MAPLE SESSION in 171 CLASS

arithmetic: MAPLE can perform arithmetic operations presizely

```
> 1/2 + 1/7;
```

$$\frac{9}{14} \quad (1)$$

(Pay attention that every command in MAPLE must end by **;)**

or approximately

```
> evalf(1/2+1/7);
```

$$0.6428571429 \quad (2)$$

By default the command "evalf" performs calculations with accuracy 10 digits after decimal point but it can be easily changed. For example the command

```
> evalf(Pi,50);
```

$$3.1415926535897932384626433832795028841971693993751 \quad (3)$$

evaluates π

$$\pi \quad (4)$$

with 50 digits after decimal points.

Solving equations. Among many commands of "solve" family MAPLE has commands "solve" for solving equations exactly (if possible) like

```
> solve(x^2+x-1,x);
```

$$-\frac{1}{2}+\frac{1}{2}\sqrt{5}, -\frac{1}{2}-\frac{1}{2}\sqrt{5} \quad (5)$$

and "fsolve" for solving them approximately

```
> fsolve(x^2+x-1,x);
```

$$-1.618033989, 0.6180339887 \quad (6)$$

The command "fsolve" finds all real solutions of polynomial equtions, for other types of equations we have to specify the interval where MAPLE will check for solutions.

```
> fsolve(cos(x)^3 + sin(x), x=Pi/2..Pi);
```

$$2.542825948 \quad (7)$$

If the screen remains empty it means the equation we are trying to solve has no real solutions in the specified interval.
Another example.

```
> fsolve(x^3+x+1);
```

$$-.6823278038 \quad (8)$$

But

```
> solve(x^3+x+1);
```

$$-\frac{1}{6}\left(108+12\sqrt{93}\right)^{1/3}+\frac{2}{\left(108+12\sqrt{93}\right)^{1/3}}, \quad (9)$$

$$\frac{1}{12}\left(108+12\sqrt{93}\right)^{1/3}-\frac{1}{\left(108+12\sqrt{93}\right)^{1/3}}+\frac{1}{2}\,I\sqrt{3}\left(-\frac{1}{6}\left(108+12\sqrt{93}\right)^{1/3}-\frac{2}{\left(108+12\sqrt{93}\right)^{1/3}}\right),$$

$$\frac{1}{12}\left(108+12\sqrt{93}\right)^{1/3}-\frac{1}{\left(108+12\sqrt{93}\right)^{1/3}}-\frac{1}{2}\,I\sqrt{3}\left(-\frac{1}{6}\left(108+12\sqrt{93}\right)^{1/3}-\frac{2}{\left(108+12\sqrt{93}\right)^{1/3}}\right)$$

MAPLE applies so called "Cardano Formulas"

Algebra with MAPLE

```
> factor(x^6+1);
```

$$\left(x^2+1\right)\left(x^4-x^2+1\right) \qquad (10)$$

```
> simplify(1/(x^2-1) + 1/(x-1)^2);
```

$$\frac{2\,x}{(x-1)^2\,(1+x)} \qquad (11)$$

For the long division of polynomials MAPLE has two commands.

```
> quo(x^5 + 1, x^2 +1,x);
```

$$x^3-x \qquad (12)$$

for the quotient, and

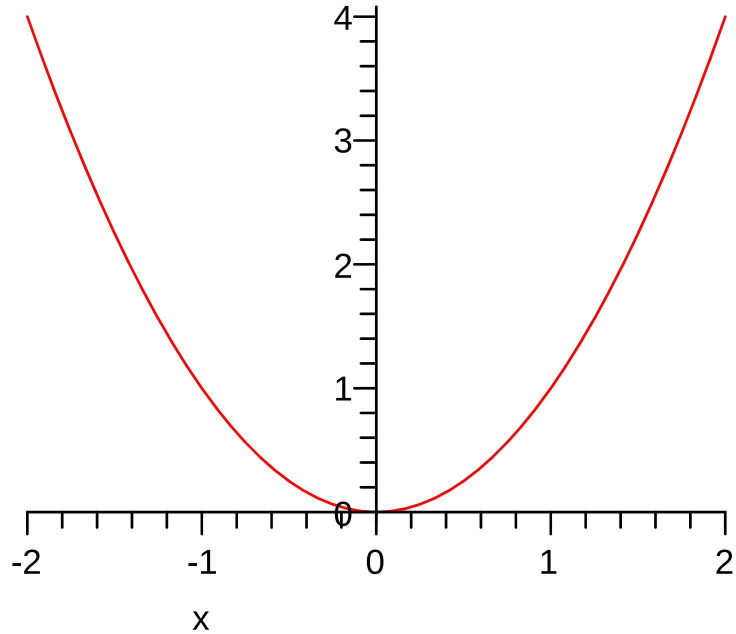
```
> rem(x^5+1, x^2+1,x);
```

$$1+x \qquad (13)$$

for the remainder.

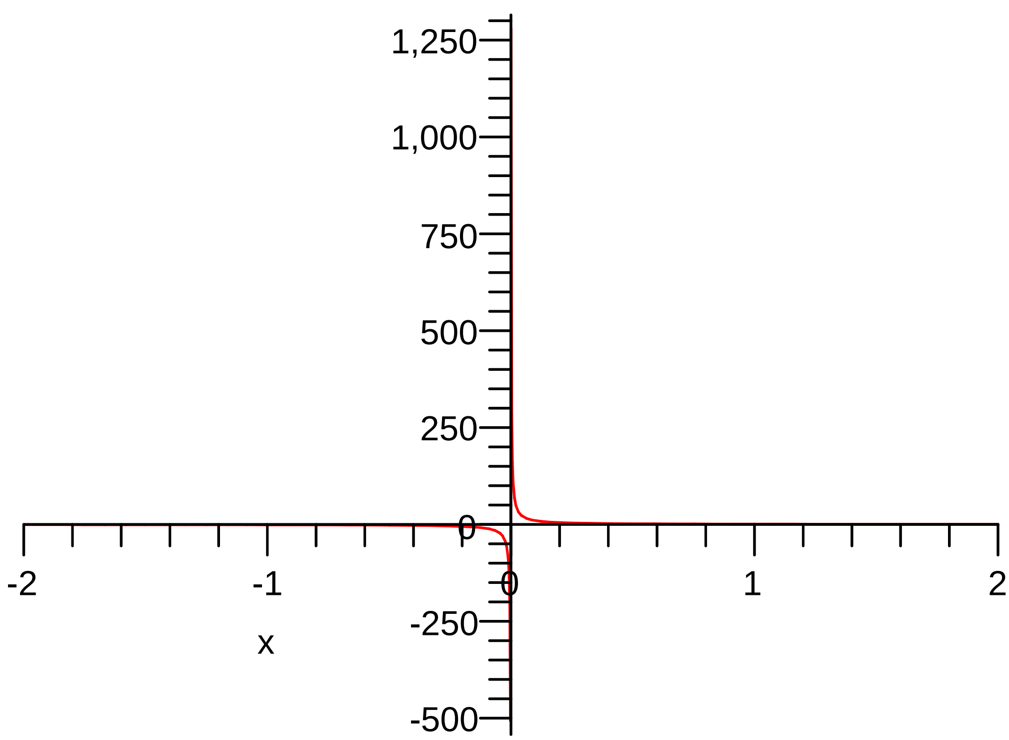
There are many commands in MAPLE for graphing. We will consider "plot" and "implicitplot"

```
> plot(x^2,x=-2..2);
```



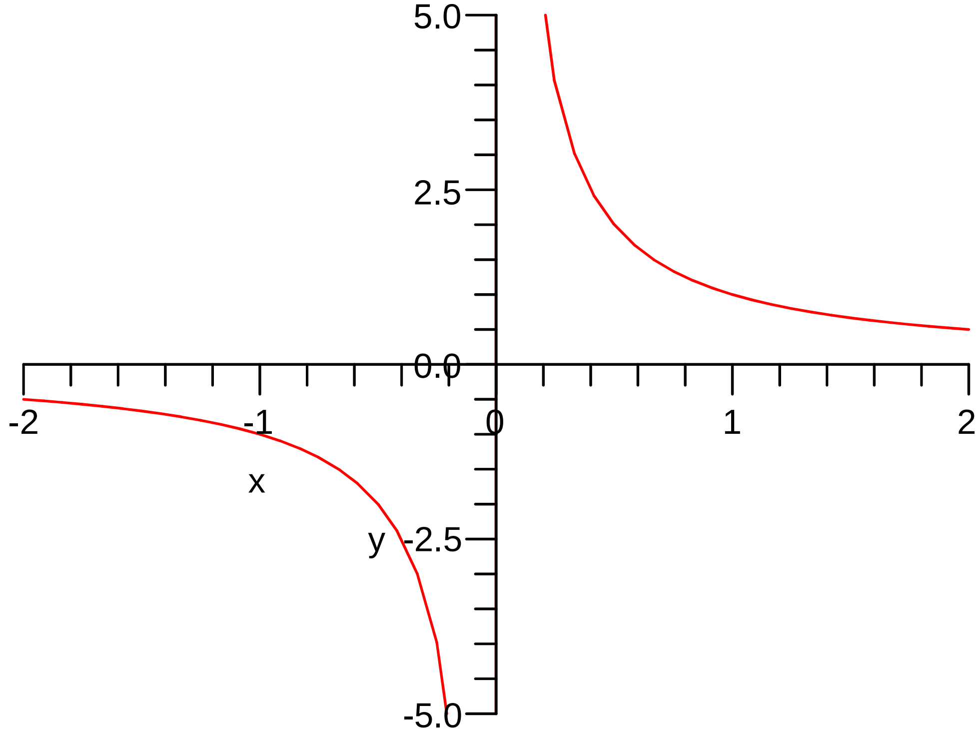
Range for x is compulsory, for y - optional, but if we try

```
> plot(1/x,x=-2..2);
```



we see practically only the vertical asymptote, thus it is better

```
> plot(1/x,x=-2..2,y=-5..5);
```



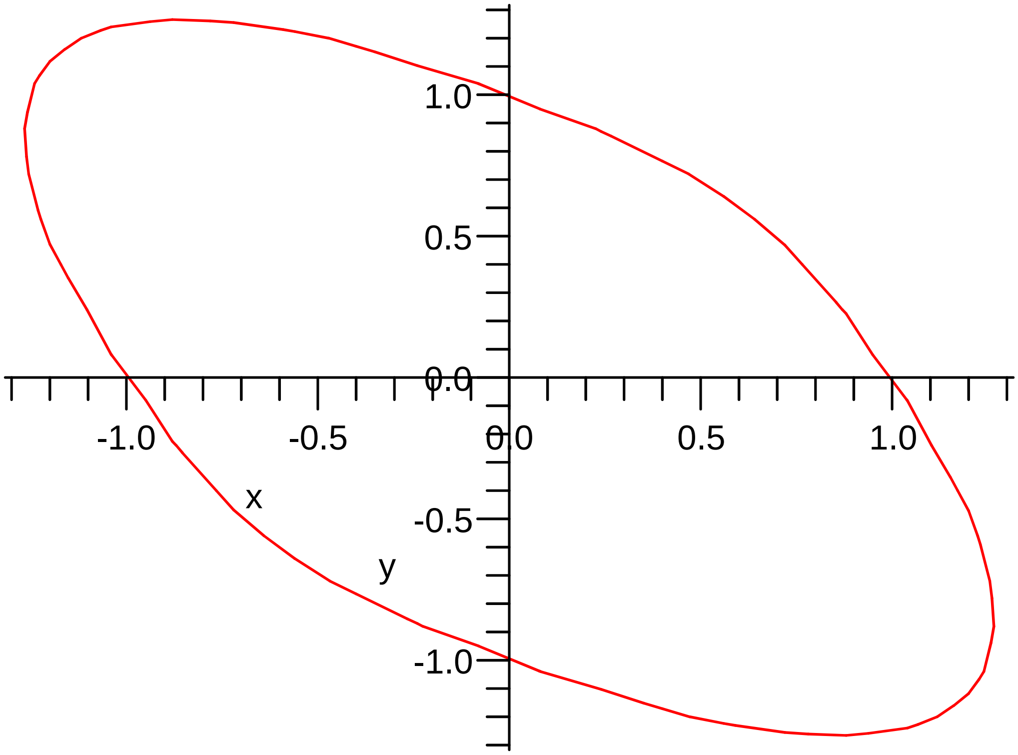
We have to experiment with ranges to get a graph we are satisfied with.

To use "implicitplot" we have first to call the package "plots"

```
> with(plots):
```

We have to do it only once during the MAPLE seccion.

```
> implicitplot(x^4+2*x*y +y^4 =1,x=-2..2,y=-2..2);
```



Pay attention that multiplication in MAPLE is indicated as * and we are not allowed to drop this sign.

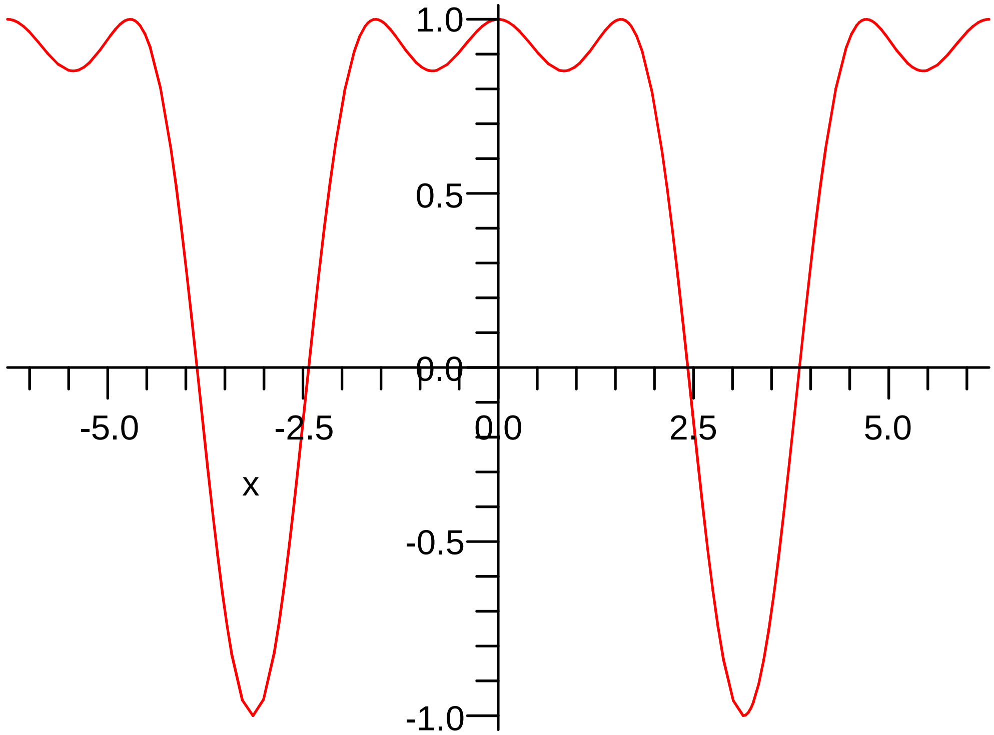
Functions.

```
> f:=x->sin(x)^2+cos(x)^3;
```

$$x \to \sin(x)^2 + \cos(x)^3 \tag{14}$$

Now f(x) will mean the expression to the right of the arrow. For example

```
> plot(f(x),x=-2*Pi..2*Pi);
```



Limits.

```
> limit((sin(x)-x)/x^3,x=0);
```

$$-\frac{1}{6} \tag{15}$$

```
> limit((1+1/x)^x,x=infinity);
```

$$e \tag{16}$$

Derivatives.

```
> diff(x^2,x);
```

$$2\,x \tag{17}$$

or

```
> diff(f(x),x);
```

$$2\sin(x)\cos(x)-3\cos(x)^2\sin(x) \tag{18}$$

For the second derivative.

```
> diff(f(x),x$2);
```

$$2\cos(x)^2-2\sin(x)^2+6\cos(x)\sin(x)^2-3\cos(x)^3 \tag{19}$$

```
> simplify(%);
```

$$4\cos(x)^2-9\cos(x)^3-2+6\cos(x) \tag{20}$$

sign % tells MAPLE to use the result of the previous command.

Now we will analyze and graph the function

```
> g:=x->(x^5+x+1)/(x^4-1);
```

$$x \rightarrow \frac{x^5+x+1}{x^4-1} \quad (21)$$

The x-intercept

```
> fsolve(x^5+x+1);
```

$$-.7548776662 \quad (22)$$

The slant asymptote

```
> quo(x^5+x+1,x^4-1,x);
```

$$x \quad (23)$$

The derivative

```
> diff(g(x),x);
```

$$\frac{5\,x^4+1}{x^4-1} - \frac{4\,(x^5+x+1)\,x^3}{(x^4-1)^2} \quad (24)$$

```
> simplify(%);
```

$$\frac{x^8-8\,x^4-1-4\,x^3}{(x^4-1)^2} \quad (25)$$

The critical points

```
> fsolve(x^8-8*x^4-1-4*x^3,x);
```

$$-1.536693553,\ 1.792689941 \quad (26)$$

The second derivative

```
> diff(g(x),x$2);
```

$$\frac{20\,x^3}{x^4-1} - \frac{8\,(5\,x^4+1)\,x^3}{(x^4-1)^2} + \frac{32\,(x^5+x+1)\,x^6}{(x^4-1)^3} - \frac{12\,(x^5+x+1)\,x^2}{(x^4-1)^2} \quad (27)$$

```
> simplify(%);
```

$$\frac{4\,x^2\,(6\,x^5+10\,x+5\,x^4+3)}{(x^4-1)^3} \quad (28)$$

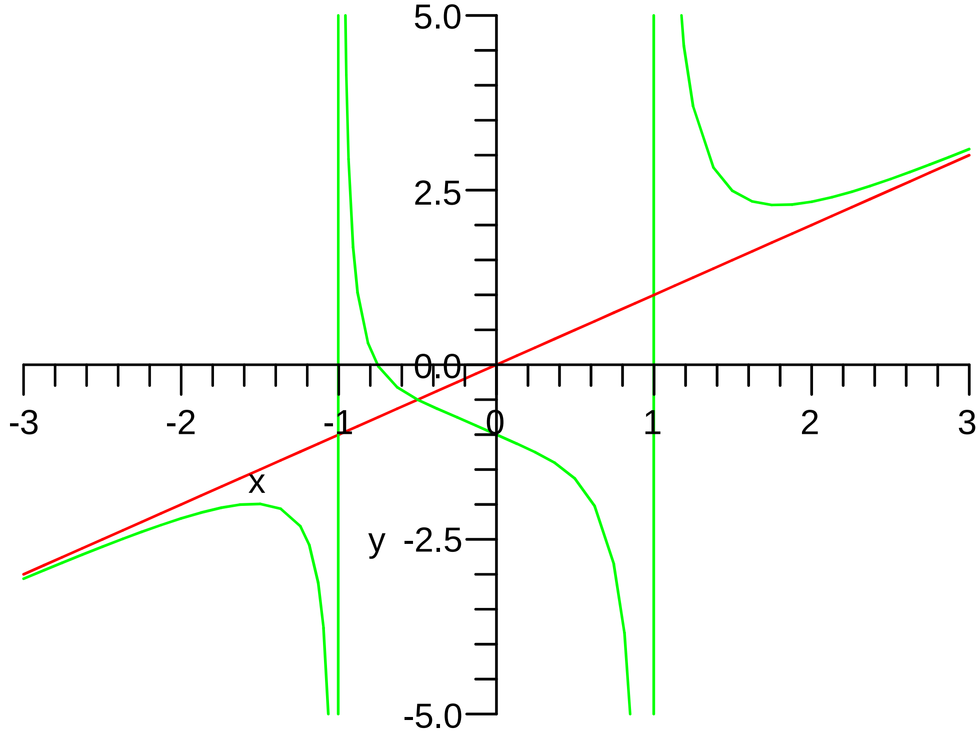
The inflection points (0 is not an inflection point because the second derivative does not change sign)

```
> fsolve(6*x^5+10*x+5*x^4+3,x);
```

$$-.3026721347 \quad (29)$$

The graph shows the function and its slant asymptote.

```
> plot({g(x),x},x=-3..3,y=-5..5);
```



Taylor and McLaurin polynomials. The next command shows McLaurin polynomial of order 7 for sin

```
> taylor(sin(x),x=0,8);
```

$$x-\frac{1}{6}x^3+\frac{1}{120}x^5-\frac{1}{5040}x^7+O(x^8) \quad (30)$$

The last expression O(x^8) shows the order of magnitude of the error. To get rid of it we can perform the command

```
> convert(%,polynom);
```

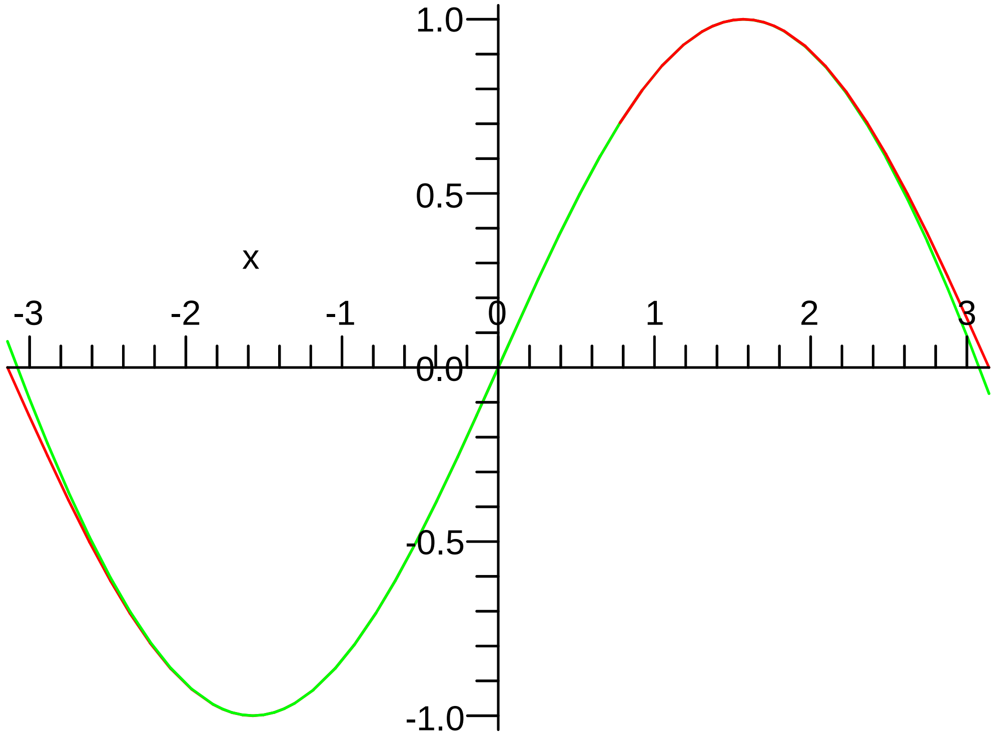
$$x-\frac{1}{6}x^3+\frac{1}{120}x^5-\frac{1}{5040}x^7 \quad (31)$$

To see how good is the approximation let us graph this polynomial and sin on the interval from minus pi to pi.

```
> h:=x->x-1/6*x^3+1/120*x^5-1/5040*x^7;
```

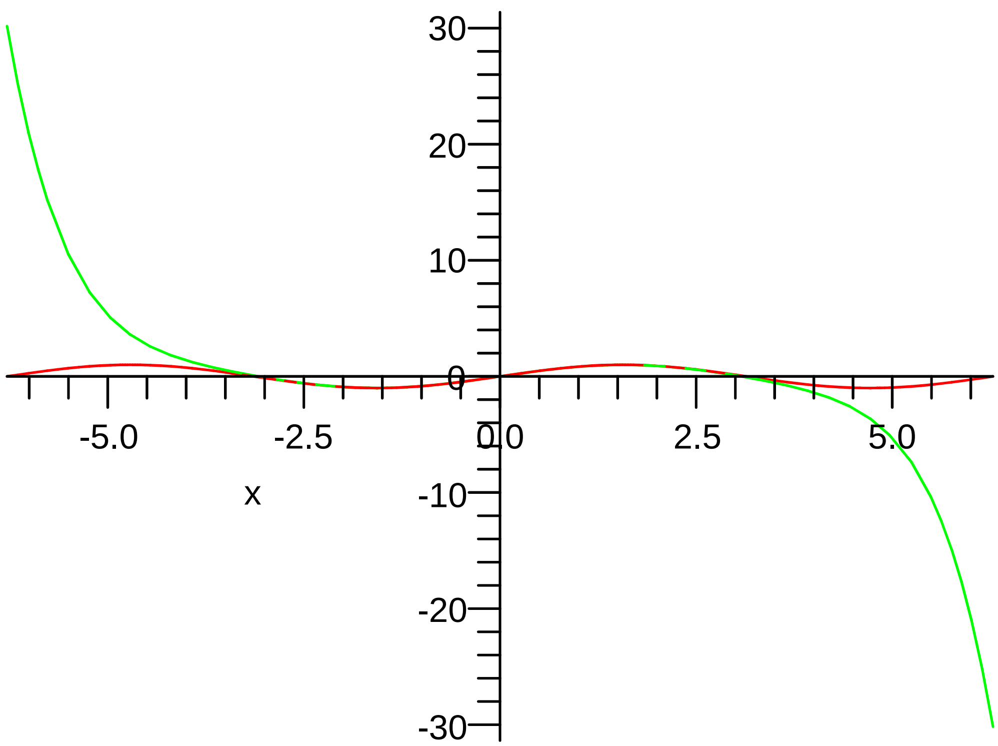
$$x\rightarrow x-\frac{1}{6}x^3+\frac{1}{120}x^5-\frac{1}{5040}x^7 \quad (32)$$

```
> plot({sin(x),h(x)},x=-Pi..Pi);
```



But on a larger interval we will clearly see the difference

```
> plot({sin(x),h(x)},x=-2*Pi..2*Pi);
```



so we can take a McLaurin polynomial of a higher degree

```
> taylor(sin(x),x=0,14);
```

$$x-\frac{1}{6}x^3+\frac{1}{120}x^5-\frac{1}{5040}x^7+\frac{1}{362880}x^9-\frac{1}{39916800}x^{11}+\frac{1}{6227020800}x^{13}+\mathrm{O}(x^{14}) \quad (33)$$

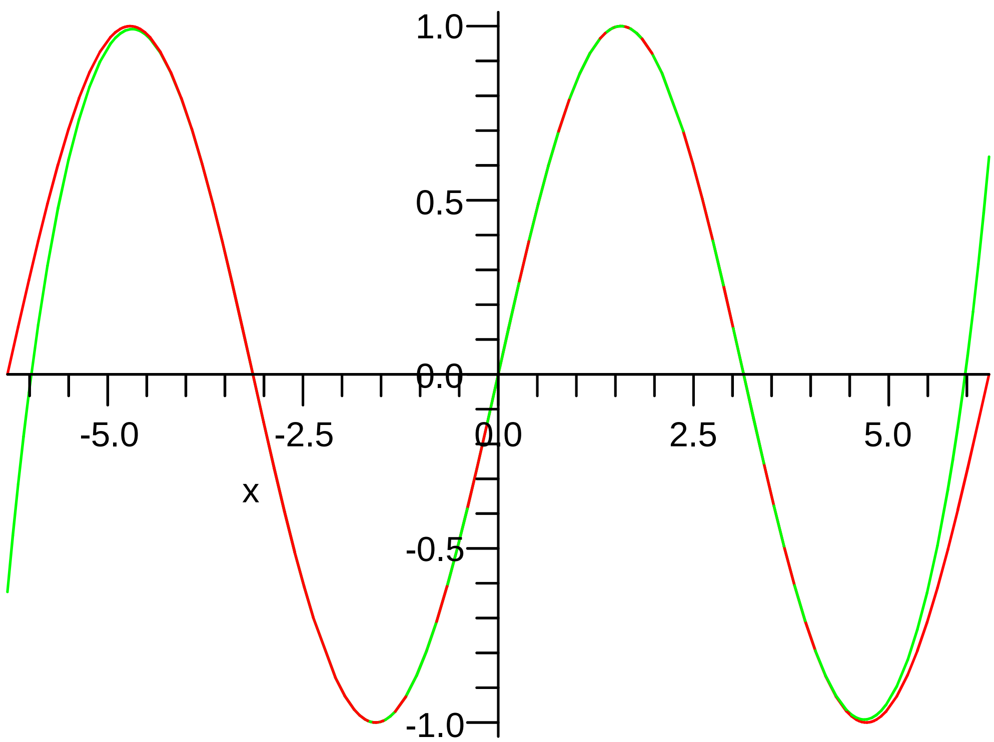
```
> convert(%,polynom);
```

$$x-\frac{1}{6}x^3+\frac{1}{120}x^5-\frac{1}{5040}x^7+\frac{1}{362880}x^9-\frac{1}{39916800}x^{11}+\frac{1}{6227020800}x^{13} \quad (34)$$

```
> s:=x->x-1/6*x^3+1/120*x^5-1/5040*x^7+1/362880*x^9-1/39916800*
x^11+1/6227020800*x^13;
```

$$x\rightarrow x-\frac{1}{6}x^3+\frac{1}{120}x^5-\frac{1}{5040}x^7+\frac{1}{362880}x^9-\frac{1}{39916800}x^{11}+\frac{1}{6227020800}x^{13} \quad (35)$$

```
> plot({sin(x),s(x)},x=-2*Pi..2*Pi);
```



Another example. For the rational function g we introduced above find the Taylor polynomial of order 6 at point 2.

```
> taylor(g(x),x=2,6);
```

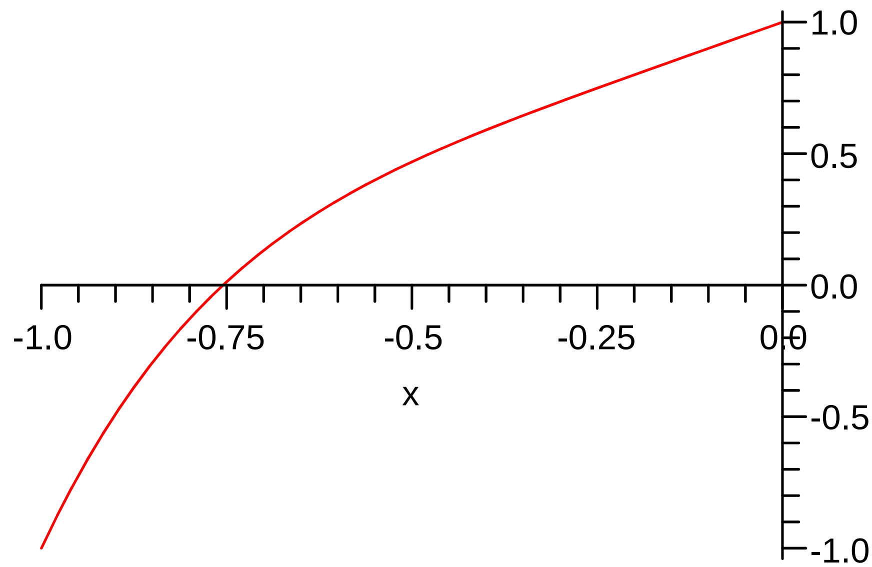
$$\frac{7}{3}+\frac{19}{45}(x-2)+\frac{472}{675}(x-2)^2-\frac{7544}{10125}(x-2)^3+\frac{114913}{151875}(x-2)^4-\frac{1723226}{2278125}(x-2)^5+\mathrm{O}\left((x-2)^6\right) \quad (36)$$

```
> convert(%,polynom);
```

$$\frac{67}{45}+\frac{19}{45}x+\frac{472}{675}(x-2)^2-\frac{7544}{10125}(x-2)^3+\frac{114913}{151875}(x-2)^4-\frac{1723226}{2278125}(x-2)^5 \quad (37)$$

The Newton's Method. We will find the real solution of the equation x^5+x+1=0 with 50 correct digits after decimal point. First we graph to find a reasonable initial approximation.

```
> plot(x^5+x+1,x=-1..0);
```



so -0.8 seems good enough as approximation number 0.
Now we make preparations for Newton's method

```
> l:=x->x^5+x+1;
```

$$x \rightarrow x^5 + x + 1 \tag{38}$$

```
> m:=x->x-l(x)/D(l)(x);
```

$$x \rightarrow x - \frac{l(x)}{\mathrm{D}(l)(x)} \tag{39}$$

The first approximation.

```
> evalf(m(-0.8),51);
```

$$-.758110236220472440944881889763779527559055118110236 \tag{40}$$

And now we just repeat iterations.

```
> evalf(m(%),51);
```

$$-.754894763899693724269538143609522118282297806904317 \tag{41}$$

```
> evalf(m(%),51);
```

$$-.754877666725986900301627529071180985598185569156888 \tag{42}$$

```
> evalf(m(%),51);
```

$$-.754877666246692760426158952850656417149354771097577 \tag{43}$$

```
> evalf(m(%),51);
```

$$-.754877666246692760049508896358528692127206704319112 \tag{44}$$

```
> evalf(m(%),51);
```

$$-.754877666246692760049508896358528691894606617772792 \tag{45}$$

```
> evalf(m(%),51);
```

$$-.754877666246692760049508896358528691894606617772793 \quad (46)$$

We can stop now. To check:

```
> evalf(l(%),51);
```

$$0. \quad (47)$$

```
> ?
```