161 Precalculus 1 **Review 3**

Problem 1 Perform computations and present the result as a complex number in the standard form $a+bi$.

$$\frac{(3+2i)(4-3i)}{(6+5i)(5-6i)}$$

Solution

$$\frac{(3+2i)(4-3i)}{(6+5i)(5-6i)}=\frac{12-9i+8i-6i^2}{30-36i+25i-30i^2}.$$

Recalling that $i^2=-1$ **we see that the above expression is equal to** $\frac{18-i}{60-11i}$.

Finally we perform the division

$$\frac{18-i}{60-11i}=\frac{(18-i)(60+11i)}{(60-11i)(60+11i)}=\frac{1091+138i}{60^2+11^2}=\frac{1091}{3721}+\frac{138}{3721}i.$$

Problem 2 Find all the solutions of the equation $x^6 - x^3 + 6 = 0$**.**

Solution The left part of the equation is a trinomial of quadratic type and it can be factored as

$$(x^3 - 3)(x^3 + 2)$$

Therefore we have to solve two equations of degree 3 $x^3 = 3$ **and** $x^3 = -2$**.**
Recall that all the solutions of the equation $x^3 = a$ **where** a **is a real number can be obtained by multiplying all the solutions of the equation** $x^3 = 1$ **by** $\sqrt[3]{a}$ **.**

As we discussed in class the solutions of the last equation are

$$1, \quad \frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}.$$

Therefore all six solutions of our original equation can be obtained as

$$x_1 = \sqrt[3]{3}, \quad x_2 = \sqrt[3]{3}\,\frac{-1+\sqrt{3}i}{2}, \quad x_3 = \sqrt[3]{3}\,\frac{-1-\sqrt{3}i}{2},$$
$$x_4 = -\sqrt[3]{2}, \quad x_5 = -\sqrt[3]{2}\,\frac{-1+\sqrt{3}i}{2}, \quad x_6 = -\sqrt[3]{2}\,\frac{-1-\sqrt{3}i}{2}.$$

Problem 3 Solve the radical equation $\sqrt{x}+\sqrt{x+1}=\sqrt{x+2}$.

Solution. Squaring both parts of the equation we get

$$x+2\sqrt{x(x+1)}+x+1=x+2.$$

Or equivalently

$$2\sqrt{x(x+1)}=1-x.$$

Squaring both parts again we obtain

$$4x(x+1)=x^2-2x+1,$$

which is equivalent to the quadratic equation

$$3x^2+6x-1=0$$

By quadratic formula

$x=\dfrac{-6\pm\sqrt{6^2-4\cdot 3\cdot(-1)}}{2\cdot 3}=\dfrac{-6\pm\sqrt{48}}{6}=-1\pm\dfrac{2\sqrt{3}}{3}$**. Clearly, the negative value of** x **is not a solution of the original equation, but the positive value is (e.g. we can use our calculators to see that** $-1+(2/3)\sqrt{3}\approx 0.1547$ **and that** $\sqrt{0.1547}+\sqrt{1.1547}\approx\sqrt{2.1547}$**). Finally**

$$x=-1+\frac{2\sqrt{3}}{3}.$$

Problem 4 Using your calculator and the method of halving the interval locate the positive solution of the equation $x^3 + x^2 - 1 = 0$ with accuracy 0.01.

Solution Let $f(x) = x^3 + x^2 - 1$. Then $f(0) = -1 < 0$ and $f(1) = 1 > 0$. Therefore the solution is in the interval $[0,1]$. The successive steps are shown in the table below.

Interval	Midpoint	Sign of the value of f at the midpoint
[0, 1]	1/2	-
[1/2, 1]	3/4	-
[3/4, 1]	7/8	+
[3/4, 7/8]	13/16	+
[3/4, 13/16]	25/32	+
[3/4, 25/32]	49/64	+
[3/4, 49/64]	97/128	+
[3/4, 97/128]		

The length of the last interval is $1/128 < 0.01$. We have located the positive solution of our equation in the interval

$$3/4, 97/128 = [0.75, 0.7578125]$$

Problem 5 Using the rational roots test, the factor theorem, and the synthetic division find all the solutions of the polynomial equation and factor the polynomial completely.

$$6x^4 - x^3 - 5x^2 + 11x - 6 = 0$$

Solution. By the rational roots test if there are rational solutions of the equation above then their numerators can be only the numbers $\pm 1, \pm 2, \pm 3, \pm 6$**, and their denominators can be only from the list** $1, 2, 3, 4, 6$**. Therefore the list of all possible rational roots is**

$$\pm 1, \pm 2, \pm 3, \pm 6, \pm 1/2, \pm 3/2, \pm 1/3, \pm 2/3, \pm 1/4, \pm 3/4, \pm 1/6.$$

Plugging in the numbers from the list into the original equation (either by using the synthetic division or a calculator) we successively eliminate the numbers $\pm 1, \pm 2, \pm 3, \pm 6, \pm 1/2, 3/2.$ **But** $-3/2$ **is a solution of the original equation. Indeed the synthetic division goes as follows**

-3/2	**6**	**-1**	**-5**	**11**	**-6**
		-9	**15**	**-15**	**6**
	6	**-10**	**10**	**-4**	**0**

whence $-3/2$ **is indeed a root of the original equation and**

$$6x^4 - x^3 - 5x^2 + 11x - 6 =$$

$$= (x + 3/2)(6x^3 - 10x^2 + 10x - 4) = (2x + 3)(3x^3 - 5x^2 + 5x - 2)$$

We have reduced the problem to solving the cubic equation

$$3x^3 - 5x^2 + 5x - 2 = 0.$$

The list of possible rational roots for this equation is much shorter : $\pm 1/3, \pm 2/3.$ **We easily eliminate** $\pm 1/3$ **but when we come to** $2/3$ **we hit another root.**

2/3	**3**	**-5**	**5**	**-2**
		2	**-2**	**2**
	3	**-3**	**3**	**0**

Therefore

$$3x^3 - 5x^2 + 5x - 2 = (x - 2/3)(3x^2 - 3x + 3) =$$
$$= (3x - 2)(x^2 - x + 1).$$

It remains to solve the quadratic equation $x^2 - x + 1 = 0$**. By the quadratic formula** $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{1 \pm \sqrt{3}i}{2}$**. Finally, the complete list of solutions of the original equation is**

$$-\frac{3}{2}, \frac{2}{3}, \frac{1+\sqrt{3}i}{2}, \frac{1-\sqrt{3}i}{2}$$

The polynomial can be factored as

$$6x^4 - x^3 - 5x^2 + 11x - 6 = (3x+2)(2x-3)\left(x - \frac{1+\sqrt{3}i}{2}\right)\left(x - \frac{1-\sqrt{3}i}{2}\right)$$