32. We assume $q > 0$. Using the notation $\lambda = q/L$ we note that the (infinitesimal) charge on an element dx of the rod contains charge $dq = \lambda\, dx$. By symmetry, we conclude that all horizontal field components (due to the dq's) cancel and we need only "sum" (integrate) the vertical components. Symmetry also allows us to integrate these contributions over only half the rod ($0 \le x \le L/2$) and then simply double the result. In that regard we note that $\sin\theta = R/r$ where $r = \sqrt{x^2 + R^2}$.

(a) Using Eq. 22-3 (with the 2 and $\sin\theta$ factors just discussed) the magnitude is

$$\left|\vec{E}\right| = 2\int_0^{L/2}\left(\frac{dq}{4\pi\varepsilon_0 r^2}\right)\sin\theta = \frac{2}{4\pi\varepsilon_0}\int_0^{L/2}\left(\frac{\lambda\, dx}{x^2+R^2}\right)\left(\frac{y}{\sqrt{x^2+R^2}}\right)$$

$$= \frac{\lambda R}{2\pi\varepsilon_0}\int_0^{L/2}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{(q/L)R}{2\pi\varepsilon_0}\cdot\left.\frac{x}{R^2\sqrt{x^2+R^2}}\right|_0^{L/2}$$

$$= \frac{q}{2\pi\varepsilon_0 LR}\frac{L/2}{\sqrt{(L/2)^2+R^2}} = \frac{q}{2\pi\varepsilon_0 R}\frac{1}{\sqrt{L^2+4R^2}}$$

where the integral may be evaluated by elementary means or looked up in Appendix E (item #19 in the list of integrals). With $q = 7.81\times10^{-12}$ C , $L = 0.145$ m and $R = 0.0600$ m, we have $|\vec{E}| = 12.4$ N/C .

(b) As noted above, the electric field $\vec{E}$ points in the $+y$ direction, or $+90°$ counterclockwise from the $+x$ axis.