

Worksheet on rational exponents.

Recall that for a rational exponent, the numerator means the power and the

denominator means the root. So, for example: $27^{\frac{2}{3}}$ means

- a) The cube root of 27 squared, or
- b) The square of the cube root of twenty-seven.
- a) 27 squared is $27 \times 27 = 729$, and the cube root of 729 (sometimes written $\sqrt[3]{729}$) is 9, which you find either using the calculator or by realizing that $9^3 = 9 \times 9 \times 9 = 729$.
- b) The cube root of 27 is 3 (because $3^3 = 3 \times 3 \times 3 = 27$), and $3^2 = 9$.

In either case we see that $27^{\frac{2}{3}} = 9$, although taking the root first is usually easier if for no other reason than the numbers are smaller.

Compute the following numbers:

1. $8^{\frac{4}{3}}$

2. $16^{\frac{1}{2}}$

3. $36^{\frac{3}{2}}$

4. $32^{\frac{2}{5}}$

5. $(144 \times 121)^{\frac{1}{2}}$

Answer:

$$(144 \times 121)^{\frac{1}{2}} = 144^{\frac{1}{2}} \times 121^{\frac{1}{2}} = 12 \times 11 = 132$$

6. $(16 \times 81)^{\frac{1}{4}}$

7. $\left(\frac{49}{64}\right)^{\frac{1}{2}}$

8. $\left(\frac{125}{8}\right)^{\frac{1}{3}}$

9. $(9 + 16)^{\frac{1}{2}}$

Here you must first add, then take the square root.

10. $9^{\frac{1}{2}} + 16^{\frac{1}{2}}$

Here you take the square roots first, then add.

11. Which answer is bigger, the sum of the square roots (#10) or

the square root of the sum (#9)?

Why?

To annoy you further, we can also have negative exponents as well as positive ones. This is not such a big deal if we remember that a negative

exponent just means the reciprocal. For example: $\left(\frac{16}{9}\right)^{-\frac{3}{2}}$ looks bad, but if

we tease out the meaning it is comprehensible. The exponent has three parts:

a) a minus sign,

b) a 2 in the denominator

c) a 3 in the numerator. In turn they mean

a) the reciprocal, b) the square root, and c) the cube (the third power).

Taking care of these, one at a time we get:

$$\left(\frac{16}{9}\right)^{-\frac{3}{2}} = \left(\frac{9}{16}\right)^{\frac{3}{2}} \text{ (taking the reciprocal gets rid of the pesky}$$

minus sign in the exponent) $\left(\frac{9}{16}\right)^{\frac{3}{2}} = \left(\frac{3}{4}\right)^3$ (taking the square root gets rid of

the 2 in the denominator, and finally

$$\left(\frac{3}{4}\right)^3 = \frac{27}{64}. \text{ Not so bad.}$$

Compute:

5. $\left(\frac{16}{81}\right)^{-\frac{3}{4}}$

6. $36^{-\frac{3}{2}}$

1. $\left(\frac{9}{4}\right)^{-\frac{1}{2}}$

2. $32^{\frac{2}{5}}$

3. $\left(\frac{49}{64}\right)^{-\frac{1}{2}}$

4. $\left(\frac{125}{8}\right)^{-\frac{1}{3}}$