1 a. Convert the given degree measure to radians: 75°, 14.23°, and $187^\circ\,35'\,13''$.
Solution we use the connection between the degree measure and the radian measure $s^\circ = \frac{\pi}{180} s$ radians. Therefore

$$75^\circ = \frac{\pi}{180}75 = \frac{5\pi}{12}, \quad 14.23^\circ = \frac{\pi}{180}14.23 \approx 0.2484,$$

$$187^\circ 35' 13'' = \left(187 + \frac{35}{60} + \frac{13}{3600}\right)^\circ = \left(187 + \frac{35}{60} + \frac{13}{3600}\right)\frac{\pi}{180} \approx 3.2740.$$

b. Convert the given radian measure to degrees: $\pi/16$, -33.5π, 12.7.

Solution here we use the formula $r = \left(\frac{180r}{\pi}\right)^\circ$.

$$\frac{\pi}{16} = \left(\frac{\pi}{16}\cdot\frac{180}{\pi}\right)^\circ = \left(\frac{45}{4}\right)^\circ = 21.25^\circ, \quad -33.5\pi = \left(-33.5\pi\cdot\frac{180}{\pi}\right)^\circ = -6030^\circ,$$

$$12.7 = \left(12.7\cdot\frac{180}{\pi}\right)^\circ \approx 727.6564^\circ.$$

2. Assume that $\sec t = -3$ and that t is an angle in the second quadrant. Find **exact (no calculators!)** values of (a) $\tan(\pi+t)$, (b) $\cos(2\pi - t)$, and (c) $\csc(\pi - t)$.
Solution (a) tangent is a π-periodic function whence $\tan(\pi+t) = \tan t$. Next we can use the identity $\tan^2 t = \sec^2 t - 1 = (-3)^2 - 1 = 8$. In the second quadrant tangent is negative whence $\tan(\pi + t) = \tan t = -\sqrt{8}$.
(b) Cosine is a 2π-periodic function whence $\cos(2\pi - t) = \cos(-t)$. Cosine is an even function and therefore $\cos(-t) = \cos t$. Finally, $\cos t = \frac{1}{\sec t} = -\frac{1}{3}$ and $\cos(2\pi - t) = -\frac{1}{3}$.
(c) We can use the formula $\csc(\pi+u) = -\csc u$ whence $\csc(\pi - t) = -\csc(-t)$. Next recall that cosecant is an odd function, $\csc(-t) = -\csc t$ and therefore $-\csc(-t) = \csc t$.
Finally, to find the value of $\csc t$ we can use for example the formula
$\csc(t) = \frac{\cot t}{\cos t} = \frac{1}{\tan t}\cdot\sec t = \left(-\frac{1}{\sqrt{8}}\right)(-3) = \frac{3}{\sqrt{8}} = \frac{3\sqrt{8}}{8}$ and so $\csc(\pi - t) = \frac{3\sqrt{8}}{8}$.

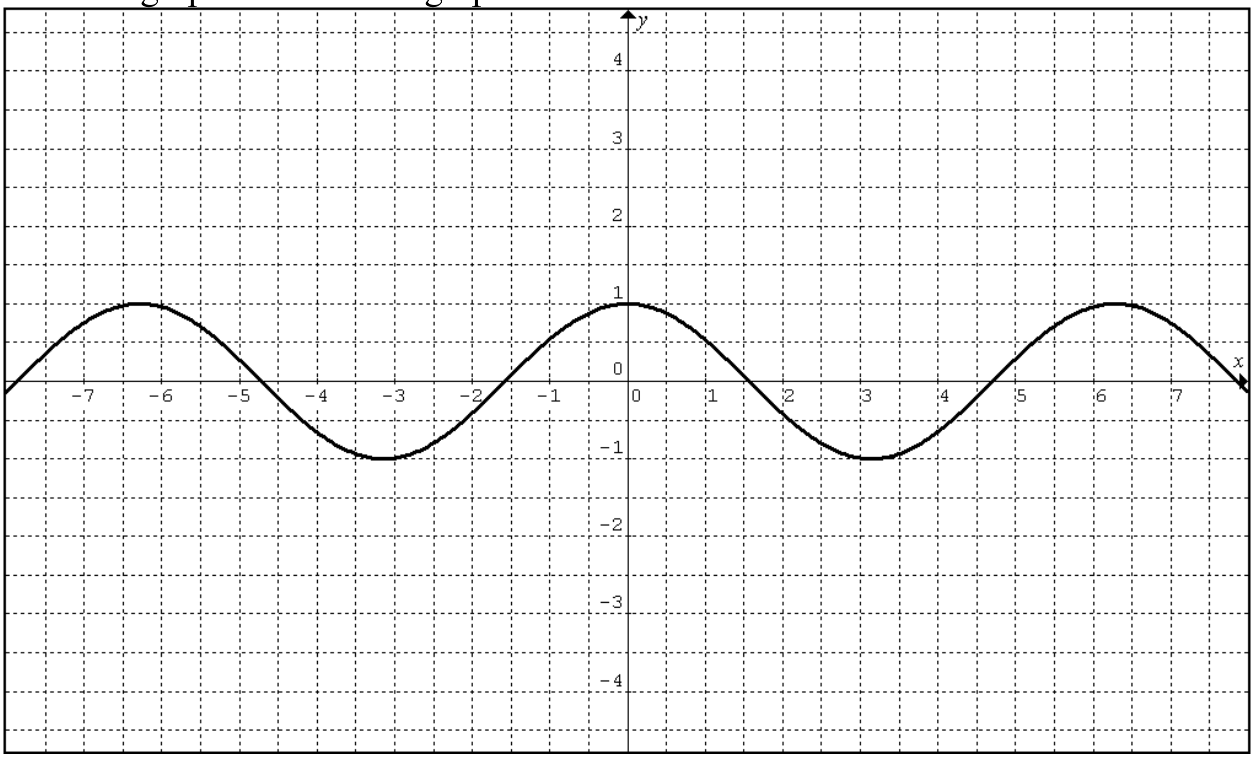
3. Find the amplitude, the period, and the phase shift. Graph the function.

$$2\cos(\pi x - \pi/2).$$

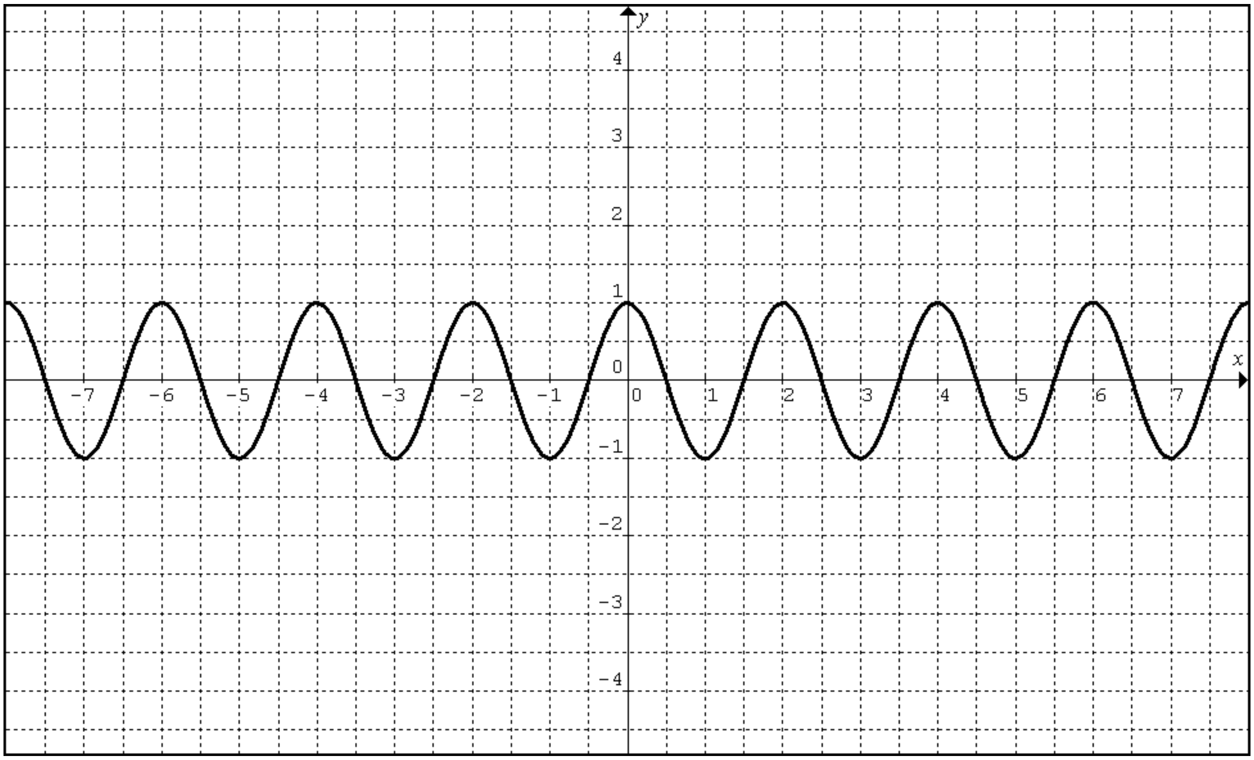
Solution In notations accepted in our book (page 207) we have $A = 2$ whence the amplitude of the function is 2; $B = \pi$ and therefore the period of the function is $P = \frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$; the phase shift is computed as $C = \frac{\pi/2}{B} = \frac{\pi/2}{\pi} = \frac{1}{2}$.

We will graph the function using elementary transformations of the standard graph of $\cos x$. The order in which we apply transformations can be different.

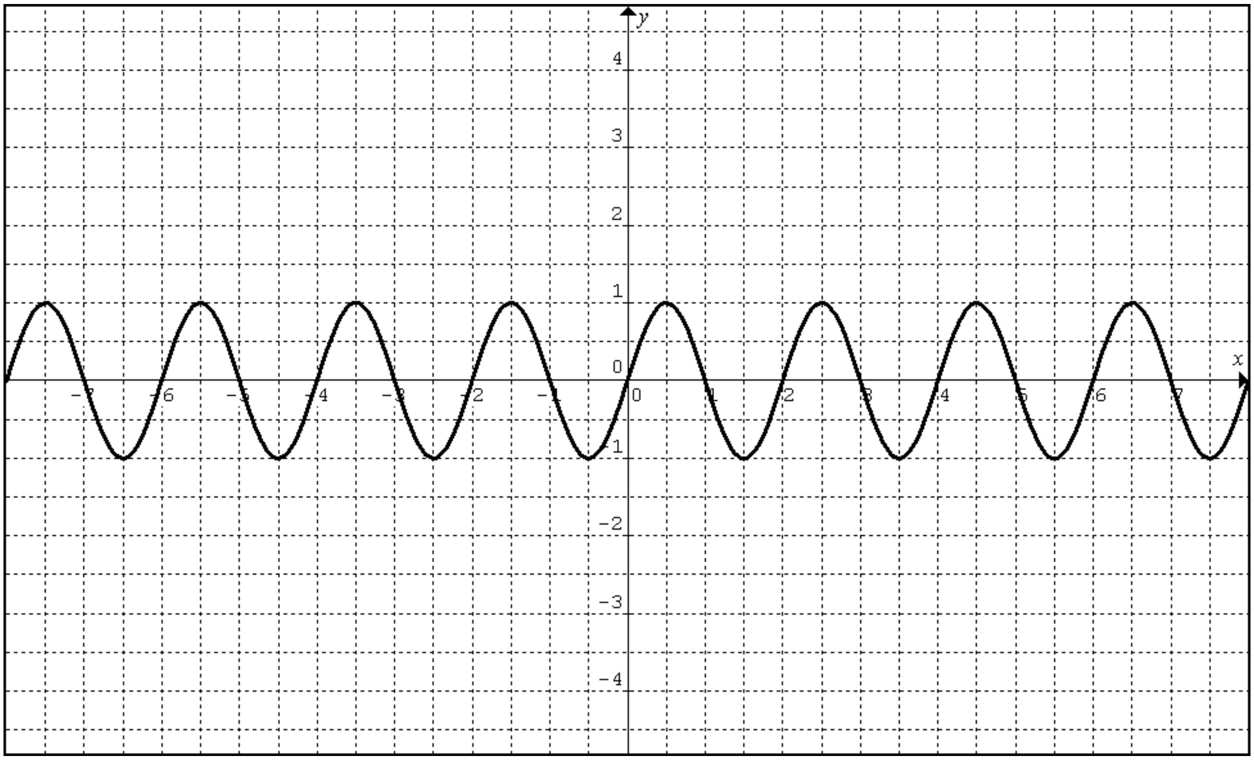
The first graph below is the graph of $\cos x$.



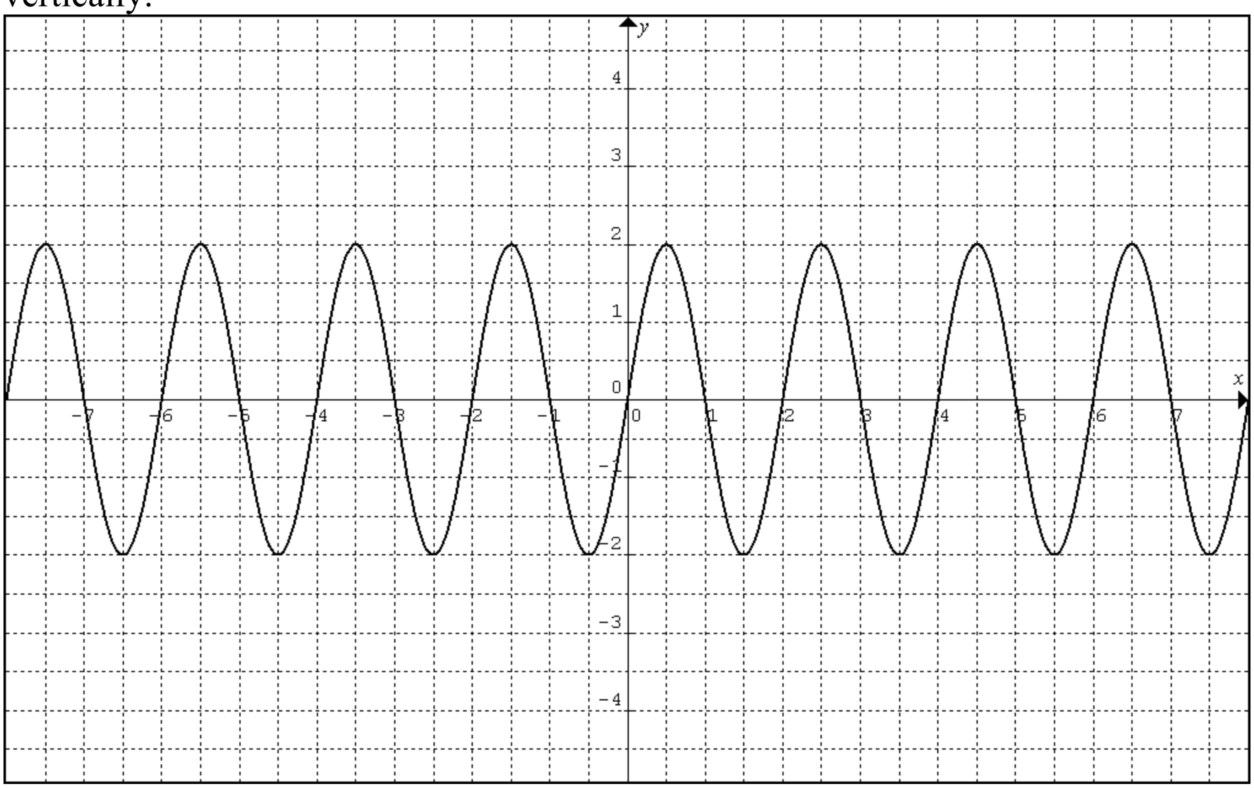
Next we graph the function $\cos(\pi x)$. Notice that the graph becomes compressed horizontally $\frac{1}{\pi}$ times; in particular the period is now 2.



Next we graph $\cos(\pi x - \pi/2)$. Notice horizontal shift by ½ to the right.



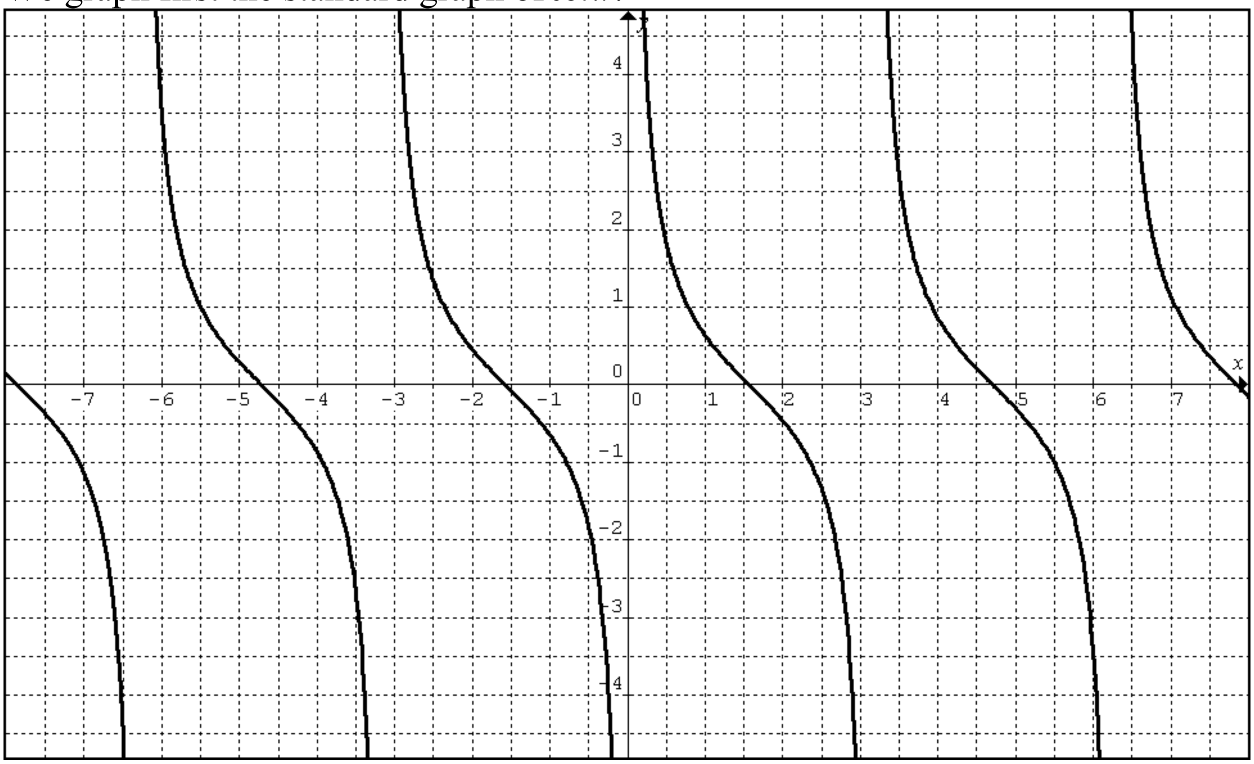
Finally, we graph $2\cos(\pi x - \pi/2)$. Our previous graph becomes stretched twice vertically.



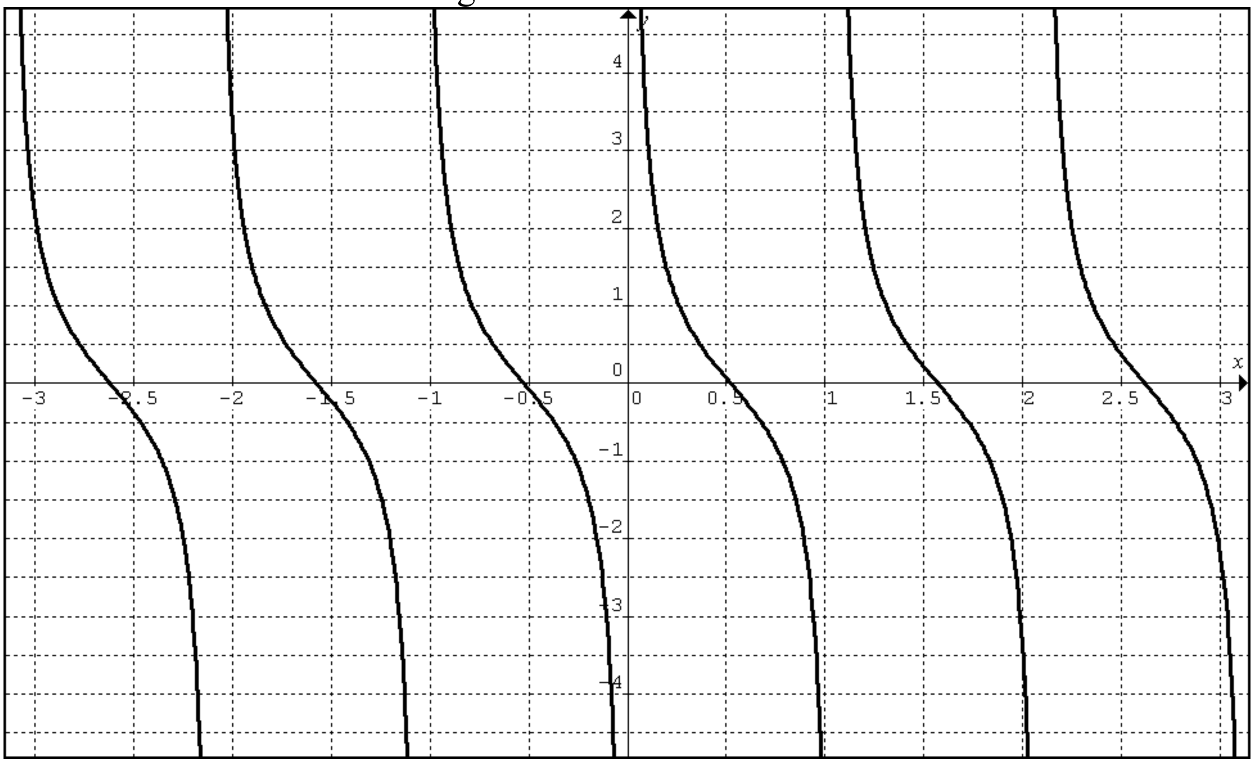
4. Find the period and the shift. Graph the function $\cot(3x + \pi/4)$.

Solution Cotangent is a π-periodic function and therefore the period P of the function $\cot(3x + \pi/4)$ is $\pi/3$. The shift C is computed as $C = -\frac{\pi}{4} \div 3 = -\frac{\pi}{12}$.

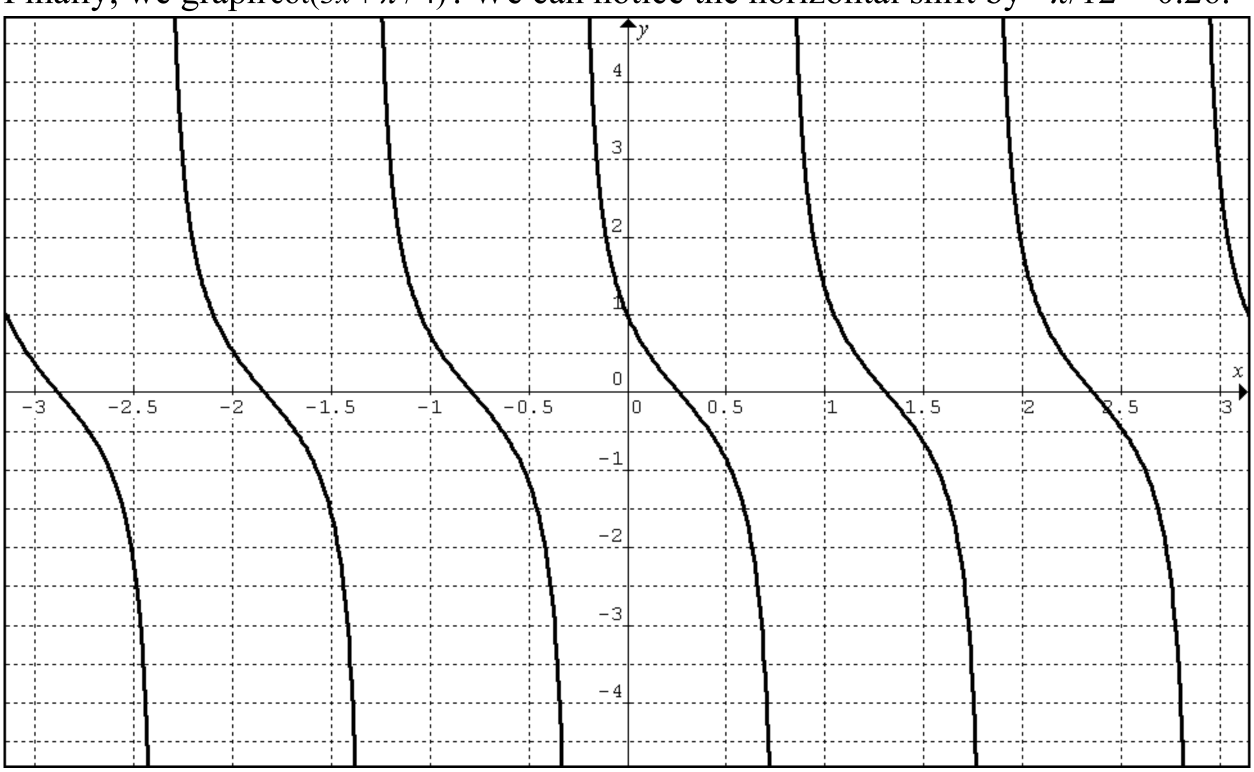
We graph first the standard graph of $\cot x$.



Next we graph $\cot(3x)$. The graph is horizontally compressed three times. The horizontal scale has been changed.



Finally, we graph $\cot(3x+\pi/4)$. We can notice the horizontal shift by $-\pi/12 \approx 0.26$.



5. Find the exact value of $\arctan(\tan(899.25\pi))$).

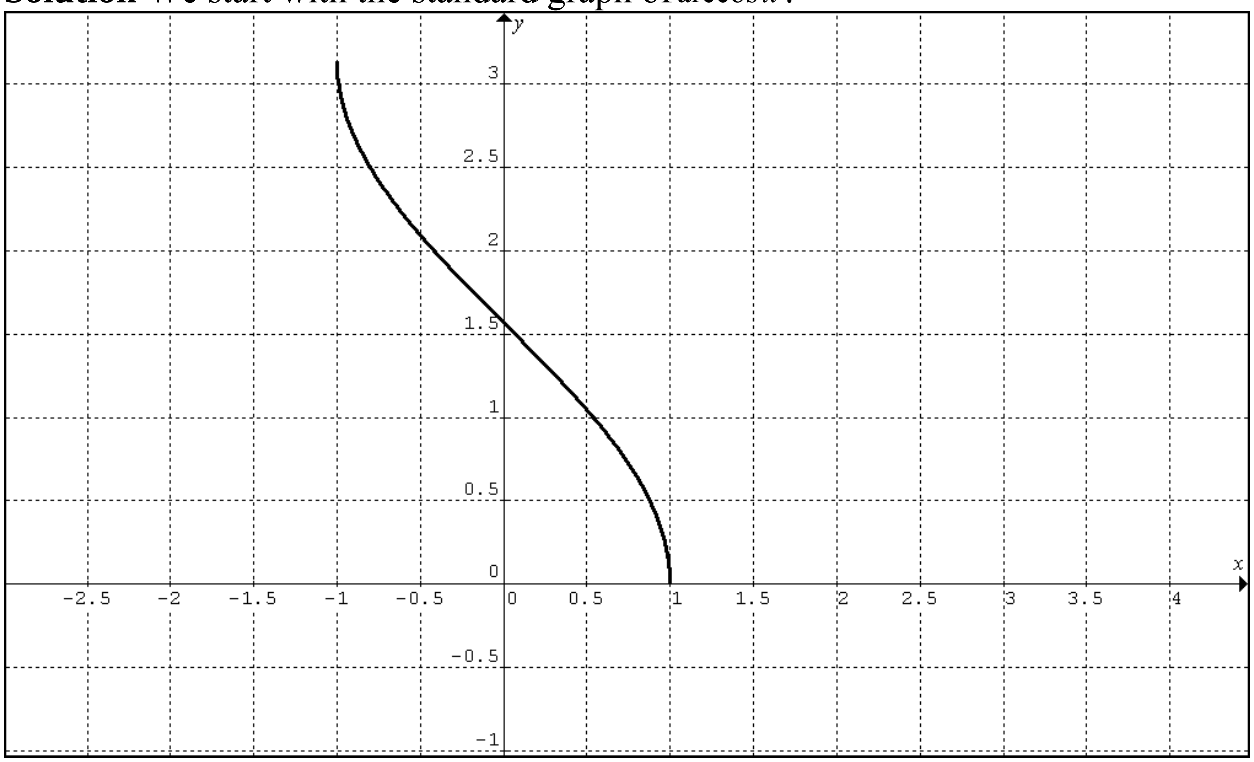
Solution The identity $\arctan(\tan x) = x$ is true only for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, i.e. $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

Therefore it will be not correct to write that $\arctan(\tan(899.25\pi)) = 899.25\pi$. But, because the function tangent is π-periodic we can write that

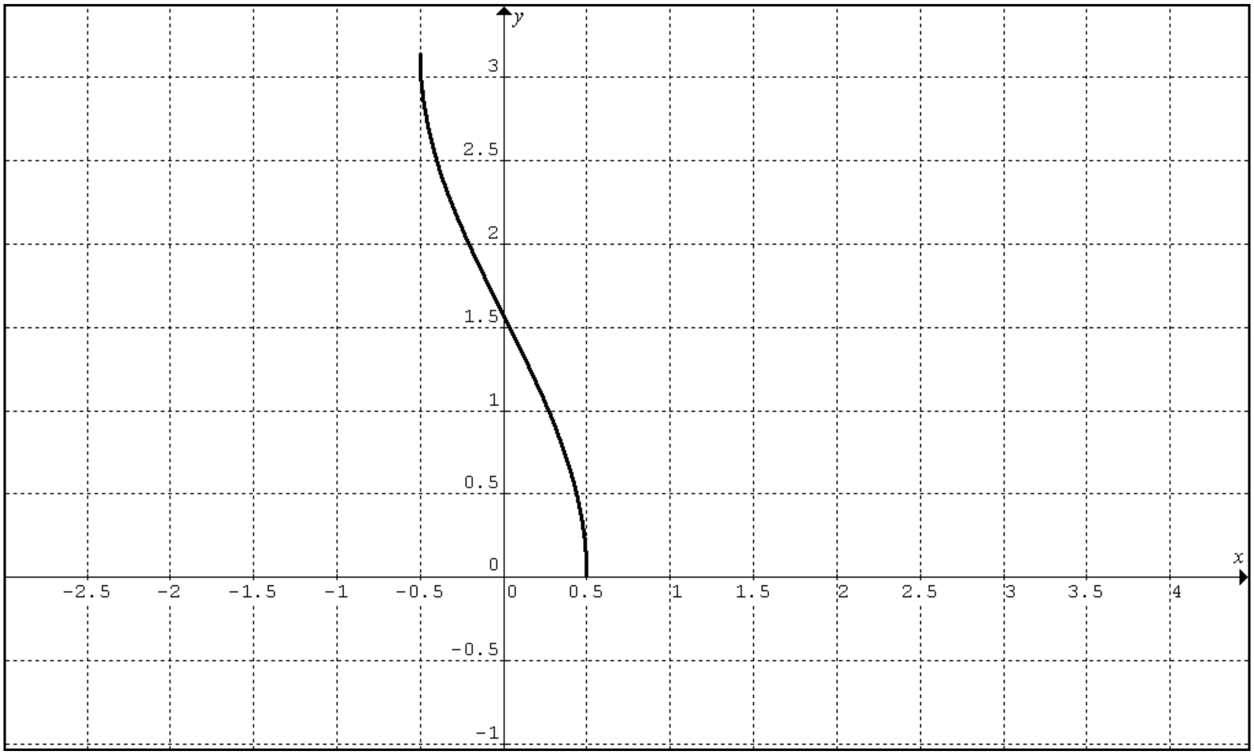
$\tan(899.25\pi) = \tan(.25\pi)$ whence $\arctan(\tan(899.25\pi)) = \arctan(\tan(.25\pi)) = .25\pi = \frac{\pi}{4}$.

6. Graph $2\arccos(2x-3)-1$.

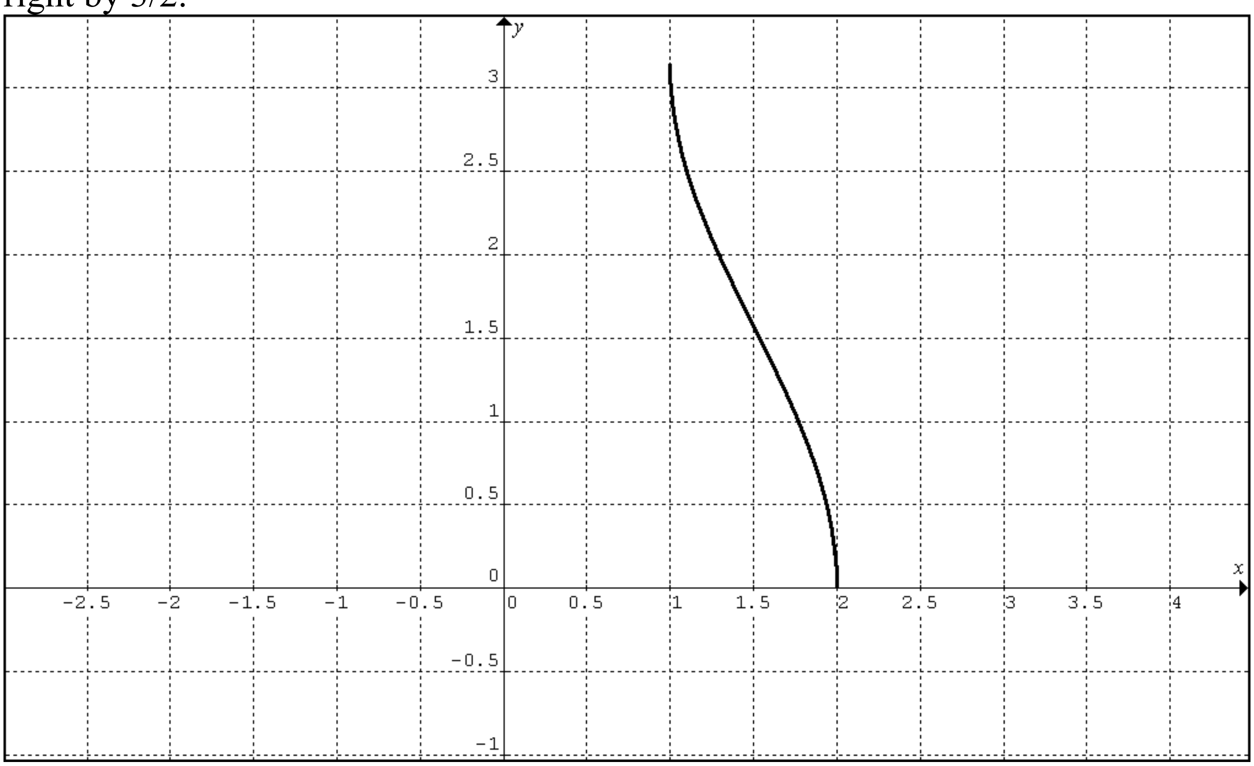
Solution We start with the standard graph of $\arccos x$.



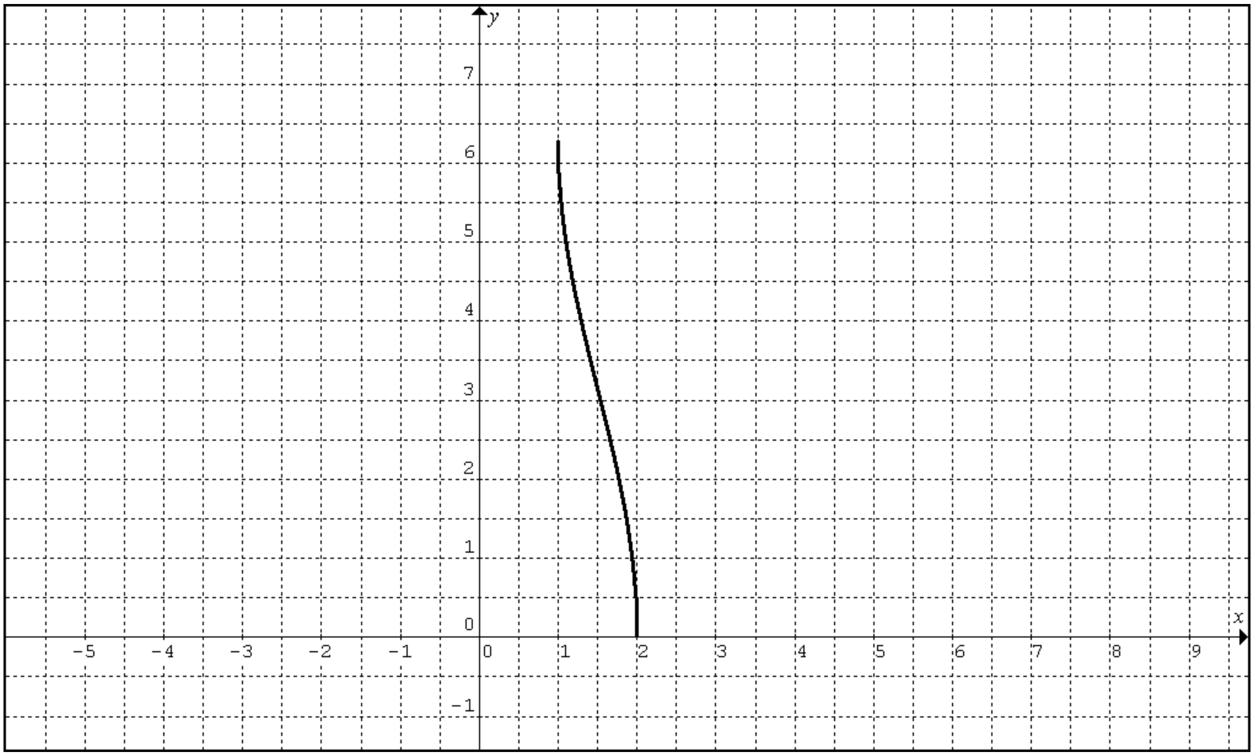
Next we graph $\arccos 2x$. The original graph is compressed twice in the horizontal direction.



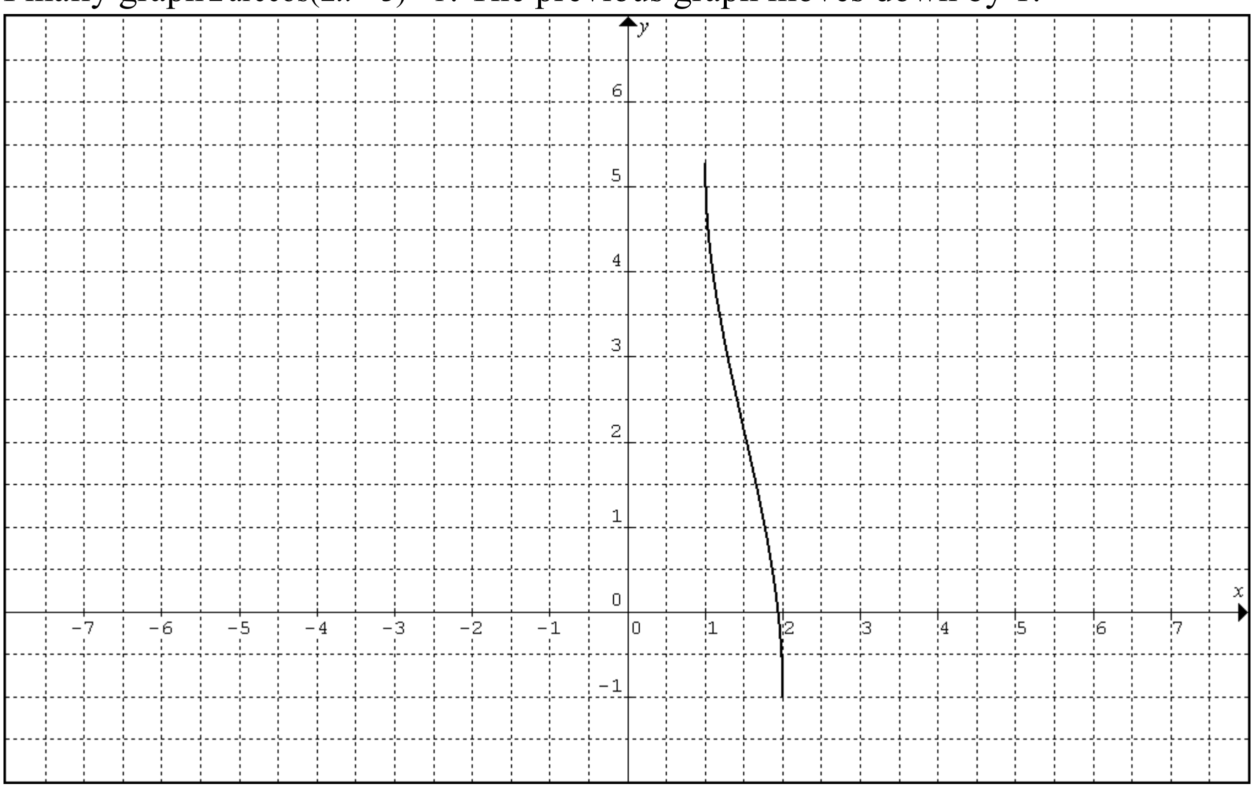
Now we graph $\arccos(2x-3)=\arccos[2(x-3/2)]$. The previous graph is moved to the right by 3/2.



Next: graph $2\arccos(2x-3)$. The previous graph is stretched twice vertically.



Finally graph $2\arccos(2x-3)-1$. The previous graph moves down by 1.



7. Simplify $\cos(\arcsin(\sqrt{1-x^2}))$.
Solution By the first Pythagorean identity

$$\cos^2(\arcsin(\sqrt{1-x^2}))+\sin^2(\arcsin(\sqrt{1-x^2}))=1$$

Next, because $\sin(\arcsin u)=u$, we have

$$\cos^2(\arcsin(\sqrt{1-x^2}))+1-x^2=1$$

whence

$$\cos^2(\arcsin(\sqrt{1-x^2}))=x^2.$$

Notice that $\cos(\arcsin(\sqrt{1-x^2}))\geq 0$ because $\arcsin$ takes values from $-\pi/2$ to $\pi/2$ and $\cos$ is non-negative in quadrants I and IV. Therefore

$$\cos(\arcsin(\sqrt{1-x^2}))=\sqrt{x^2}=|x|, \quad -1\leq x\leq 1.$$

8. Compute the exact value (no calculators!) of $\sec(\arctan(2/3))$.
Solution By one of the Pythagorean identities we have

$$\sec^2(\arctan(2/3))=1+\tan^2(\arctan(2/3))=1+(2/3)^2=13/9.$$

We have $\sec(\arctan(2/3))>0$, because $0<\arctan(2/3)<\pi/2$ and therefore $\sec(\arctan(2/3))=\sqrt{13/9}=\dfrac{\sqrt{13}}{3}$.

9. Prove that $\ln|\sec x+\tan x|=-\ln|\sec x-\tan x|$.
Solution First notice that $|\sec x+\tan x|\cdot|\sec x-\tan x|=|\sec^2 x-\tan^2 x|=|1|=1$. Therefore by properties of logarithms

$$\ln|\sec x+\tan x|+\ln|\sec x-\tan x|=\ln(|\sec x+\tan x|\cdot|\sec x-\tan x|)=\ln 1=0,$$

and our statement is proved.

10. Find the distance in kilometers between two cities located on the same meridian if the latitude of the first city is $59^\circ 36' 18'' N$ and the latitude of the second one is $19^\circ 13' 24'' S$. Assume that the radius of Earth is approximately 6300 km.
Solution The angle between the radii going from the center of Earth to the cities is $59^\circ 36' 18''+19^\circ 13' 24''=78^\circ 55' 42''$. The radian measure of this angle is $\theta=(78+55/60+42/3600)\cdot\pi/180\approx 1.3776$. The distance between the cities is $R\theta\approx 6300\cdot 1.3776\approx 8679 km$.