171 Calculus 1 Review 2

In the following problems find the derivative. Simplify if possible.

1. $f(x)=3^{2x}\csc(5x)$.
We combine the product and the chain rules and use the formulas
$\dfrac{d(a^x)}{dx}=a^x\ln a$ and $\dfrac{d(\csc x)}{dx}=-\csc x\cot x$. Thus we obtain

$$\frac{df}{dx}=3^{2x}\times\ln 3\times 2\csc(5x)-3^{2x}\csc(5x)\cot(5x)\times 5=3^{2x}\csc(5x)[2\ln 3-5\cot(5x)].$$

2. $f(x)=\dfrac{x^4-x^2+1}{x^4+x^2+1}$. We use the quotient rule $\left(\dfrac{f}{g}\right)'=\dfrac{f'g-fg'}{g^2}$.

$$f'(x)=\frac{(4x^3-2x)(x^4+x^2+1)-(x^4-x^2+1)(4x^3+2x)}{(x^4+x^2+1)^2}=$$
$$=\frac{4x^7+4x^5+4x^3-2x^5-2x^3-2x-4x^7+4x^5-4x^3-2x^5+2x^3-2x}{(x^4+x^2+1)^2}=$$
$$=\frac{4x^5-4x}{(x^4+x^2+1)^2}=\frac{4x(x^4-1)}{(x^4+x^2+1)^2}.$$

3. $f(x)=\sqrt{\cot(e^{\sin x})}$. The problem is on the application of the chain rule.

$$\frac{df}{dx}=\frac{1}{2\sqrt{\cot(e^{\sin x})}}(-\csc^2(e^{\sin x}))e^{\sin x}\cos x.$$

4. $f(x)=\dfrac{\ln x+\sin x}{\ln x+\cos x}$. Like in Problem 2 we apply the quotient rule.

$$\frac{df}{dx}=\frac{(1/x+\cos x)(\ln x+\cos x)-(\ln x+\sin x)(1/x-\sin x)}{(\ln x+\cos x)^2}=$$
$$=\frac{(1/x)\ln x+(1/x)\cos x+\cos x\ln x+\cos^2 x-(1/x)\ln x+\sin x\ln x-\sin x(1/x)+\sin^2 x}{(\ln x+\cos x)^2}=$$
$$=\frac{(1/x)(\cos x-\sin x)+\ln x(\cos x+\sin x)+1}{(\ln x+\cos x)^2}$$

5. Use implicit differentiation to find the slope-intercept equation of the tangent line to the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ at the point $(1, 3\sqrt{3}/2)$.

After differentiating both parts of the relation we get

$\frac{x}{2}+\frac{2y}{9}\frac{dy}{dx}=0$. Plugging in $x=1$ and $y=3\sqrt{3}/2$ we obtain $\frac{1}{2}+\frac{\sqrt{3}}{3}\frac{dy}{dx}=0$ whence $\frac{dy}{dx}(1,3\sqrt{3}/2)=-\frac{\sqrt{3}}{2}$. An equation of the tangent line at the point $(1,3\sqrt{3}/2)$ in the point-slope form is $y-\frac{3\sqrt{3}}{2}=-\frac{\sqrt{3}}{2}(x-1)$ whence the slope-intercept form is $y=-\frac{\sqrt{3}}{2}x+2\sqrt{3}$.

6. $f(x)=\frac{x^6(x-1)^8}{(x-2)^{10}(x-3)^{12}}$. We use the formula for logarithmic differentiation $\frac{df}{dx}=f(x)\frac{d(\ln(f(x)))}{dx}$. We have $\ln(f(x))=6\ln x+8\ln(x-1)-10\ln(x-2)-12\ln(x-3)$ whence $\frac{d(\ln(f(x)))}{dx}=\frac{6}{x}+\frac{8}{x-1}-\frac{10}{x-2}-\frac{12}{x-3}=-\frac{4(2x^3-15x+9)}{x(x-1)(x-2)(x-3)}$

Finally,

$$\frac{df}{dx}=-\frac{4x^5(x-1)^7(2x^3-15x+9)}{(x-2)^{11}(x-3)^{13}}.$$