Things to know about exponentials and logs:

- Properties of exponential functions: Domain and Range. Typical graph. Graph of $f(x) = e^x$
- Properties of logarithmic functions: Examples such as $f(x) = \log x, \quad f(x) = \ln x$ and their graphs.
- $y = \log_a x$ means $a^y = x$
- The number $e = \left(1 + \frac{1}{n}\right)^n$ as $n \to \infty$
- Natural logarithm: $\ln x = \log_e x \quad y = \ln x$ means $e^y = x$
- Common logarithm $\log x = \log_{10} x \qquad y = \log x$ means $10^y = x$
- Logarithms are the inverse of exponentials. That is, $a^{\log_a x} = x, \qquad \log_a a^x = x$

For example $\ln e^x = x, \qquad e^{\ln x} = x$ and $\log 10^x = x, \quad 10^{\log x} = x$

- Properties of logarithms.

$$\log uv = \log u + \log v$$

$$\log \frac{u}{v} = \log u - \log v$$

$$\log u^n = n \log u$$

For example: $\log 2000 = \log 2 + \log 1000 = \log 2 + 3$

- Change of base formula: $\log_a x = \frac{\log_b x}{\log_b a}$

For example: $\log_9 27 = \frac{\ln 27}{\ln 9}$

Another example: Solve $1.06^t = 2 \qquad t = \log_{1.06} 2 = \frac{\ln 2}{\ln 1.06}$

- Compound interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
- Continuous compounding: $A = Pe^{rt}$
- Exponential growth and decay: $A = A_0 e^{rt}$
- Important: Since $e^{\ln x} = x$ and $\ln x^n = n \ln x$ it follows that $x^n = e^{n \ln x}$

In other words, $A^B = e^{B \ln A}$

For example: $2^{\sqrt{2}} = e^{\sqrt{2} \ln 2}$ and $x^{\frac{1}{x}} = e^{\frac{\ln x}{x}}$