

## Calculus with MAPLE in 171 class. Session 2.

We start with an optimization problem.

Problem 58, page 339.

The intensity of light at point P is given by the expression

```
> In:=x->1/(x^2+d^2) + 1/((10-x)^2+d^2);
```

$$In := x \rightarrow \frac{1}{x^2 + d^2} + \frac{1}{(10-x)^2 + d^2} \quad (1)$$

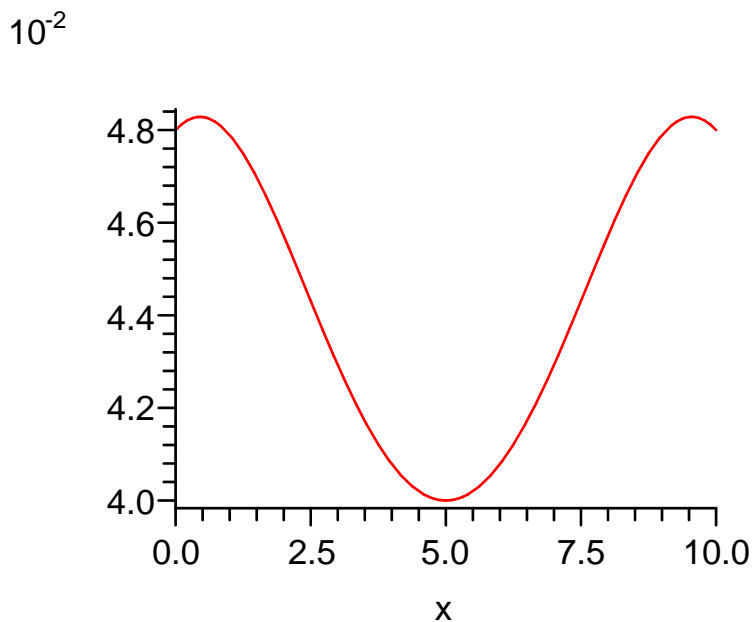
Indeed, by the Pithagoras' theorem the denominators in the expression above represent the squares of the distances from the first and the second source of light to P, respectively.

We will now graph  $In(x)$  when the distance  $d$  is equal to 5.

```
> d:=5;
```

$$d := 5 \quad (2)$$

```
> plot(In(x), x=0..10);
```

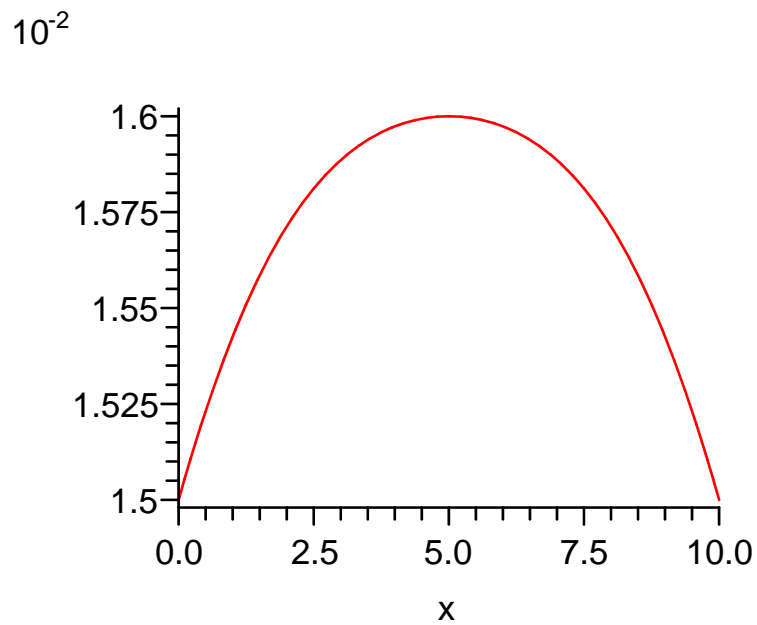


And if  $d = 10$ ,

```
> d:=10;
```

$$d := 10 \quad (3)$$

```
> plot(In(x), x=0..10);
```



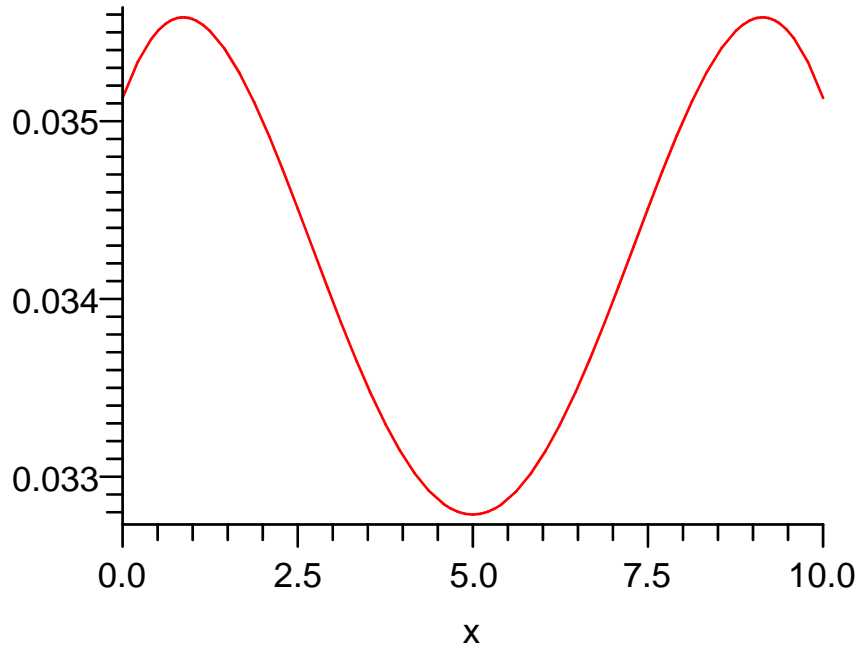
so in the first case the intensity is the smallest at the equal distance from two sources, but in the second it is the greatest at a similar point. Let us experiment with the value of  $d$  a little bit more.

```
> d:=6;
```

```
d:=6
```

(4)

```
> plot(In(x), x=0..10);
```

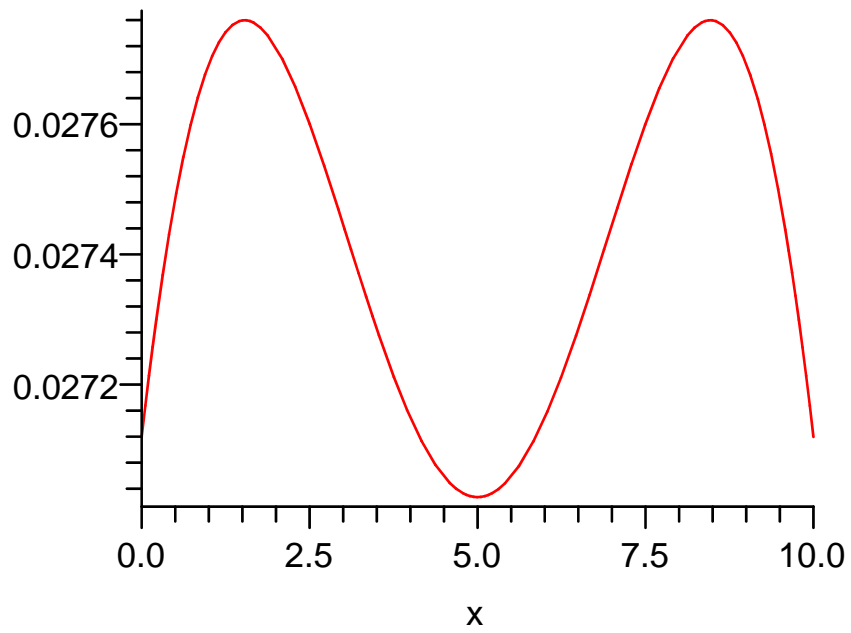


```
> d:=7;
```

*d:=7*

(5)

```
> plot(In(x),x=0..10);
```



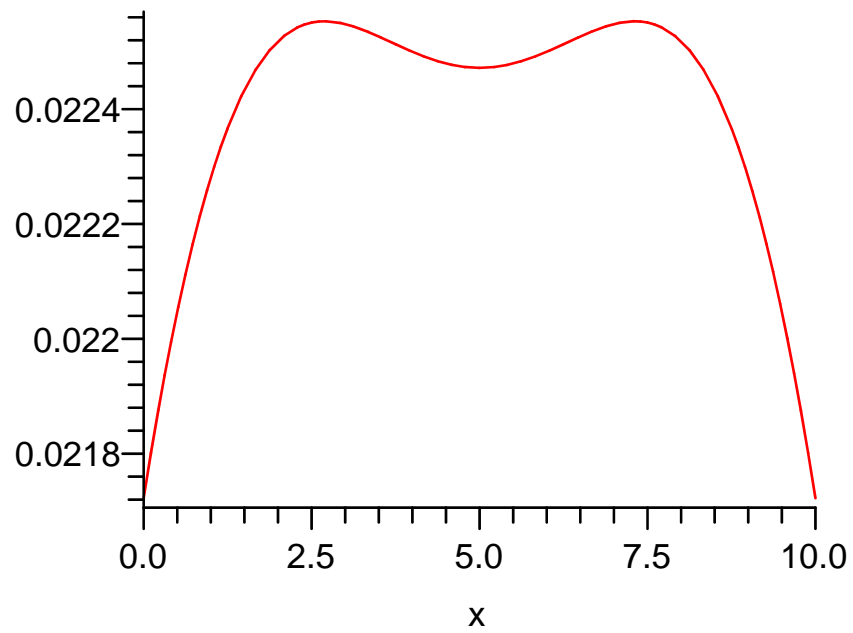
```
> d:=8;
```

(6)

$d := 8$

(6)

```
> plot(In(x), x=0..10);
```

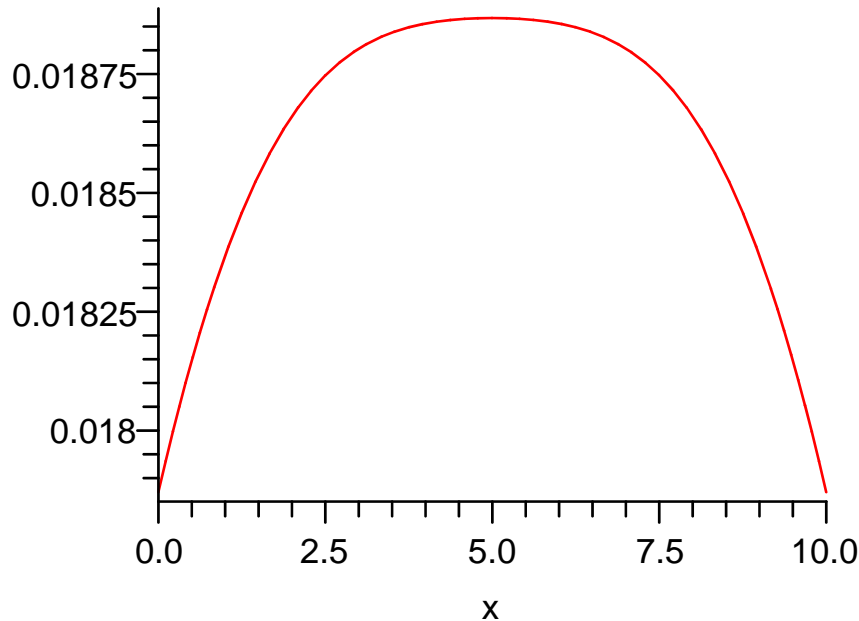


```
> d:=9;
```

$d := 9$

(7)

```
> plot(In(x), x=0..10);
```



So, the change happens for some  $d$  between 8 and 9. To analyze what happens in more details notice that for  $d = 8$  the function  $\ln(x)$  has three critical points, but for  $d = 9$  it has just one critical point,  $x=5$ . Therefore when  $d$  takes the transition value three critical points merge into one ( $x=5$ ) which will be a root of the first derivative of multiplicity at least 3. So we will look at the first derivative of  $\ln(x)$ . First, because we assigned to  $d$  some numerical value, we have to change the letter in the definition of  $\ln(x)$ .

**> In:=x->1/(x^2+s^2) + 1/((10-x)^2+s^2);**

$$\ln := x \rightarrow \frac{1}{x^2+s^2} + \frac{1}{(10-x)^2+s^2} \quad (8)$$

The first derivative.

**> diff(In(x), x);**

$$-\frac{2x}{(x^2+s^2)^2} - \frac{-20+2x}{((10-x)^2+s^2)^2} \quad (9)$$

**> simplify(%);**

$$\frac{4(5000x-2000x^2+300x^3+100xs^2-25x^4-30x^2s^2+x^5+2x^3s^2+xs^4-5s^4)}{(x^2+s^2)^2(100-20x+x^2+s^2)^2} \quad (10)$$

Let us factor the denominator. **factor(5000\*x-2000\*x^2+300\*x^3+100\*x\*s^2-25\*x^4-30\*x^2\*s^2+x^5+2\*x^3\*s^2+x\*s^4-5\*s^4);**

(11)

$$(x-5) (x^4-20x^3+200x^2+2x^2s^2-1000x-20xs^2+s^4) \quad (11)$$

Now notice that when  $s$  takes the transition value  $x=5$  is a root of the second factor. So we can find the transition value of  $s$  in the following way.

> `x:=5;`

$$x := 5 \quad (12)$$

> `solve(x^4-20*x^3+200*x^2+2*x^2*s^2-1000*x-20*x*s^2+s^4,s);`

$$5 \text{ I, } -5 \text{ I, } 5\sqrt{3}, -5\sqrt{3} \quad (13)$$

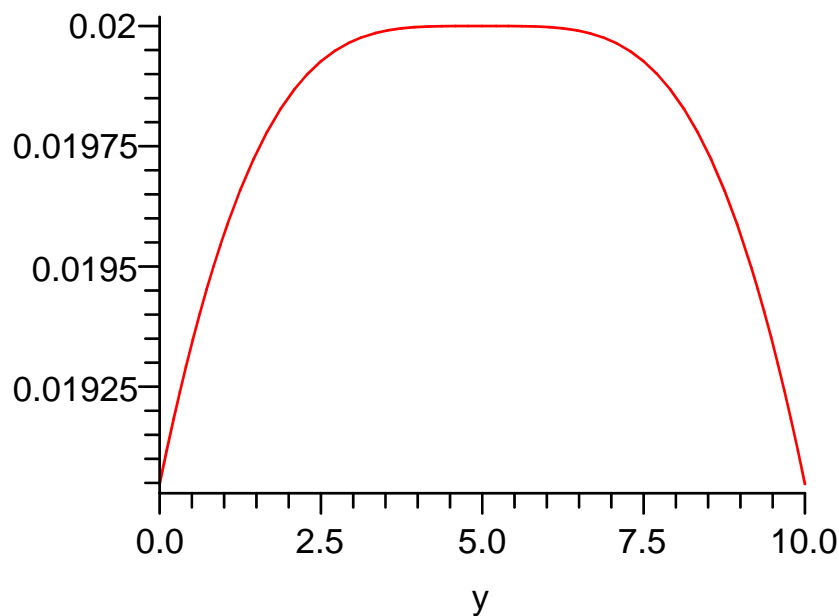
Only the positive solution

> `s:=5*sqrt(3);`

$$s := 5\sqrt{3} \quad (14)$$

makes sense. Let us look at the graph of  $\ln(x)$  for this transition value.

> `plot(ln(y),y=0..10);`



Now some examples with integrals.

> `int((y^2+1)^20*y,y);`

$$\begin{aligned} & \frac{1}{42} y^{42} + \frac{1}{2} y^{40} + 5 y^{38} + \frac{95}{3} y^{36} + \frac{285}{2} y^{34} + \frac{969}{2} y^{32} + 1292 y^{30} + \frac{19380}{7} y^{28} + 4845 y^{26} \\ & + \frac{20995}{3} y^{24} + 8398 y^{22} + 8398 y^{20} + \frac{20995}{3} y^{18} + 4845 y^{16} + \frac{19380}{7} y^{14} + 1292 y^{12} \\ & + \frac{969}{2} y^{10} + \frac{285}{2} y^8 + \frac{95}{3} y^6 + 5 y^4 + \frac{1}{2} y^2 \end{aligned} \quad (15)$$

As we see MAPLE does not recognize possibility to make a simple substitution. We can make substitutions with MAPLE if we tell it what to do

```
> with(student) :
> changevar (y^2+1=u, Int ( (y^2+1)^20*y, y) , u) ;
```

$$\int \frac{1}{2} u^{20} du \quad (16)$$

Similarly we change variable in definite integrals.

```
> changevar (sin (y)=u, Int (y*arcsin (y) , y=-1..1) , u) ;
```

$$\int_{-\sin(1)}^{\sin(1)} \frac{\arcsin(u) \arcsin(\arcsin(u))}{\sqrt{1-u^2}} du \quad (17)$$

Now an example on computing area between curves. We will find the area between

```
> y1:=t->t^3-3*t+1;
```

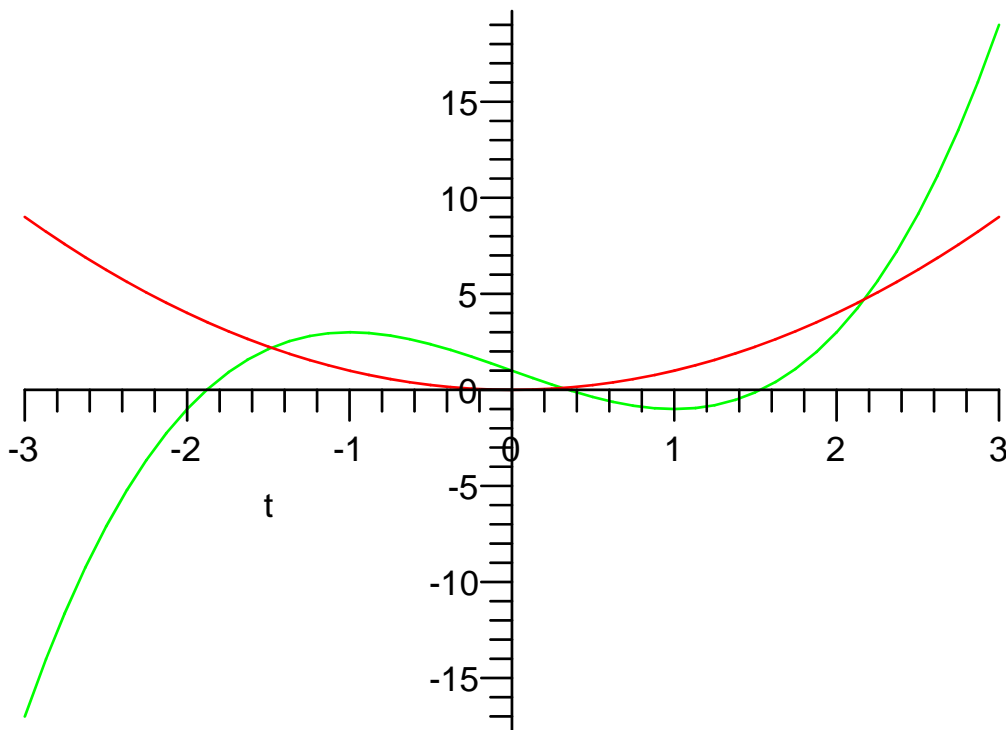
$$y1 := t \rightarrow t^3 - 3t + 1 \quad (18)$$

and

```
> y2:=t^2;
```

$$y2 := t^2 \quad (19)$$

```
> plot ({y1 (t) , y2 (t) } , t=-3..3) ;
```



To find points of intersection we perform the command

```
> fsolve(t^3-3*t+1=t^2);  
      -1.481194304, 0.3111078175, 2.170086487
```

(20)

```
> Area:=int(abs(y1(t)-y2(t)),t=-1.481194304..2.170086487);  
      Area:=5.558024874
```

(21)