161 Precalculus 1 **Review 4**

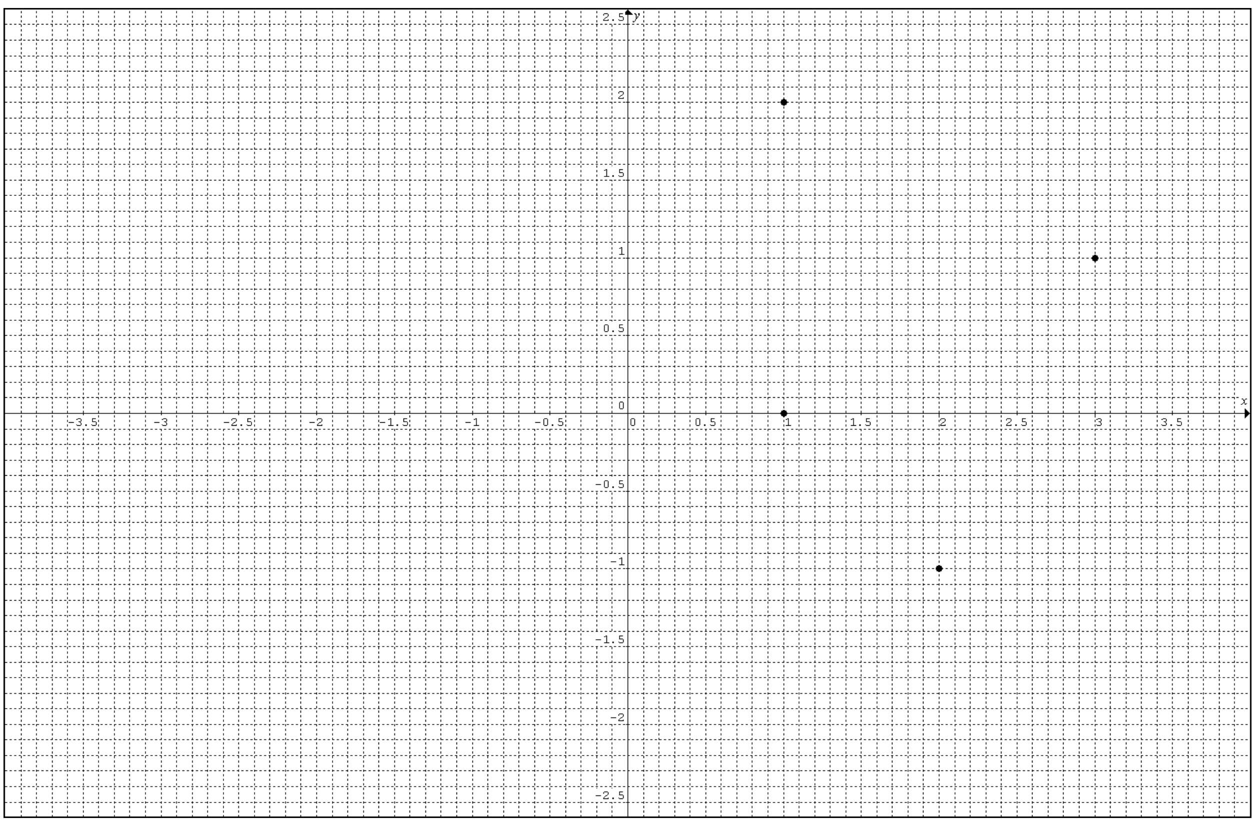
Problem 1

(a) A relation is given as the set of pairs

$$R = \{(1,0),(2,-1),(3,1),(1,2)\}$$

Find the domain and the range of this relation, graph the relation, and decide whether this relation is a function.

Solution. The domain is the set of inputs $\{1,2,3\}$. The range is the set of outputs $\{-1,0,1,2\}$. The graph of the relation is shown below.



The relation is not a function because to the same input 1 correspond two different outputs 0 and 2. Also we see that the graph does not pass the vertical line test.

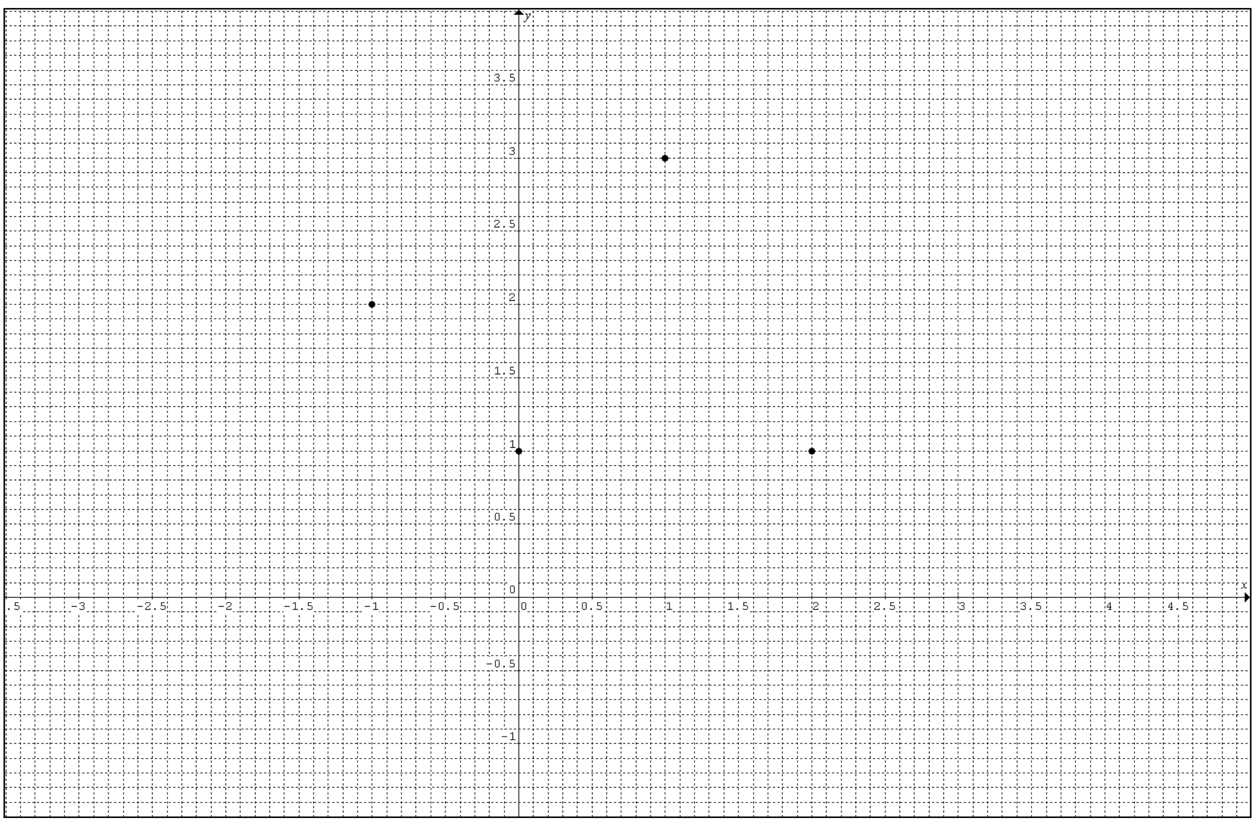
(b) Describe the relation inverse to the relation from Problem 1. Is this relation a function? Graph it.

Solution We get the inverse relation if we interchange the input and the output in each pair.

$$R^{-1} = \{(0,1),(-1,2),(1,3),(2,1)\}.$$

This relation is a function: to each input corresponds only one output.

The graph below passes the vertical line test.

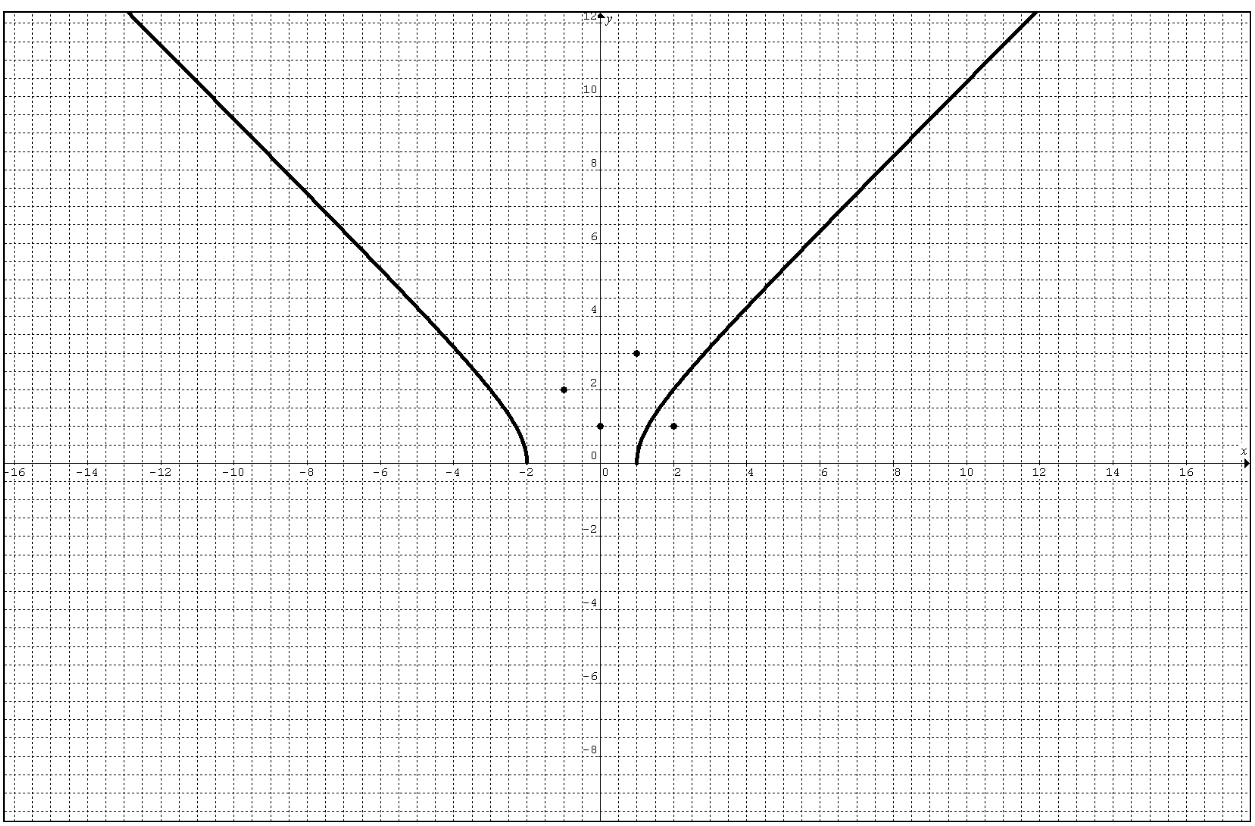


Problem 2 Consider the following two functions

$$f(x)=\sqrt{x}, g(x)=x^2+x-2$$

Describe the composition $f\circ g$ **. Find the domain and the range of this composition. Graph the composition.**

Solution $f(g(x))=\sqrt{g(x)}=\sqrt{x^2+x-2}$ **. To find the domain of this function notice that the expression under the square root cannot be negative; thus we have to solve the inequality** $x^2+x-2\geq 0$ **. To solve it we factor the left part and write the equation as** $(x-1)(x+2)\geq 0$ **. The left part will be nonnegative either on the interval** $(-\infty,-2]$ **or on the interval** $[1,\infty)$ **. Therefore the domain is** $(-\infty,-2]\cup[1,\infty)$ **. On this domain the quadratic function** g **takes all nonnegative values whence the composition** $f\circ g=\sqrt{g}$ **also takes all nonnegative values and its range is** $[0,\infty)$ **.**



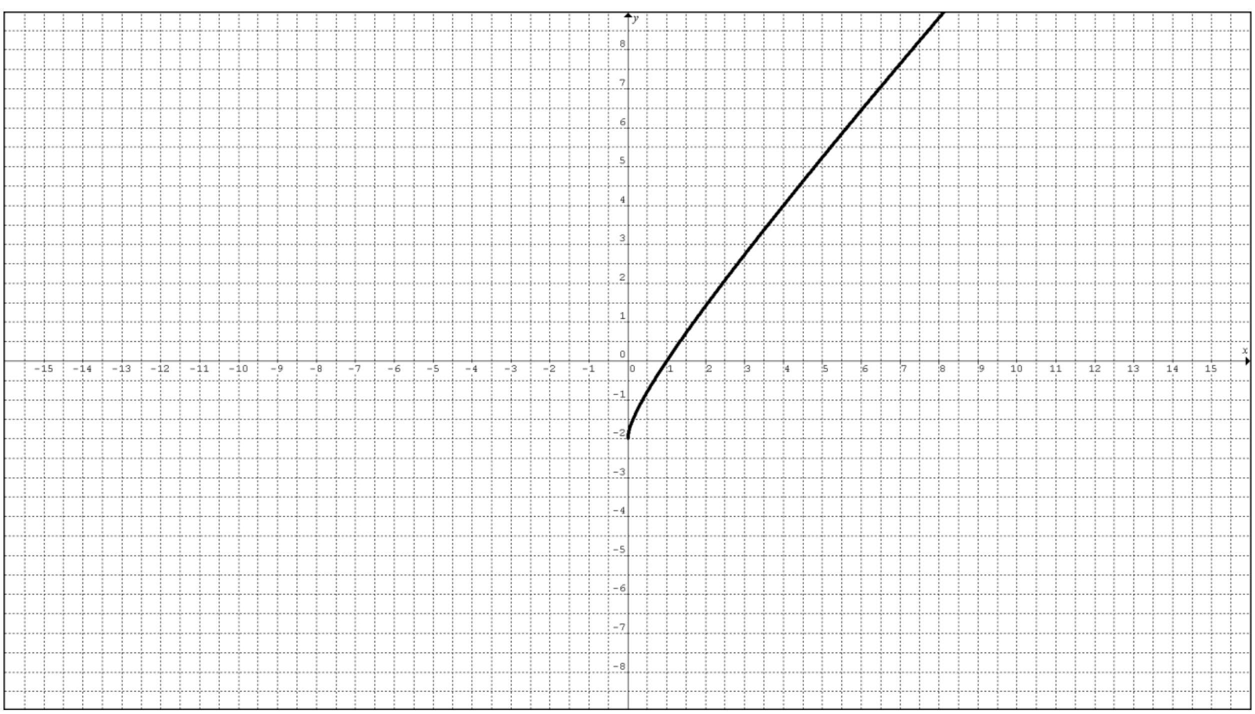
Problem 3 Consider the composition $g \circ f$ where g and f are functions from Problem 2. Describe the composition; find its domain and its range; graph the composition function.

Solution $g(f(x) = (\sqrt{x})^2 + \sqrt{x} - 2 = x + \sqrt{x} - 2$. **Clearly this expression is defined if and only if** $x \geq 0$ **and therefore the domain of the composition is** $[0, \infty)$. **The question about the range can be reduced to the following: what values does the quadratic trinomial** $u^2 + u - 1$ **take if** $u \geq 0$**? To answer this question notice that the vertex of this quadratic trinomial is at**

$u_v = -\frac{b}{2a} = -\frac{1}{2}$. **To the right of the vertex the quadratic function is increasing because** $a > 0$ **and therefore the range of the composition is**

$$[g(f(0), \infty) = [-2, \infty).$$

The graph of the composition is shown below



Problem 4 Consider the function $f(x)=\sqrt{x^3+1}$. Find its domain and its range. Prove that the function f is one-to-one and find its inverse f^{-1}.

Solution The function f is defined if $x^3+1\geq 0$. Factoring the left part of this inequality we write it as $(x+1)(x^2-x+1)\geq 0$. Because $x^2-x+1=\left(x-\frac{1}{2}\right)^2+\frac{3}{4}>0$ the last inequality is equivalent to $x+1\geq 0$ or $x\geq -1$. Thus the domain of f is $[-1,\infty)$. The range of f is clearly $[0,\infty)$.

To prove that f is one-to-one and to find its inverse let $y=\sqrt{x^3+1}$. Notice that $y\geq 0$. By squaring both parts we obtain $y^2=x^3+1$, and $x=\sqrt[3]{y^2-1}$. Thus, if the output y is given the input x is defined in the unique way which proves that f is one-to-one. Finally we get an expression for f^{-1} by interchanging the input and the output.

$$f^{-1}(x)=\sqrt[3]{x^2-1},\ x\geq 0.$$

Problem 5 Graph the functions f and f^{-1} from Problem 4 in the same coordinate system.

Solution