

66. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocities (and angles) in this problem are positive. Mechanical energy conservation applied to the particle (before impact) leads to

$$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$$

for its speed right before undergoing the completely inelastic collision with the rod. The collision is described by angular momentum conservation:

$$mvd = (I_{\text{rod}} + md^2)\omega$$

where  $I_{\text{rod}}$  is found using Table 10-2(e) and the parallel axis theorem:

$$I_{\text{rod}} = \frac{1}{12}Md^2 + M\left(\frac{d}{2}\right)^2 = \frac{1}{3}Md^2.$$

Thus, we obtain the angular velocity of the system immediately after the collision:

$$\omega = \frac{md\sqrt{2gh}}{(Md^2/3) + md^2}$$

which means the system has kinetic energy  $(I_{\text{rod}} + md^2)\omega^2/2$  which will turn into potential energy in the final position, where the block has reached a height  $H$  (relative to the lowest point) and the center of mass of the stick has increased its height by  $H/2$ . From trigonometric considerations, we note that  $H = d(1 - \cos\theta)$ , so we have

$$\frac{1}{2}(I_{\text{rod}} + md^2)\omega^2 = mgH + Mg\frac{H}{2} \Rightarrow \frac{1}{2} \frac{m^2d^2(2gh)}{(Md^2/3) + md^2} = \left(m + \frac{M}{2}\right)gd(1 - \cos\theta)$$

from which we obtain

$$\begin{aligned} \theta &= \cos^{-1}\left(1 - \frac{m^2h}{(m + M/2)(m + M/3)}\right) = \cos^{-1}\left(1 - \frac{h/d}{(1 + M/2m)(1 + M/3m)}\right) \\ &= \cos^{-1}\left(1 - \frac{(20 \text{ cm}/40 \text{ cm})}{(1+1)(1+2/3)}\right) = \cos^{-1}(0.85) \\ &= 32^\circ. \end{aligned}$$