

20. (a) With energy in Joules and length in meters, we have

$$\Delta U = U(x) - U(0) = -\int_0^x (6x' - 12)dx' .$$

Therefore, with $U(0) = 27$ J, we obtain $U(x)$ (written simply as U) by integrating and rearranging:

$$U = 27 + 12x - 3x^2 .$$

(b) We can maximize the above function by working through the $dU/dx = 0$ condition, or we can treat this as a force equilibrium situation — which is the approach we show.

$$F = 0 \Rightarrow 6x_{eq} - 12 = 0$$

Thus, $x_{eq} = 2.0$ m, and the above expression for the potential energy becomes $U = 39$ J.

(c) Using the quadratic formula or using the polynomial solver on an appropriate calculator, we find the negative value of x for which $U = 0$ to be $x = -1.6$ m.

(d) Similarly, we find the positive value of x for which $U = 0$ to be $x = 5.6$ m