MAPLE IN 172 CLASS

Antiderivatives.

We use the command "int" for finding antiderivatives.

```
> int(x^2,x);
```

$$\frac{x^3}{3} \tag{1}$$

If we want to see the integral on the screen before finding the antiderivative, we can use the command "Int"

```
> Int(sqrt(x^2+x+1),x);
```

$$\int \sqrt{x^2+x+1}\, dx \tag{2}$$

```
> int(sqrt(x^2+x+1),x);
```

$$\frac{(2x+1)\sqrt{x^2+x+1}}{4} + \frac{3}{8}\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right) \tag{3}$$

Pay attention that

```
> arcsinh(x)=ln(x+sqrt(x^2+1));
```

$$\operatorname{arcsinh}(x) = \ln\left(x+\sqrt{x^2+1}\right) \tag{4}$$

Two examples when MAPLE does not recognize a possibility of substitution.

```
> Int(x*(x^2+1)^20,x);
```

$$\int x\left(x^2+1\right)^{20} dx \tag{5}$$

```
> int(x*(x^2+1)^20,x);
```

$$\frac{1}{42}x^{42}+\frac{1}{2}x^{40}+5x^{38}+\frac{95}{3}x^{36}+\frac{285}{2}x^{34}+\frac{969}{2}x^{32}+1292x^{30}+\frac{19380}{7}x^{28}+4845x^{26} + \frac{20995}{3}x^{24}+8398x^{22}+8398x^{20}+\frac{20995}{3}x^{18}+4845x^{16}+\frac{19380}{7}x^{14}+1292x^{12} + \frac{969}{2}x^{10}+\frac{285}{2}x^{8}+\frac{95}{3}x^{6}+5x^{4}+\frac{1}{2}x^{2} \tag{6}$$

We have to help MAPLE by telling it what substitution to use.

First we call the package "student". We have to do it only once during a session.

```
> with(student):
> changevar(x^2+1=u,Int((x^2+1)^20*x,x),u);
```

$$\int \frac{u^{20}}{2}\, du \tag{7}$$

```
> int(u^20/2,u);
```

$$\frac{u^{21}}{42} \quad (8)$$

```
> u^21/42 = (x^2+1)^21/42;
```

$$\frac{u^{21}}{42} = \frac{(x^2+1)^{21}}{42} \quad (9)$$

In the example above MAPLE at least was able to evaluate the integral though not in the best possible way.
But let us look at the integral

```
> Int(arcsin(x)^2,x);
```

$$\int \arcsin(x)^2 \, dx \quad (10)$$

```
> int(arcsin(x)^2,x);
```

$$\arcsin(x)^2 x - 2x + 2 \arcsin(x) \sqrt{1-x^2} \quad (11)$$

MAPLE tells us that it cannot evaluate this integral. Let us perform a substitution.

```
> changevar(arcsin(x) = u,Int(arcsin(x)^2,x,u));
```

$$\int u^2 \sqrt{1-\sin(u)^2} \, du \quad (12)$$

Still not good. If we try to evaluate it

```
> int(u^2*(1-sin(u)^2)^(1/2),u);
```

$$-\frac{\frac{1}{2} \mathrm{I} \left(2 + 2 \mathrm{I} u - u^2 - 2 \mathrm{e}^{2 \mathrm{I} u} + 2 \mathrm{I} \mathrm{e}^{2 \mathrm{I} u} u + u^2 \mathrm{e}^{2 \mathrm{I} u}\right)}{\sqrt{\mathrm{e}^{2 \mathrm{I} u}}} \quad (13)$$

the answer is not much of a help. But if we help MAPLE again by noticing that

```
> sqrt(1-sin(u)^2)=cos(u);
```

$$\sqrt{1-\sin(u)^2} = \cos(u) \quad (14)$$

then finally MAPLE can do it

```
> int(cos(u)*u^2,u);
```

$$u^2 \sin(u) - 2 \sin(u) + 2 u \cos(u) \quad (15)$$

PARTIAL FRACTIONS

Suppose we want to represent the rational function

```
> f:=x->(x^5 +3*x^4+x^2+3)/((x-1)^2*(x^2+x+1)^2);
```

$$f := x \to \frac{x^5+3\,x^4+x^2+3}{(x-1)^2\,(x^2+x+1)^2} \qquad (16)$$

as a sum of partial fractions.

```
> convert(f(x),parfrac,x);
```

$$\frac{8}{9\,(x-1)^2} + \frac{1}{3\,(x-1)} + \frac{22+6\,x}{9\,(x^2+x+1)} + \frac{x}{3\,(x^2+x+1)^2} \qquad (17)$$

But if

```
> g:=x->1/(x^4+1);
```

$$g := x \to \frac{1}{x^4+1} \qquad (18)$$

and we try

```
> convert(g(x), parfrac, x);
```

$$\frac{1}{x^4+1} \qquad (19)$$

we see that MAPLE cannot do it without our help.

```
> convert(g(x),parfrac,x,sqrt(2));
```

$$\frac{2+x\sqrt{2}}{4\,(x^2+x\sqrt{2}+1)} + \frac{2-x\sqrt{2}}{4\,(x^2-x\sqrt{2}+1)} \qquad (20)$$

Of course, we can integrate g directly with the command int

```
> int(g(x),x);
```

$$\frac{1}{8}\sqrt{2}\,\ln\left(\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}\right) + \frac{1}{4}\sqrt{2}\,\arctan\left(x\sqrt{2}+1\right) + \frac{1}{4}\sqrt{2}\,\arctan\left(x\sqrt{2}-1\right) \qquad (21)$$

Integration by parts. In the command intparts below we write first the integral and then the function we take as "u".

```
> intparts(Int(2^x*sin(x),x),2^x);
```

$$-2^x\cos(x) - \int \left(-2^x \ln(2)\cos(x)\right) \mathrm{d}x \qquad (22)$$

```
> intparts(Int(x^n*exp(x),x),x^n);
```

$$x^n\,\mathrm{e}^x - \int \frac{x^n\,n\,\mathrm{e}^x}{x}\,\mathrm{d}x \qquad (23)$$

An example of the Weierstrass' rationalizing substitution.

```
> Int((sin(x)-3*cos(x))/(2*sin(x)+cos(x)),x);
```

$$\int \frac{\sin(x)-3\cos(x)}{2\sin(x)+\cos(x)}\,\mathrm{d}x \qquad (24)$$

```
> changevar(tan(x/2)=u,Int((sin(x)-3*cos(x))/(2*sin(x)+cos(x)),x),u)
;
```

$$\int \frac{2\sin(2\arctan(u))-6\cos(2\arctan(u))}{(2\sin(2\arctan(u))+\cos(2\arctan(u)))\left(1+u^2\right)}\,\mathrm{d}u \tag{25}$$

as we see MAPLE does not help us with the susstitution, but when it evaluates the antiderivative directly it makes use of it.

```
> int((sin(x)-3*cos(x))/(2*sin(x)+cos(x)),x);
```

$$\frac{7}{5}\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)-\frac{7}{5}\ln\left(-4\tan\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)^2-1\right)-\frac{x}{5} \tag{26}$$

```
>
```