is continuous on $[a , b]$, the **definite integral of $f$ from $a$ to $b$** is
   \[ \int_{a}^{b} f(x) \, dx = \]
30. Let \( F(x) = \int_{0}^{x} \sin t \, dt \). \( F \) is a function of what variable? 

31. For the function defined above, what is \( F'(x) \)?

32. For the function defined above, what is \( F(0) \)?

33. Evaluate \( \int_{1}^{2} \frac{1}{t^4} \, dt \)

34. Find the general antiderivative of \( f(x) = 2\sqrt{x} + 2x + \frac{1}{\sqrt{1-x^2}} \)

35. Evaluate \( \int_{0}^{1} \left( 2\sqrt{x} + 2x + \frac{1}{\sqrt{1-x^2}} \right) \, dx \)

36. Evaluate \( \int_{0}^{\frac{\pi}{2}} (\sin x + x \cos x) \, dx \) Hint: look at your answer to problem 12.