

1 Course: Maths 016 , Arithmetic

Maths 016 is a course in arithmetic. Catalogue Description: Integers, fractions, decimals, scientific notation, ratio and proportion, percents, geometry and measurement, applications, approximations, use of a scientific calculator. Credit will not apply toward graduation.

2 Meeting Place and Time

1:25 pm - 2:20 pm MWF West Building W3-56.

3 Instructor Information

I am (Dr.) David A. SANTOS. I have been teaching full-time at CCP since the autumn of 1999. I received an AB in Mathematics from the University of Chicago in 1987 and a Ph.D. in Mathematics from the University of Michigan (Ann Arbor) in 1993. My office is on the second floor (uppermost level) of the Bonnell Building: B2-14. My office phone is (215) 751-8698. My email is dsantos@ccp.edu. My website is at <http://faculty.ccp.edu/faculty/dsantos>. I like camels (the quadrupeds, not the cigarettes!).

4 Office Hours

I have a very tight schedule this semester, and hence, apologise for the inconvenience of my hours. This semester they will be,

Mon-Wed-Fri	10:10—10:40 AM
Mon-Wed-Fri	3:30—5:00 AM

5 Course Pre-requisites and Goals

The formal pre-requisite for this course is maths 016, arithmetic. In practice what you need to know are basic integer and fraction arithmetic, although we will review this latter topic briefly.

This is a fairly fast-paced course. There will be little time in class to review or to go over the homework. You are expected to have done the assigned homework before the next class.

From years of teaching experience, I can tell you that successful students in this class are those who attend regularly, and regularly do their homework.

6 Class Website

My webpage at CCP is at

<http://faculty.ccp.edu/faculty/dsantos>.

Click on the Arithmetic link to see old exams and assignments that I have given when I have taught this course previously. Also, you will find there handouts, exams, and assignments for the current semester as they are given in class. Thus if you happen to miss a class when I have given a handout, you can refer to the website. This is particularly useful as I tend to dispose of any extra copies that are not picked-up by students on the day that I distribute handouts—this helps me keep the clutter in my office to a minimum.

7 Textbook

I will provide a series of lecture notes free. They can also be freely downloaded from my website.

8 Topics Outline

Disclaimer: this is a *very fast-paced course*. There will be *little time—if any*—for review. *What follows is an approximate outline of the pace of the course. We may go faster or slower, contingent on class response.*

Week I Diagnostic Test. Egyptian and Roman Numerals.

Week II Systems of Numeration. The Decimal Scale. Symbolical Expression.

Week III Addition, Subtraction, and Multiplication of Natural Numbers. **Review. Exam I.**

Week IV Primes and Factorisation. Division.

Week V Long Division. Roots and Geometric Progressions.

Week VI Long Division. Roots and Geometric Progressions. **Review. Exam II.**

Week VII Fractions. Addition and Subtraction of Fractions.

Week VIII Multiplication and Division of Fractions.

Week IX Ratio and Proportion. **Review. Exam III.**

Week X Decimals

Week XI Decimals. Addition, Subtraction and Multiplication of Decimals. Division of Decimals.

Week XII The Integers.

Week XIII The Rational Numbers. **Review. Exam IV.**

Week XIV Review.

Week XIV Comprehensive Final Exam.

9 How your final grade is determined

The way your final grade is computed **is not negotiable**. There will be four topical exams, each worth 20 marks. There will be a *comprehensive final exam* (worth 30 marks). **None** of these grades will be dropped. The grading scale is as follows. Thus you will have a total of

$$4 \times 20 + 30 = 110$$

possible marks. This is a PASS/FAIL course. Those scoring 77 marks or above will pass, those scoring 76 marks or below will fail.

10 Missing an Exam

You must present a valid medical excuse for missing an exam, within a week of having missed the exam. Otherwise you will receive a grade of 0. You must arrange a time, *at my convenience*, for a make up. You will not be given the same exam the class took, neither will I weigh more other exams to make a grade up.

11 Missing the Final Exam

Here you are in trouble! I will give you an **I**, incomplete grade, that must be made up during the first to weeks of the next semester, otherwise it turns into an **F**. Don't miss the final!!!

12 Absences

If you are absent (without a valid excuse) four or more days during the semester, I have the right to drop you from class. This does not mean that I will drop you, only that I *may*. If you should drop the class, do so before the deadline passes. Verify with the registrar's office that you are indeed dropped from class, they tend to make mistakes. I tend to give "F"'s to students who stop shewing up for class and do not earn enough marks to get a "P" if they stop shewing up.

If you expect to miss class and have a good excuse, please leave a message in my voice mail, or, preferably, send me an email. This will be especially important if you miss an exam. For very good reasons, which might amuse you to know, I tend not to reply to phone

calls. I do, however, reply to all email. My email is dsantos@ccp.edu.

If you miss a class, it is your responsibility to get the lecture notes from a classmate. I never have written plans of my lectures since I lecture contingent on the response that I get from the audience.

13 Tutoring

The maths lab in B2-36 may have tutors for maths016. This depends on the availability of personnel, which is something I cannot control. Ask repeatedly in that office whether tutors or workshops are given for this class.

14 Extra Credit

The need of extra credit arises from your inability to cope with the minimum class burden given. In such a case, this means that you have not acquired the necessary know-how in Arithmetic to be deemed proficient in the course, hence you must repeat the course. Extra credit will **NEVER** be given, so do not ask for it!

Many a student with histrionic abilities come to me at the end of the semester telling me that they need the class to graduate, that this is their last class at CCP, that they need a good grade in order to keep or get a scholarship, etc. **Don't bother me with this! I don't care! This is not my concern!** Every student will pass the class the student like every other student: through hard work and through passing the exams. Do not expect me to make special allowances for you: don't even suggest it, you will only make me angry.

15 Practice Exams and Review for Tests

Old exams are available at my website. Also, the book's appendix A has old examination questions. I will not have time to review for tests in class.

16 Appropriate Classroom Behaviour

- Please be on time for class. It is distracting to me as to your other classmates to have people come in late.
- Please switch off pagers and cell phones before entering the classroom.
- Abstain from bringing friends or small children to the classroom.

- Be courteous and considerate to your other classmates and to me.
- Do not ask for extra credit, as none will be given.
- Mathematics is a cumulative subject. Do not expect to understand subsequent material if you have not mastered current material.
- Homework may be assigned, but not collected. Do the assigned homework.
- The best way to ask a question is something like: “How did you get from the second step to the third step?” or “What does it mean to complete the square?” Asseverations like “I don’t understand” do not help me answer your queries. Moreover, not understanding is a psychological problem, not a mathematical one, and I am only paid to help you with your mathematics.
- I am very patient, but I will not tolerate boorish, badgering behaviour. If it comes to me having to call security to remove you from the classroom, and if I ask you to leave, please do leave. You should be able to return the next day, and I will not penalise you for a bad behaviour day. I don’t keep grudges! Beware: if I must call security, an incident report will be written and it will go into your record. Let us keep our teacher-student relationship civil!

17 Statement on Disability

Students who are registered with the Centre of Disability must inform the instructor by the end

of the first week of classes if special accommodations are required.

18 Making a Complaint

It is always best to inform your instructor immediately about any concerns you may have. If you are having a problem in any class, the first step is to make an appointment with the instructor to try to resolve the problem directly. If you have been unable to resolve it with your instructor, you may use a student complaint form, available at the Chair’s Office in W2-7.

When you fill out the student Complaint Form you will be required to fill out your name, course, and other identifying characteristics, as anonymous complaints will not be paid attention to.

Upon receipt of the complaint form, the Chair will contact you and will contact me.

Complaining with the Chair will not change your grade, as only I can do that.

19 Spelling

I use British spelling. So I end my verbs in “ise” rather than “ize,” I use the ending “our” rather than “or” (as in “colour”) and I use the correct spelling “shew” rather than “show.” If you have a problem with this, tough break!

Positional Notation

6. Convert 321_4 to decimal.

- (A) 55 (B) 56 (C) 321 (D) 57 (E) none of these
-

7. Convert 321 to base-4.

- (A) 11011_4 (B) 10011_4 (C) 11001_4 (D) 1231_4 (E) none of these
-

8. Which integer follows the hexadecimal integer $A2FF_{16}$? In other words, what is $A2FF_{16} + 1_{16}$?

- (A) $A2A00_{16}$ (B) $A300_{16}$ (C) $B300_{16}$ (D) $A2FF_{16}$ (E) none of these
-

9. $11_2 + 10_2 =$

- (A) 100_2 (B) 110_2 (C) 111_2 (D) 101_2 (E) none of these
-

10. Which of the following is the *wrong* description for the '6' in 654321?

- (A) 600000 units
(B) 6000 hundreds
(C) 600 thousands
(D) 60 ten thousands
(E) 6 millions

Symbolical Expression

11. Let x be the unknown quantity. Translate into symbols: “a number diminished by two.”

- (A) $2x$ (B) $x - 2$ (C) $x + 2$ (D) $\frac{x}{2}$ (E) none of these
-

12. Let s be the unknown quantity. Translate into symbols: “twice a number is reduced by its square.”

- (A) $s^2 + 2s$ (B) $s^2 - 2s$ (C) $3s - s^2$ (D) $2s - s^2$ (E) none of these
-

13. If my age by 31 December 2020 will be s , what was my age by 31 December 2005?

- (A) $s - 15$ (B) $s + 15$ (C) $15 - s$ (D) $s - 14$ (E) none of these
-

14. If my age by 31 December 2005 was s , what will my age be by 31 December 2020?

- (A) $s - 15$ (B) $s + 15$ (C) $15 - s$ (D) $s - 14$ (E) none of these
-

15. I am thinking of a rule that converts the number 3 into the number 20. Which of the following could *not* be my rule?

- (A) add seventeen
- (B) multiply by five, then add five
- (C) add one, then multiply by five
- (D) square, then add eleven
- (E) cube, then add three

Addition and Subtraction of Natural Numbers

16. Collect like terms: $(2a + 7b) + (3b + 8a)$

- (A) $20ab$ (B) $100ab$ (C) $10 + a + b$ (D) $10a + 10b$ (E) none of these
-

17. Collect like terms: $(2x + 3) + (3x - 1)$

- (A) $5x + 4$ (B) $5x + 2$ (C) $x + 2$ (D) $5x - 2$ (E) none of these
-

18. In the **difference**

$$\begin{array}{r} 5 \bullet 2 \bullet \\ - \bullet 3 2 1 \\ \hline 3 8 \bullet 2 \end{array}$$

what is the sum of the missing digits?

- (A) 4 (B) 5 (C) 6 (D) 7 (E) none of these
-

19. In the magic square, the three numbers in each row, in each column, and in each diagonal add up to the same number. When the magic square below is completed, which of the following numbers is *not* used?

13		
	10	
9		7

- (A) 6 (B) 8 (C) 12 (D) 14 (E) 15
-

20. For a given charity, Adam contributes a dollars, Betty contributes b dollars and Carl contributes c dollars. Dwight, Erin, and Frances are **thieves**, and so from the contributions, Dwight steals d dollars, Erin steals e dollars and Frances steals f dollars. What is the available amount of dollars that the charity has?

- (A) $a + b + c - d - e - f$ (B) $a + b + c + d + e + f$ (C) $abc - def$ (D) $\frac{abc}{def}$ (E) none of these

Brief Answers and Solutions

1. D.
2. A.
3. D.
4. B.
5. B. $\heartsuit\heartsuit\clubsuit\clubsuit = \clubsuit\clubsuit\clubsuit\clubsuit = \spadesuit\spadesuit$.
6. D. $321_4 = 3 \cdot 4^2 + 2 \cdot 4 + 1 = 3 \cdot 16 + 2 \cdot 4 + 1 = 48 + 8 + 1 = 57$.
7. C.
8. B.
9. D.
10. E.
11. B.
12. D.
13. A.
14. B.
15. E.
16. D. $(2a + 7b) + (3b + 8a) = 2a + 8a + 7b + 3b = 10a + 10b$.
17. B. $(2x + 3) + (3x - 1) = 2x + 3x + 3 - 1 = 5x + 2$.
18. A. The difference is $5123 - 1321 = 3802$, hence the sum of the missing digits is $1 + 3 + 1 + 0 = 4$.
19. E. The completed magic square follows.

13	6	11
8	10	12
9	14	7

20. A. Add the contributions and take away the money stolen.

NAME: _____

Instructions: You have the full period to complete this exam. You are allowed to use your book, your notes, and your calculator. If you don't understand a question, please ask me for clarification. I may answer it or not, depending on whether I consider it a fair question.

Arithmetic progressions

1. Consider the arithmetic progression 5, 14, 23, 32, . . . , where 5 is on the first position, 14 in the second position, etc. Which number occupies the 41st position?

(A) 374 (B) 365 (C) 369 (D) 360 (E) none of these

2. Which of the following numbers does not belong to the arithmetic progression 5, 14, 23, 32, . . . ?

I : 1805 *II* : 905 *III* : 338 *IV* : 335

(A) 1805 (B) 905 (C) 338 (D) 335 (E) all these numbers belong to the progression

Multiplication

3. $(1 + 2 \cdot 3)^2 =$

(A) 37 (B) 729 (C) 49 (D) 27 (E) none of these

4. Multiply: $(4x^3)(5x^2)$.

(A) $20x^5$ (B) $9x^5$ (C) $20x^6$ (D) $20x^9$ (E) none of these

5. Multiply: $(x + 1)(x + 2)$.

(A) $x^2 + 2x + 2$ (B) $x^2 + 3x + 2$ (C) $2x + 4$ (D) $x^2 + 2$ (E) none of these

6. In Microsoft Excell, the columns are ordered by letters. The first 26 columns are A to Z . Columns 27 to 52 are labelled AA to ZZ , etc. How many letters have you used when labelling the first 52 columns from A to ZZ ?

(A) 26 (B) 52 (C) 78 (D) 104 (E) none of these

7. Suppose you write all the integers from 1 to 100 in a row:

12345678910111213...9899100.

How many digits have you used?

(A) 100 (B) 192 (C) 102 (D) 1000 (E) none of these

Primes and Factorisation

8. What is the prime factorisation of 90?

(A) $2 \cdot 3 \cdot 5$ (B) $2 \cdot 3^2 \cdot 5$ (C) $2^2 \cdot 3 \cdot 5$ (D) $2^2 \cdot 3^2 \cdot 5$ (E) none of these

9. What is the prime factorisation of 300?

(A) $2^2 \cdot 3^2 \cdot 5$ (B) $2 \cdot 3^2 \cdot 5$ (C) $2^2 \cdot 3 \cdot 5^2$ (D) $2 \cdot 3^2 \cdot 5^2$ (E) none of these

10. Find $\text{lcm}(90, 300)$.

(A) 30 (B) 300 (C) 900 (D) 90 (E) none of these

11. Find $\text{gcd}(90, 300)$.

(A) 30 (B) 300 (C) 900 (D) 90 (E) none of these

Division

12. $72 \div 24 \div 3 =$

 (A) 1 (B) 4 (C) 2 (D) 3 (E) none of these

13. $72 \div 24 \cdot 2 =$

 (A) 4 (B) 1 (C) 3 (D) 6 (E) none of these

14. You start writing, in sequence,

red, pink, commie, hippie, tosser, red, pink, commie, hippie, tosser, red, pink, commie, hippie, tosser, . . .

Which word is the 2008th in this sequence?

 (A) red (B) pink (C) commie (D) hippie (E) tosser

15. When a number is multiplied by 3 and 5 is added to the result, we get 26. What was the original number?

 (A) 10 (B) 8 (C) 5 (D) 7 (E) none of these

16. Which value of the digit d makes the number $12d$ divisible by 9?

(A) 9

(B) 6

(C) 3

(D) 0

(E) none of these

17. Which value of the digit d makes the number $12d$ divisible by 5?

(A) 1

(B) 2

(C) 3

(D) 0

(E) none of these

Long Division

18. $1002008016032 \div 32 =$

(A) 313127505010

(B) 313127505001

(C) 31312750501

(D) 30103010207050501

(E) none of these

19. Divide: $(6x^9) \div (2x^3)$

(A) $3x^3$

(B) $3x^6$

(C) $4x^3$

(D) $4x^6$

(E) none of these

20. Divide: $\frac{20x^2 + 10x^3}{2x}$

(A) $10x^2 + 5x$

(B) $18x + 8x^2$

(C) $10 + 5x$

(D) $10x + 5x^2$

(E) none of these

Brief Answers and Solutions

1. B. The n th term has the form $5 + 9(n - 1)$. The 41st is thus $5 + 9(40) = 5 + 360 = 365$.
2. D. Every term leaves remainder 5 upon division by 9. But $335 = 333 + 2 = 9(37) + 2$, which leaves remainder 2.
3. C. $(1 + 2 \cdot 3)^2 = (1 + 6)^2 = 7^2 = 49$.
4. A. $(4x^3)(5x^2) = 20x^{3+2} = 20x^5$.
5. B. We have

$$\begin{array}{r} x + 1 \\ x + 2 \\ \hline x^2 + x \\ + 2x + 2 \\ \hline x^2 + 3x + 2 \end{array}$$

6. C. One needs 26 letters for the first 26 columns and then $2 \cdot 26 = 52$ letters for the next 26 columns. Hence, one needs $26 + 52 = 78$ letters altogether.
7. B. One needs 9 digits for the one-digit numbers, $2 \cdot 90 = 180$ digits for the two-digit numbers and then 3 digits to write 100. Hence the total is $9 + 180 + 3 = 192$.
8. B. $90 = 9 \cdot 10 = 3 \cdot 3 \cdot 2 \cdot 5 = 2 \cdot 3^2 \cdot 5$.
9. C. $300 = 10 \cdot 30 = 2 \cdot 5 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5^2$.
10. C. $\text{lcm}(90, 300) = 2^2 \cdot 3^2 \cdot 5^2 = 4 \cdot 9 \cdot 25 = 900$.
11. A. $\text{gcd}(90, 300) = 2 \cdot 3 \cdot 5 = 30$.
12. A. $72 \div 24 \div 3 = 3 \div 3 = 1$.
13. D. $74 \div 24 \cdot 2 = 3 \cdot 2 = 6$.
14. C. There are five repeating words. Upon division by 5, associate the remainders to the words: You start writing, in sequence, red = 1, pink = 2, commie = 3, hippie = 4, tosser = 0. Since $2008 = 2005 + 3 = 5 \cdot 401 + 3$, leaves remainder 3 upon division by 5, the 2008th word is *commie*.
15. D. Prior to 26 we had $26 - 5 = 21$. Prior to 21 we had $21 \div 3 = 7$, whence the original number was a 7.
16. B. A number is divisible by 9 if and only if its digital sum is divisible by 9. We have $12d \rightarrow 1 + 2 + d = 3 + d$. To make this divisible by 9 we let $d = 6$.
17. D. A number is divisible by 5 if and only if its last digit is 0 or 5. In the choices we only have 0 available.
18. C.
19. B. $(6x^9) \div (2x^3) = 3x^{9-3} = 3x^6$.
20. D. $\frac{20x^2 + 10x^3}{2x} = \frac{20x^2}{2x} + \frac{10x^3}{2x} = 10x + 5x^2$.

NAME: _____

Instructions: You have the full period to complete this exam. You are allowed to use your book, your notes, and your calculator. If you don't understand a question, please ask me for clarification. I may answer it or not, depending on whether I consider it a fair question.

Roots

1. $\sqrt{9} + \sqrt{16}$

- (A) $\sqrt{25}$ (B) 7 (C) 12 (D) $12\frac{1}{2}$ (E) none of these

2. $\sqrt{9} \cdot \sqrt{16}$

- (A) 36 (B) 7 (C) 12 (D) $12\frac{1}{2}$ (E) none of these

Fractions

3. How many of the following statements are true?

$$I : \frac{3}{4} = \frac{5}{6} \quad II : \frac{3}{4} < \frac{5}{6} \quad III : \frac{6}{8} = \frac{15}{20} \quad IV : 4\frac{1}{3} = \frac{4}{3}$$

- (A) none (B) exactly one (C) exactly two (D) exactly three (E) all four

4. What fraction of an hour is 18 minutes?

- (A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{10}$ (D) $\frac{5}{6}$ (E) none of these

5. Express $\frac{6}{132}$ in least terms.

- (A) $\frac{1}{11}$ (B) $\frac{1}{22}$ (C) $\frac{1}{44}$ (D) $\frac{1}{66}$ (E) none of these

6. Convert the improper fraction to a mixed number: $\frac{43}{8}$

(A) $8\frac{5}{8}$

(B) $8\frac{3}{5}$

(C) $5\frac{3}{8}$

(D) $8\frac{3}{8}$

 (E) none of these

7. Convert the mixed number to an improper fraction: $2\frac{3}{4}$

(A) $\frac{6}{4}$

(B) $\frac{10}{4}$

(C) $\frac{11}{4}$

(D) $\frac{4}{11}$

 (E) none of these

8. How many of the following assertions are **false**?

$$I : \frac{1}{2} = \frac{2}{1} \quad II : \frac{3}{5} < \frac{4}{7} \quad III : \frac{1}{2} < \frac{1}{3} \quad IV : \frac{3}{1} < \frac{2}{2}$$

 (A) exactly one (B) exactly two (C) exactly three (D) all four (E) none of them

Addition and Subtraction of Fractions

9. Add the fractions: $\frac{2}{5} + \frac{3}{4}$

(A) $\frac{5}{9}$

(B) $\frac{1}{4}$

(C) $\frac{5}{20}$

(D) $\frac{23}{20}$

 (E) none of these

10. Subtract the fractions: $\frac{11}{20} - \frac{2}{15}$

(A) $\frac{9}{5}$

(B) $\frac{5}{12}$

(C) $\frac{9}{12}$

(D) $\frac{3}{4}$

 (E) none of these

11. Calculate: $3\frac{3}{4} - 2\frac{1}{2}$.

(A) $\frac{14}{15}$

(B) $1\frac{3}{5}$

(C) $1\frac{15}{14}$

(D) $1\frac{5}{4}$

(E) none of these

12. Calculate: $\frac{1}{3} - \frac{1}{6} + \frac{1}{9}$.

(A) $\frac{1}{12}$

(B) $\frac{5}{18}$

(C) $\frac{1}{18}$

(D) $\frac{5}{18}$

(E) none of these

Multiplication and Division of Fractions

13. Find the reciprocal of $2\frac{3}{4}$.

(A) $2\frac{4}{3}$

(B) $\frac{10}{4}$

(C) $\frac{11}{4}$

(D) $3\frac{4}{11}$

(E) none of these

14. Simplify as a mixed numeral in lowest terms:

$$\left(2\frac{2}{3}\right) \cdot \left(3\frac{3}{4}\right).$$

(A) 10

(B) $6\frac{6}{12}$

(C) $\frac{5}{4}$

(D) $8\frac{3}{4}$

(E) none of these

15. Calculate: $\frac{4}{5} \cdot \frac{5}{6} + \frac{4}{5} \div \frac{5}{6}$

(A) $\frac{4}{3}$

(B) $\frac{16}{75}$

(C) $\frac{49}{15}$

(D) $\frac{122}{75}$

(E) none of these

16. Calculate: $\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{2} - \frac{1}{3}}$

(A) 5

(B) $\frac{1}{5}$

(C) 1

(D) $\frac{2}{5}$

(E) none of these

17. Calculate: $\frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}$

(A) $\frac{3}{2}$

(B) $\frac{3}{5}$

(C) $\frac{5}{3}$

(D) $\frac{3}{5}$

(E) none of these

Ratio and Proportion

18. Solve the proportion for n

$$\frac{5}{6} = \frac{60}{n}$$

(A) 72

(B) 50

(C) $\frac{1}{2}$

(D) 12

(E) none of these

19. A recipe for five people takes four eggs. Assuming proportional ingredients available, how many people will twenty eggs serve?

(A) 21

(B) 10

(C) 25

(D) 15

(E) none of these

20. John takes 8 hours to fill a hole, Atish takes 4 hours to fill the same hole, and David takes 2 hours to fill the same hole. If the three are working together, how many hours will it take them to fill the hole?

(A) $1\frac{1}{8}$ hours

(B) $1\frac{1}{2}$ hours

(C) $\frac{7}{8}$ hours

(D) $1\frac{1}{7}$ hours

(E) none of these

Brief Answers and Solutions

1. B. $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$.

2. C. $\sqrt{9} \cdot \sqrt{16} = 3 \cdot 4 = 12$.

3. C. Since $\frac{3}{4} = \frac{9}{12}$ and $\frac{5}{6} = \frac{10}{12}$, I is false and II is true. III is true since $\frac{6}{8} = \frac{3}{4} = \frac{15}{20}$. IV is false, since $4\frac{1}{3} = \frac{13}{3}$.

4. C. We need $\frac{18}{60} = \frac{3}{10}$.

5. B. $\frac{6}{132} = \frac{6 \div 6}{132 \div 6} = \frac{1}{22}$.

6. C. $\frac{43}{8} = \frac{5}{8 \overline{)43}}$
 $\frac{40}{3}$

7. C. $2\frac{3}{4} = \frac{2 \cdot 4 + 3}{4} = \frac{11}{4}$.

8. D.

9. D. $\frac{2}{5} + \frac{3}{4} = \frac{8}{20} + \frac{15}{20} = \frac{23}{20}$.

10. B. $\frac{11}{20} - \frac{2}{15} = \frac{33}{60} - \frac{8}{60} = \frac{25}{60} = \frac{5}{12}$.

11. E. $3\frac{3}{4} - 2\frac{1}{2} = \frac{15}{4} - \frac{5}{2} = \frac{15}{4} - \frac{10}{4} = \frac{5}{4}$.

12. B, D. Calculate: $\frac{1}{3} - \frac{1}{6} + \frac{1}{9} = \frac{6}{18} - \frac{3}{18} + \frac{2}{18} = \frac{5}{18}$.

13. E. $2\frac{3}{4} = \frac{2 \cdot 4 + 3}{4} = \frac{11}{4}$, and so the reciprocal sought is $\frac{4}{11}$.

14. A. $(2\frac{2}{3}) \cdot (3\frac{3}{4}) = (\frac{8}{3}) \cdot (\frac{15}{4}) = \frac{120}{12} = 10$.

15. D. $\frac{4}{5} \cdot \frac{5}{6} + \frac{4}{5} \div \frac{5}{6} = \frac{2}{3} + \frac{24}{25} = \frac{122}{75}$.

16. A. $\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{2} - \frac{1}{3}} = \frac{\frac{5}{6}}{\frac{1}{6}} = 5$.

17. B, D. $\frac{1}{1 + \frac{1}{1 + \frac{1}{2}}} = \frac{1}{1 + \frac{1}{\frac{3}{2}}} = \frac{1}{1 + \frac{2}{3}} = \frac{1}{\frac{5}{3}} = \frac{3}{5}$.

18. A. $\frac{5}{6} = \frac{60}{72}$.

19. C. $\frac{5 \text{ people}}{4 \text{ eggs}} \cdot (20 \text{ eggs}) = \frac{5 \cdot 20}{4} \text{ people} = 25 \text{ people}$.

20. D. Working together, in one hour, they accomplish $\frac{1}{8} + \frac{1}{4} + \frac{1}{2} = \frac{1+2+4}{8} = \frac{7}{8}$ of the task. Thus the whole task will be accomplished in $\frac{8}{7} = 1\frac{1}{7}$ hours.

NAME: _____

Instructions: You have the full period to complete this exam. You are allowed to use your book, and your Calculators will not be permitted for this exam. If you don't understand a question, please ask me for clarification. I may answer it or not, depending on whether I consider it a fair question.

Decimals1. Convert to a decimal: $2\frac{3}{4}$. (A) 2.34 (B) 2.25 (C) 2.75 (D) 2.034 (E) none of these2. Convert to a decimal: $\frac{9}{11}$. (A) $0.8\bar{1}$ (B) $0.9\bar{11}$ (C) 0.81818181818 (D) $0.8\bar{1}$ (E) none of these

3. Convert to an improper fraction: 2.34.

 (A) $\frac{117}{100}$ (B) $\frac{234}{50}$ (C) $\frac{3}{2}$ (D) $\frac{117}{50}$ (E) none of these

4. Determine the largest number among the following:

 $A : \frac{1}{6}$ $B : 0.16\bar{7}$ $C : 0.\bar{17}$ $D : 0.\bar{16}$ $E : 0.1\bar{7}$ (A) (B) (C) (D) (E)

Addition, Subtraction and Multiplication of Decimals

5. $9.0909 + 2.0202 =$

- (A) 10.1111 (B) 11.0111 (C) 11.111 (D) 11.1111 (E) none of these
-

6. Convert to a decimal: $11\% + 11\%$.

- (A) 0.22 (B) 22.0 (C) 2.2 (D) 0.0022 (E) none of these
-

7. Convert to a decimal: $(11\%) \cdot (11\%)$.

- (A) 0.121 (B) 121.0 (C) 1.21 (D) 0.0121 (E) none of these
-

8. Convert to a decimal: $\frac{1}{8} + 0.25 =$

- (A) 0.43 (B) 0.37 (C) 0.375 (D) 0.5 (E) none of these
-

9. Convert to a decimal: $(0.1)^2 \cdot 0.2 + 0.1 \cdot (0.2)^2 =$

- (A) 0.06 (B) 0.006 (C) 0.0028 (D) 0.6 (E) none of these
-

10. What percent of 44 is 11?

- (A) 11% (B) 484% (C) 400% (D) 25% (E) none of these
-

Division of Decimals

11. $0.102 \div 0.03 =$

- (A) 3.4 (B) 34 (C) 0.034 (D) 0.34 (E) none of these
-

12. $1.1 \div 0.3 =$

- (A) $3.\overline{06}$ (B) $3.\overline{6}$ (C) $3.0\overline{6}$ (D) $3.0\overline{36}$ (E) none of these
-

Integer Addition and Subtraction

13. If the following numbers were written from smallest to largest, which one would be in the middle?

$$A : 7 \quad B : -3 \quad C : -4 \quad D : 0 \quad E : -1$$

- (A) (B) (C) (D) (E)
-

14. $3 - 9 + 27 - 81 =$

- (A) 60 (B) -42 (C) -60 (D) -114 (E) none of these
-

15. $-1 + |-2| =$

- (A) -3 (B) 1 (C) -1 (D) 3 (E) none of these
-

Integer Multiplication and Division

16. $\frac{5^2 - (-3)^2}{5 - (-3)} =$

 (A) 2 (B) -2 (C) 8 (D) 16 (E) none of these

17. $(2)(-8) - (-2)(-2)(3) =$

 (A) 4 (B) -4 (C) 28 (D) -28 (E) none of these

18. $5 - 4^2 =$

 (A) -11 (B) 11 (C) 21 (D) -21 (E) none of these

Rational Numbers

19. $2 - \left(-\frac{1}{2}\right)^3 =$

 (A) $\frac{15}{8}$ (B) $\frac{11}{6}$ (C) $\frac{17}{8}$ (D) $\frac{13}{6}$ (E) none of these

20. $\frac{2^3 - (-1)^3}{2 - (-1)}$

 (A) 3 (B) 1 (C) -3 (D) -1 (E) none of these

Brief Answers and Solutions

1. C. $2\frac{3}{4} = 2 + \frac{3}{4} = 2 + 3 \div 4 = 2 + 0.75.$
2. D. $\frac{9}{11} = 9 \div 11 = 0.8181818181 \dots = 0.\overline{81}.$
3. D. $2.34 = \frac{234}{100} = \frac{117}{50}.$
4. E. $0.\overline{16} < \frac{1}{6} < 0.1\overline{67} < 0.\overline{17} < 0.1\overline{7}.$
5. D. $9.0909 + 2.0202 = 11.1111$
6. A. $11\% + 11\% = 0.11 + 0.11 = 0.22.$
7. D. $(11\%) \cdot (11\%) = (0.11)(0.11) = 0.0121.$
8. C. $\frac{1}{8} + 0.25 = 0.125 + 0.25 = 0.375.$
9. B. $(0.1)^2 \cdot 0.2 + 0.1 \cdot (0.2)^2 = (0.1)(0.1)(0.2) + (0.1)(0.1)(0.2) = 0.002 + 0.004 = 0.006.$
10. D. $\frac{11}{44} = \frac{11 \cdot 100}{44} \cdot \frac{1}{100} = 25\%.$
11. A. $0.102 \div 0.03 = 10.2 \div 3 = 3.4.$
12. B. $1.1 \div 0.3 = 11 \div 3 = 3.\overline{6}.$
13. E. $-4 < -3 < -1 < 0 < 7.$
14. C. $3 - 9 + 27 - 81 = -6 + 27 - 81 = 21 - 81 = -60.$
15. B. $-1 + |-2| = -1 + 2 = 1.$
16. A. $\frac{5^2 - (-3)^2}{5 - (-3)} = \frac{25 - 9}{8} = \frac{16}{8} = 2$
17. D. $(2)(-8) - (-2)(-2)(3) = -16 - 12 = -28.$
18. A. $5 - 4^2 = 5 - 16 = -11.$
19. C. $2 - \left(-\frac{1}{2}\right)^3 = 2 - \left(-\frac{1}{8}\right) = 2 + \frac{1}{8} = \frac{17}{8}.$
20. A. $\frac{2^3 - (-1)^3}{2 - (-1)} = \frac{8 - (-1)}{2 + 1} = \frac{9}{3} = 3.$

6. Write the prime factorisation of 72 and 70. Find $\gcd(72, 70)$. Find $\text{lcm}(72, 70)$.

7. If today is Saturday, what day will it be 666 days from today?

8. Determine a digit d so that the 3-digit integer $21d$ be divisible by 9.

9. Compute: $\frac{2^3 + 2^3 + 2^3 + 2^3 + 2^3 + 2^3}{2^2 + 2^2 + 2^2}$.

10. Compute: $\frac{1111^5 + 1111^5 + 1111^5 + 1111^5}{1111^3 + 1111^3}$.

Fractions

11. What is a half of a fifth of 100?

12. Compute and express your answer as an improper fraction: $\frac{2}{3} + \frac{3}{4} + \frac{5}{12}$.

13. Compute and express your answer as a mixed number: $(1\frac{4}{5})(3\frac{1}{2})$.

14. Compute: $\frac{4}{3 + \frac{1}{2}}$.

15. Compute: $\frac{6}{5} \cdot \frac{5}{4} + \frac{6}{5} \div \frac{5}{4}$.

16. What fraction of an hour does 6 minutes constitute?
17. David spent $\frac{2}{5}$ of his money on a storybook. The storybook cost \$35. How much money did he have at first?
18. Tito was pigging-out on cookies, and in the course of three days, he ate 120 cookies. On the first day, he ate $\frac{1}{6}$ of the cookies. On the second day, he ate $\frac{3}{5}$ of the remaining cookies. How many cookies did he eat on the third day?
19. Find n if $\frac{2n}{21} = \frac{14}{21}$.
20. A recipe for sangría uses 2 measures of lemonade for 3 measures of red wine. How many cups of lemonade and of red wine are required if 40 cups of sangría are desired?

Decimals

21. Compute and write your answer as a decimal: $\frac{1}{4} + 1.7 + 5\%$.

22. Compute and write your answer as a decimal: $\left(\frac{4}{10}\right)\left(\frac{3}{100}\right)\left(\frac{2}{1000}\right)\left(\frac{1}{10000}\right)$.

23. Compute: $(0.3)(0.2) + (0.1)^2$.

24. Convert to a fraction: $\frac{1}{1 + \frac{1}{0.1}}$.

25. Back in the good old days of 1999, a euro was worth \$1.05. Now, the value of the dollar has fallen, and a euro is worth 20% more of what it was in 1999. How much is worth a euro now, in dollars?

26. How much does it cost to fill a 12 gallon tank, if gas costs \$3.47 a gallon?

27. Divide: $123.123 \div 0.6$. Is the result a periodic or a finite decimal?
28. Divide: $123.123 \div 0.09$. Is the result a periodic or a finite decimal?
29. T-shirts are now at a 40% discount at the local T-shirt store. What is the discounted price of a T-shirt of original price \$19.90?
30. The area of an aerobic hall in a tri-complex is 70% of the total area (aerobic hall, tennis court, and pool). The area of a tennis court is 30% of the aerobic hall area. Find the area of a pool, if the aerobic hall area is 476 sq. feet.

Integers

31. Compute: $(1 + 2 - 5)(4 - 3 - 6) - (8 + 9 \cdot 10)$

32. Compute: $|(-1)^3 + (-2)^2 + (-4)(-5)|$

33. Calculate: $(6 - 4^2)(4 - (-4)^2)$

34. When a number is divided by 3 and to this result we add 5, we obtain 2. What was the original number?

35. A number is doubled, and then the result is decreased by 10. One obtains -12 as the final answer. What was the original number?

Rational Numbers

36. Calculate: $2 - \left(-\frac{3}{2}\right)^2$.

37. Calculate: $1 + (-0.1)^3$.

38. Calculate: $(1 - 1.1)^3 + 1$.

39. Calculate: $\frac{-\frac{2}{3} \div \frac{3}{4}}{-\frac{2}{3} + \frac{3}{4}}$.

40. Calculate: $\frac{(-2008)^1 + (-2)^2 + (-3)^3}{(-2008)^0 + (-2)^1 + (-3)^2}$.