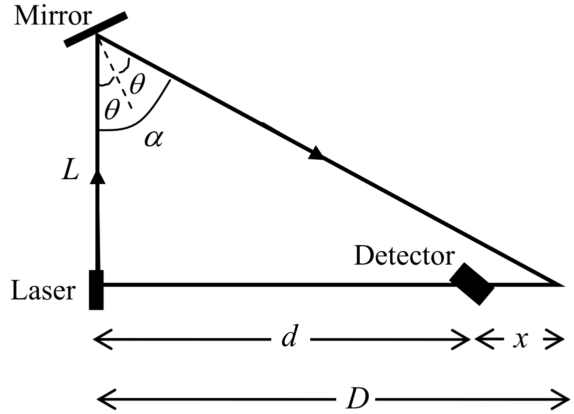
10. ***REASONING***

a. The incident ray from the laser travels a distance of $L = 50.0$ km due north to the mirror (see the drawing, which is *not* to scale). The angle θ we seek, that between the normal to the surface of the mirror and due south, is the angle of incidence. The angle of incidence must equal the angle of reflection, so the angle α between the incident and reflected rays is bisected by the normal to the surface of the mirror. Therefore, we have that

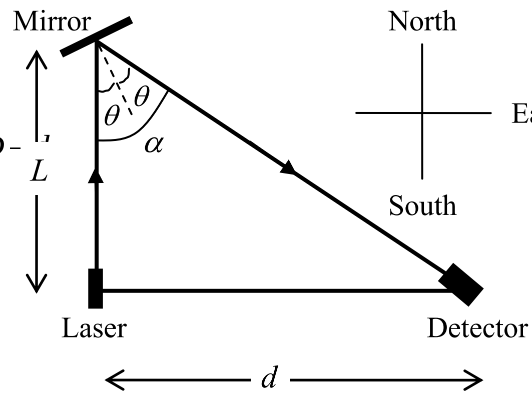


$$\theta = \tfrac{1}{2}\alpha \tag{1}$$

Because the laser, the mirror, and the detector sit at the corners of a right triangle, we will use $\alpha = \tan^{-1}(d/L)$ (Equation 1.6) to determine the angle α, where $d = 117$ m is the distance between the laser and the detector.

b. Turning the surface normal too far from due south increases both the angle of incidence and the angle of reflection, in this case by 0.004° each. Thus, the angle α between the incident and reflected rays increases by twice that amount. We will use the new angle α, with $\tan\alpha = D/L$ (Equation 1.3), to determine the distance D between the laser and the reflected beam (see the drawing). The distance x by which the reflected beam misses the detector is the distance D minus the distance d between the laser and the detector:

$$x = D - d \tag{2}$$



We note that the distance d is given in meters, while the distance L is given in kilometers, where 1 km = 10^3 m.

SOLUTION

a. Substituting $\alpha = \tan^{-1}(d/L)$ (Equation 1.6) into Equation (1) yields

$$\theta = \tfrac{1}{2}\alpha = \tfrac{1}{2}\tan^{-1}\left(\frac{d}{L}\right) = \tfrac{1}{2}\tan^{-1}\left(\frac{117\ \text{m}}{50.0\times10^{3}\ \text{m}}\right) = \boxed{0.0670^\circ}$$

b. When the mirror is misaligned, the angle of incidence is larger than the result found in part (a) by 0.004°. From Equation (1), then, the angle α between the incident and reflected rays becomes

$$\alpha = 2\theta = 2\left(0.0670^\circ + 0.004^\circ\right) = 0.142^\circ$$

Solving $\tan\alpha = D/L$ (Equation 1.3) for the distance D between the laser and the reflected beam, we obtain

$$D = L\tan\alpha \tag{3}$$

Substituting Equation (3) into Equation (2), and using $\alpha = 0.142^\circ$, we find that

$$x = D - d = L\tan\alpha - d = \left(50.0\times10^3\ \text{m}\right)\tan 0.142^\circ - 117\ \text{m} = \boxed{7\ \text{m}}$$
