Math 172

Review for the final exam.

1. Find the integrals.(32)
   (a) \( \int x \tan^{-1} x \, dx \);
   (b) \( \int \frac{1}{t^2+2t+2} \, dt \);
   (c) \( \int \cos^3 x \sin^{1/2} x \, dx \);
   (d) \( \int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}} \, dx \).

2. Find the improper integral \( \int_2^\infty \frac{1}{x^{1/2}-1} \, dx \).(10)

3. Find the area of the region enclosed by an ellipse,
   \( x = 3 \sin(t), y = 4 \cos(t), 0 \leq t \leq 2\pi \).(10)

4. Find the volume of the solid generated when the region bounded by
   \( y = 2 - x, y = \sqrt{x} \) and \( x = 0 \) is revolved about x-axis.(10)

5. Find the coordinates of the center of mass for the region described in the
   previous problem. (10)

6. Determine whether the series absolutely converges conditionally
   converges or diverges.(20)
   (a) \( \sum_{k=2}^{\infty} \frac{1}{\sqrt{k^6-4k}} \);
   (b) \( \sum_{k=1}^{\infty} (-1)^k \frac{k}{k^2+2} \);
   (c) \( \sum_{k=1}^{\infty} (-1)^k \frac{\ln k}{k} \);
   (d) \( \sum_{k=1}^{\infty} \frac{\tan^{-1} k}{k} \).

7. Find the function to which the series \( \sum_{k=1}^{\infty} (k - 1)x^{k+1} \) converges.(6)

8. Find the Taylor series about \( x = a \) for the given function; express your
   answer in sigma notation(\( \Sigma \)); then find its radius of convergence and the
   interval of convergence.(12)
   (a) \( f(x) = \frac{1}{x^2}, \) at 0;
   (b) \( f(x) = \ln x, \) at 2.