

1. Definition: The **derivative of a function f at x** , denoted by $f'(x)$, is

2. For example, $\lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} =$ _____

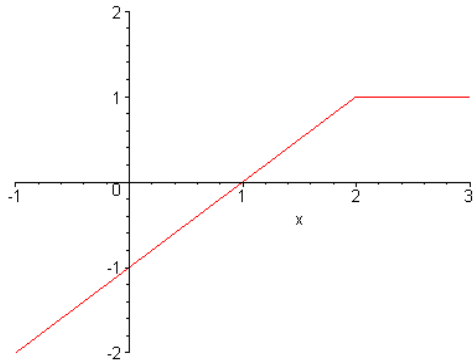
3. Find $\lim_{x \rightarrow 2} \frac{\ln x - \ln 2}{x - 2}$

4. Let $f(x) = \ln x$. $f'(2) =$ _____

5. State, as precisely as you can either in clear English or using mathematics, what it means to say $\lim_{x \rightarrow a} f(x) = L$

6. Definition: a function is **continuous** at a point a in its domain if

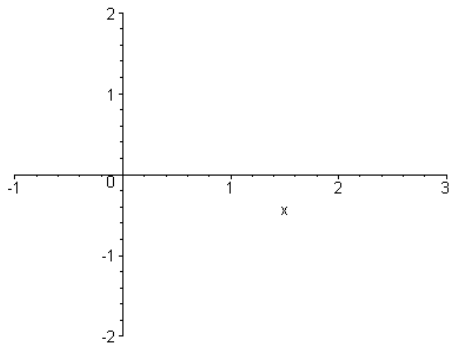
Here is a picture of the graph of a function. Let's call it the graph of $y = f(x)$



7. Is f continuous at $x = 2$?

8. Is f differentiable at $x = 2$?

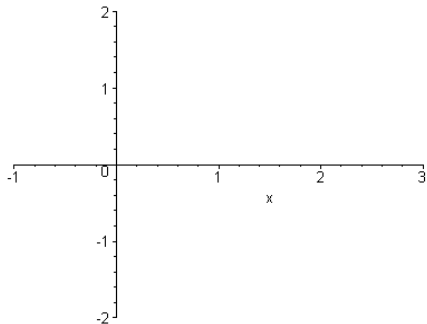
9. Draw the graph of $y = f'(x)$



10. Assuming that this function above is the derivative of another function F , i.e. $F'(x) = f(x)$, Then F has a critical point at $x = \underline{\hspace{2cm}}$.

11. F is decreasing over the interval $\underline{\hspace{2cm}}$ and increasing over the interval $\underline{\hspace{2cm}}$

12. If $F(0) = 0$ draw a picture of F .



Find the following derivatives.

$$13. \frac{d}{dx}[\sqrt{x}]$$

$$14. \frac{d}{dx}[\sqrt{\sin x}]$$

$$15. \frac{d}{dx}[e^{\sqrt{\sin x}}]$$

$$16. \frac{d}{dx}\left[\frac{x-2}{x+1}\right]$$

$$17. \frac{d}{dx}[xe^{-x}]$$

$$18. \frac{d}{dx}[x^{\sin x}]$$

19. $\frac{d}{dx}\left[\frac{1}{2^x}\right]$ Hint: this is the same as $\frac{d}{dx}\left[\left(\frac{1}{2}\right)^x\right]$

20. Let $f(x) = \sqrt{x-1}$. $f(5) = \underline{\hspace{2cm}}$, $f'(x) = \underline{\hspace{2cm}}$, $f'(5) = \underline{\hspace{2cm}}$

21. Write the equation for the line tangent to the graph of $y = \sqrt{x-1}$ at $x = 5$.

22. $f(x) = x^3 - 3x^2$ is increasing over the intervals $\underline{\hspace{4cm}}$ and decreasing over the interval $\underline{\hspace{4cm}}$

23. The above function concave up over $\underline{\hspace{4cm}}$ and concave down over $\underline{\hspace{4cm}}$

24. Let $f(x) = xe^{-x}$. By a problem above, $f'(x) = \underline{\hspace{4cm}}$. Therefore, f has a critical point at $x = \underline{\hspace{2cm}}$

25. Locate the maximum and minimum values of the function $f(x) = xe^{-x}$ on the interval $[0, 2]$.

26. What does the mean value theorem say about the function $f(x) = x^3 - 3x^2$ on the interval $[2, 3]$

Use L'Hôpital's rule if applicable to find the following limits:

27. $\lim_{x \rightarrow 0} \frac{\sin x}{1 - e^x}$

28. $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x$

29. $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2}$

30. Definition: If f is continuous on $[a, b]$, the **definite integral of f from a to b** is

$$\int_a^b f(x) dx =$$

31. Let $F(x) = \int_1^x \frac{1}{t} dt$. F is a function of what variable? _____

32. $F'(x) =$ _____

33. $F(1) =$ _____

34. What other famous function has the same derivative as F ?

35. In general, two functions with the same derivative differ by a constant. What is the constant in this case? I.e. what is the difference between F and your answer to number 34?

35. Evaluate $\int_1^4 \frac{1}{\sqrt{t}} dt$

36. Evaluate $\int_{\frac{\pi}{2}}^{\pi} \frac{\cos t}{\sqrt{\sin t}} dt$