

30. To solve the problem, we note that the first derivative of the function with respect to time gives the rate. Setting the rate to zero gives the time at which an extreme value of the variable mass occurs; here that extreme value is a maximum.

(a) Differentiating $m(t) = 5.00t^{0.8} - 3.00t + 20.00$ with respect to t gives

$$\frac{dm}{dt} = 4.00t^{-0.2} - 3.00.$$

The water mass is the greatest when $dm/dt = 0$, or at $t = (4.00/3.00)^{1/0.2} = 4.21$ s.

(b) At $t = 4.21$ s, the water mass is

$$m(t = 4.21 \text{ s}) = 5.00(4.21)^{0.8} - 3.00(4.21) + 20.00 = 23.2 \text{ g}.$$

(c) The rate of mass change at $t = 2.00$ s is

$$\begin{aligned} \left. \frac{dm}{dt} \right|_{t=2.00 \text{ s}} &= [4.00(2.00)^{-0.2} - 3.00] \text{ g/s} = 0.48 \text{ g/s} = 0.48 \frac{\text{g}}{\text{s}} \cdot \frac{1 \text{ kg}}{1000 \text{ g}} \cdot \frac{60 \text{ s}}{1 \text{ min}} \\ &= 2.89 \times 10^{-2} \text{ kg/min.} \end{aligned}$$

(d) Similarly, the rate of mass change at $t = 5.00$ s is

$$\begin{aligned} \left. \frac{dm}{dt} \right|_{t=5.00 \text{ s}} &= [4.00(5.00)^{-0.2} - 3.00] \text{ g/s} = -0.101 \text{ g/s} = -0.101 \frac{\text{g}}{\text{s}} \cdot \frac{1 \text{ kg}}{1000 \text{ g}} \cdot \frac{60 \text{ s}}{1 \text{ min}} \\ &= -6.05 \times 10^{-3} \text{ kg/min.} \end{aligned}$$