Worksheet on sigma notation

1. Write out the sum \( \sum_{k=1}^{5} 2k \)

2. Using the distributive law, show that it is the same as \( 2 \sum_{k=1}^{5} k \)

3. What is \( \sum_{k=1}^{5} 1 \)? How about \( \sum_{k=1}^{n} 1 \)? More generally \( \sum_{k=1}^{n} c \) where \( c \) is any constant?

4. Write out the sum \( \sum_{k=1}^{5} k^2 + k \)

5. Using the commutative law, show that this is the same as \( \sum_{k=1}^{5} k^2 + \sum_{k=1}^{5} k \)

6. Express in sigma notation: \( 1 - 3 + 5 - 7 + 9 - 11 + 13 - 15 + 17 \)

7. Observe that \( 1 = 1^2, 1 + 3 = 2^2, 1 + 3 + 5 = 3^2, 1 + 3 + 5 + 7 = 4^2 \)

8. What is the next sum? Express this relationship in sigma notation.

9. Show that \( \sum_{k=1}^{n} (k^2 - (k-1)^2) = n^2 \) (Don’t be confused by this; the “n” is the upper limit. Write out the first few terms without computing and you will see that the sum “telescopes”.)

10. Note that \( k^2 - (k-1)^2 = 2k - 1 \) (This has nothing to do with sums, this is elementary algebra.)

11. Conclude that \( \sum_{k=1}^{n} (2k - 1) = n^2 \) (Not much work here, just put in the equal sign.)

12. Now show that \( \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \) by the following method: Rewrite \( \sum_{k=1}^{n} (2k - 1) = n^2 \) as \( \sum_{k=1}^{n} 2k - \sum_{k=1}^{n} 1 = n^2 \) and solve for \( \sum_{k=1}^{n} k \)

13. Express in sigma notation: \( \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \) and compute this number.

14. Compute the number \( \sum_{k=1}^{5} \frac{1}{2^k} \)
15. Compute the number \( \sum_{k=1}^{6} \frac{1}{2^k} \)

16. What do you guess \( \sum_{k=1}^{7} \frac{1}{2^k} \) will be?

17. Assuming we had a good definition for an infinite sum, what should \( \sum_{k=1}^{\infty} \frac{1}{2^k} \) be?

18. Using the formulas

\[
\sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \left[ \frac{n(n+1)}{2} \right]^2
\]

Compute \( \sum_{k=1}^{20} k(k-2) \) and \( \sum_{k=1}^{10} k(k+1)(k+2) \)

19. Try computing this without getting confused between the index \( k \) and the upper limit \( n \) (which has nothing to do with the index, it is a constant).

\( \sum_{k=1}^{n} \frac{k}{n^2} \)

This is a harder set of problems, analogous to numbers 9 – 12.

20. Show that \( \sum_{k=1}^{n} k^3 - (k-1)^3 = n^3 \)

21. Using elementary algebra, show \( k^3 - (k-1)^3 = 3k^2 - 3k + 1 \)

22. Conclude that \( \sum_{k=1}^{n} 3k^2 - 3k + 1 = n^3 \)

23. Solve the above equation for \( \sum_{k=1}^{n} k^2 \)