161 Precalculus 1 **Review 6**

Problem 1 The equation of a circle is given as

$$x^2 - 8x + y^2 + 4y = 5$$

- **(a) Bring this equation to the standard form**
- **(b) Find the center and the radius of the circle**
- **(c) Graph the circle**

Solution (a) we will complete squares using the formula

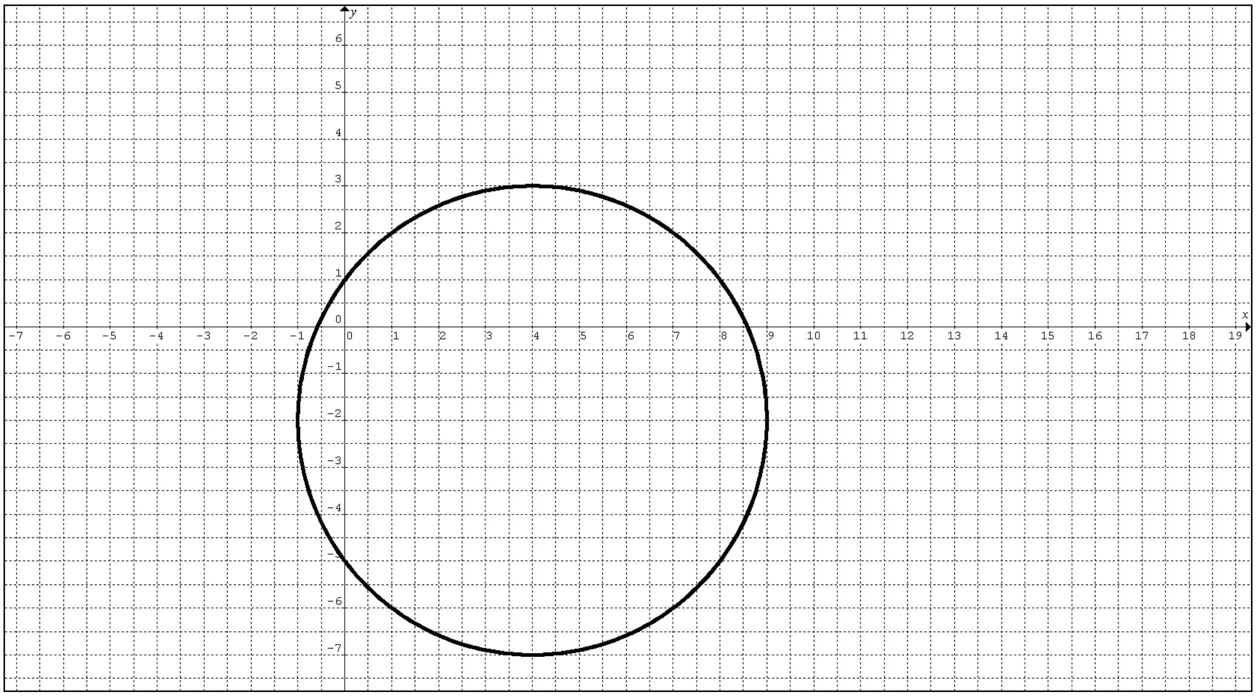
$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

Then $x^2 - 8x = (x-4)^2 - 16$ **and** $y^2 + 4y = (y+2)^2 - 4$**. Thus the standard form of the original equation is**

$$(x-4)^2 + (y+2)^2 = 25$$

(b) From the standard equation we see that the center of the circle is $(4,-2)$ **and its radius is** 5

(c) The graph of the circle is shown below



Problem 2 the vertices of an ellipse are at $(-5,0)$ and at $(5,0)$; the eccentricity of the ellipse is $\varepsilon = 0.4$

(a) Find the standard equation of the ellipse
(b) Find the position of foci and of the ends of the minor axis
(c) Graph the ellipse

Solution (a) from the information provided we see that the center of the ellipse is at the origin and that $a = 5$. Next we see that $1 - b^2/a^2 = \varepsilon^2 = 0.16$ whence $b^2 = a^2(1-0.16) = 25 \cdot 0.84 = 21$. The standard equation thus is

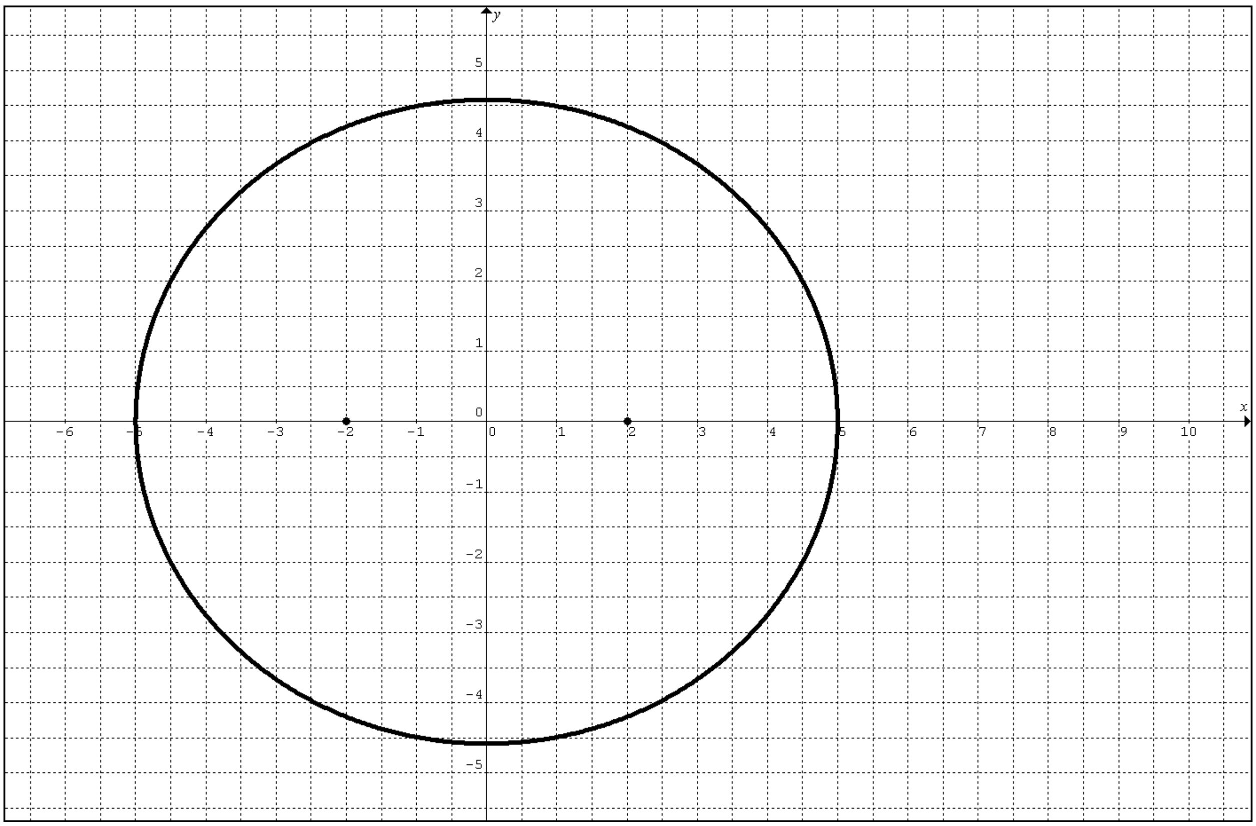
$$\frac{x^2}{25} + \frac{y^2}{21} = 1$$

(b) Foci are located at $\left(\pm\sqrt{a^2 - b^2}, 0\right)$ or at $(-2,0)$ and at $(2,0)$.

The ends of the minor axis are located at $\left(0, \pm\sqrt{21}\right)$ **or approximately at**

$(0, \pm 4.6)$

The graph of the ellipse is shown below



Problem 3 the equation of an ellipse is

$$2x^2+8x+3y^2-18y=1$$

(a) **Write this equation in the standard form**
(b) **Find the center, vertices, ends of the minor, axes, foci, and the eccentricity of this ellipse**
(c) **Graph the ellipse**

Solution we transform the right part of the equation in the following way using the completion of squares

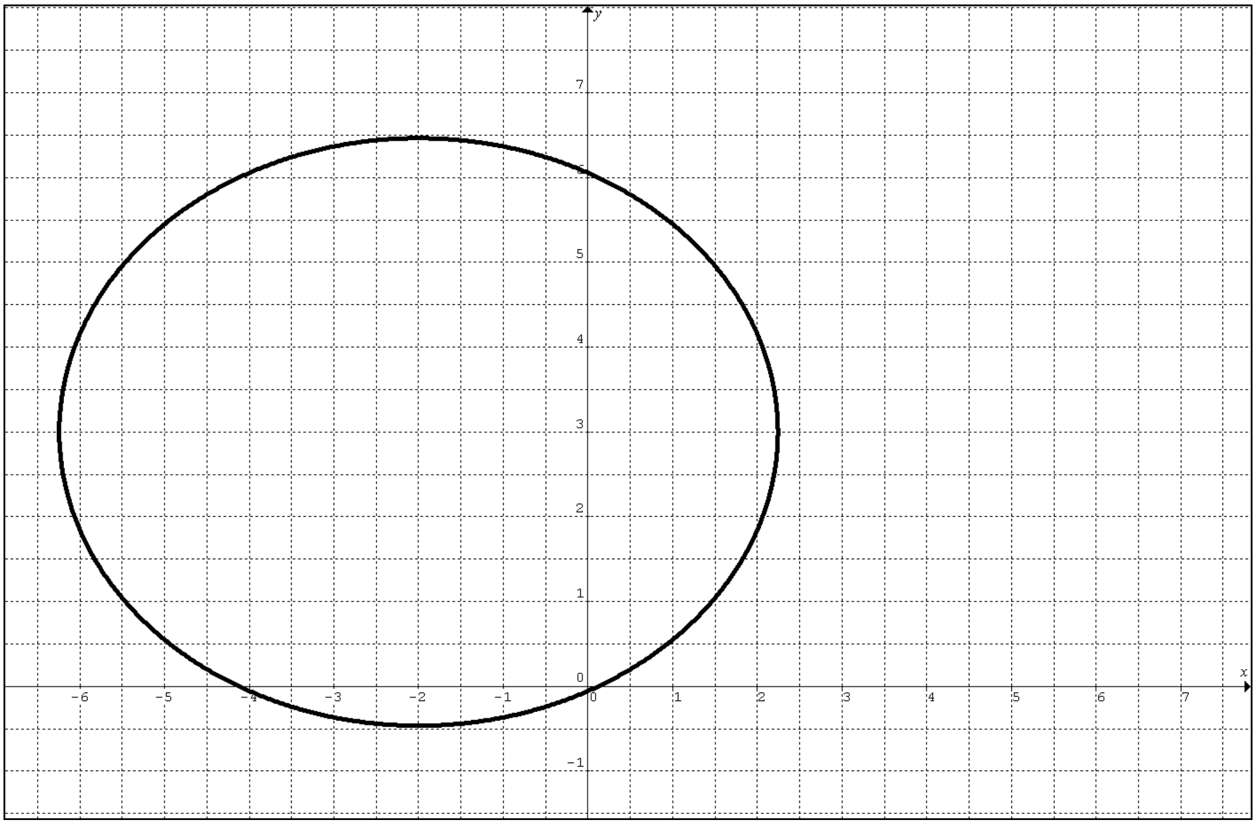
$$\begin{aligned}2x^2+8x+3y^2-18y&=2(x^2+4x)+3(y^2-6y)=\\&=2[(x+2)^2-4]+3[(y-3)^2-9]=\\&=2(x+2)^2+3(y-3)^2-35\end{aligned}$$

Therefore $2(x+2)^2+3(y-3)^2=36$ **and finally the standard equation is**

$$\frac{(x+2)^2}{18}+\frac{(y-3)^2}{12}=1$$

(b)Notice that $a=\sqrt{18}=3\sqrt{2}$, $b=\sqrt{12}=2\sqrt{3}$**, and** $f=\sqrt{a^2-b^2}=\sqrt{6}$**. The center of the ellipse is at** $(-2,3)$**, the vertices are at** $(-2\pm3\sqrt{2},3)$**, the ends of the minor axis are at** $(-2,3\pm2\sqrt{3})$**, the foci are at** $(-2\pm\sqrt{6},3)$ **and the eccentricity is** $\varepsilon=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{\frac{1}{3}}\approx0.58$

(d) The graph of the ellipse is shown below



Problem 4 consider the following equation of a parabola

$$y^2 + 4y + 4x = 8$$

(a) **Write this equation in the standard form**

(b) **Find the position of the vertex and the focus of the parabola. Find an equation of the directrix**

(c) **Graph the parabola and the directrix**

Solution completing the squares we rewrite the equation as $(y+2)^2 + 4x - 4 = 8$**. The standard form is**

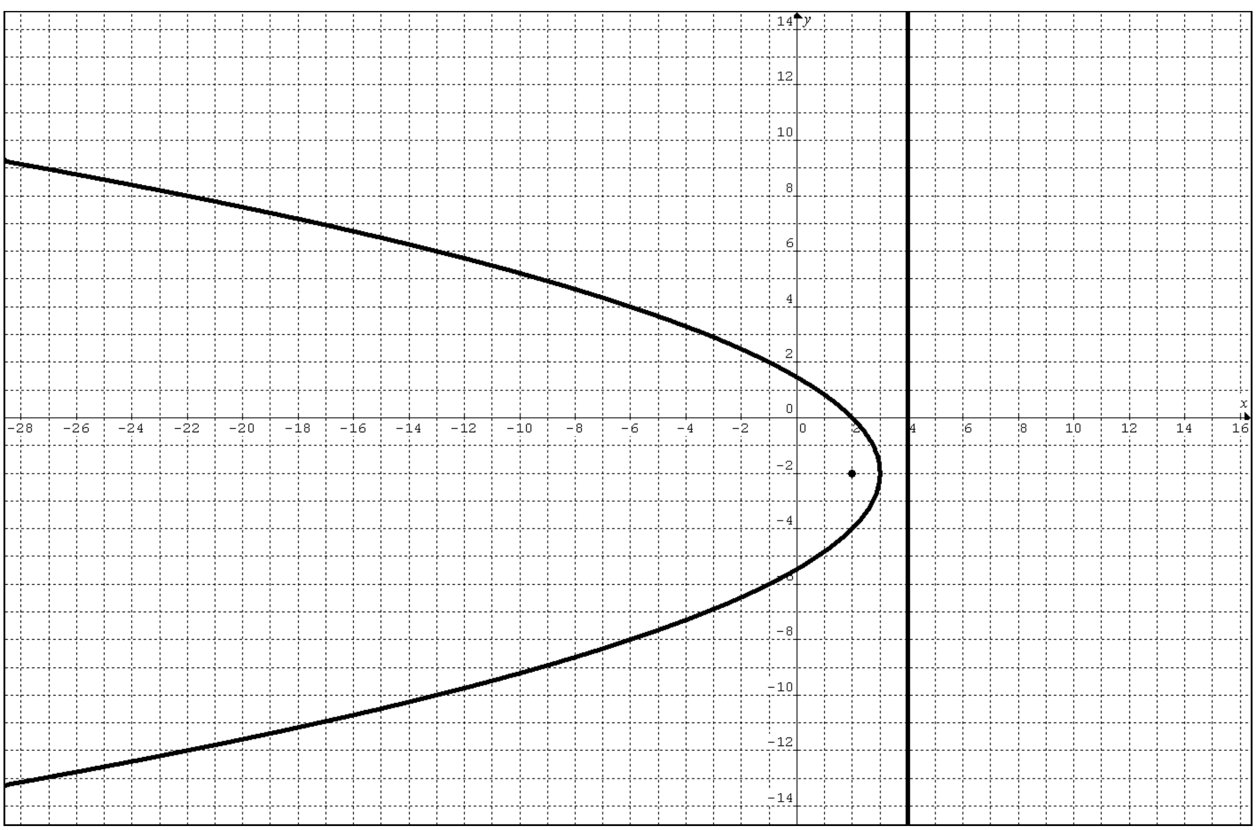
$$x-3=-\frac{(y+2)^2}{4}$$

(b) From the standard equation we see that the vertex is at $(3,-2)$**. The sign minus in the right part shows that the graph of the parabola is to the left of the vertex. The distance from the vertex to the focus and to the directrix is**

$\frac{1}{4}\div 4=\frac{1}{16}$**. The focus is at** $(47/16,-2)$**. An equation of the directrix is**

$$x=\frac{49}{16}$$

(c) The graph is shown below (the distances between the focus, the directrix, and the vertex are shown greater then in reality).



Problem 5 consider the following equation of a hyperbola

$$x^2 + 2x - 2y^2 + 8y = 9$$

(a) **Bring the equation to the standard form**
(b) **Find the location of vertices and foci and equations of asymptotes**
(c) **Graph the hyperbola and its asymptotes**

Solution (a)

$$\begin{aligned} x^2 + 2x - 2y^2 + 8y = x^2 + 2x - 2(y^2 - 4y) = \\ = (x+1)^2 - 1 - 2[(y-2)^2 - 4] = \\ = (x+1)^2 - 2(y-2)^2 + 7 \end{aligned}$$

Whence $(x+1)^2 - 2(y-1)^2 = 2$ **and the standard equation is**

$$\frac{(x+1)^2}{2} - (y-2)^2 = 1$$

The center of the hyperbola is at $(-1,2)$. **The axis of the hyperbola is horizontal. Notice that** $a = \sqrt{2}$ **and** $b = 1$ **whence** $f = \sqrt{a^2 + b^2} = \sqrt{3}$ **. The vertices are at** $(-1 \pm \sqrt{2}, 2)$ **and the foci are at** $(-1 \pm \sqrt{3}, 2)$. **The asymptotes are given by equations** $y - 2 = \pm \frac{b}{a}(x+1) = \pm \frac{\sqrt{2}}{2}(x+1)$.

(d) The graph is shown below