151 Linear Mathematics **Review 2**

1. Solve the system

$$\begin{aligned} 2x+3y+z&=9\\ 4x-y+3z&=-1\\ 3x+y-2z&=-4 \end{aligned}$$

Solution. The augmented matrix of the system is $\begin{pmatrix} 2 & 3 & 1 & 9\\ 4 & -1 & 3 & -1\\ 3 & 1 & -2 & -4 \end{pmatrix}$. To get rid of *x* in the second equation we perform the operation R_2-2R_1. The result is $\begin{pmatrix} 2 & 3 & 1 & 9\\ 4-2\cdot 2 & -1-2\cdot 3 & 3-2\cdot 1 & -1-2\cdot 9\\ 3 & 1 & -2 & -4 \end{pmatrix}=\begin{pmatrix} 2 & 3 & 1 & 9\\ 0 & -7 & 1 & -19\\ 3 & 1 & -2 & -4 \end{pmatrix}$. Next we get rid of *x* in the third equation with the help of the operation $2R_3-3R_1$. The result is $\begin{pmatrix} 2 & 3 & 1 & 9\\ 0 & -7 & 1 & -19\\ 2\cdot 3-3\cdot 2 & 2\cdot 1-3\cdot 3 & 2(-2)-3\cdot 1 & 2(-4)-3\cdot 9 \end{pmatrix}=\begin{pmatrix} 2 & 3 & 1 & 9\\ 0 & -7 & 1 & -19\\ 0 & -7 & -7 & -35 \end{pmatrix}$. Finally to bring the matrix to the triangular form we perform the operation R_3-R_2 with the result $\begin{pmatrix} 2 & 3 & 1 & 9\\ 0 & -7 & 1 & -19\\ 0 & -7-(-7) & -7-1 & -35-(-19) \end{pmatrix}=\begin{pmatrix} 2 & 3 & 1 & 9\\ 0 & -7 & 1 & -19\\ 0 & 0 & -8 & -16 \end{pmatrix}$. The corresponding system is

$$\begin{aligned} 2x+3y+z&=9\\ -7y+z&=-19\\ -8z&=-16 \end{aligned}\quad .$$

From the last equation we get $z=2$. Then we plug in this value of *z* into the second equation, $-7y+2=-19,\ -7y=-21,\ y=3$. Finally, we plug in these values of *y* and *z* into the first equation. $2x+3\cdot 3+2=9,\ 2x=-2,\ x=-1$.
The system has the unique solution $x=-1,\ y=3,\ z=2$. Plugging in these numbers into the original system we check that the solution is correct.

2. Show that the following system has no solutions.

$$\begin{aligned} 5x-4y+2z&=4\\ 5x+3y-z&=17\\ 15x-5y+3z&=24 \end{aligned}$$

Solution.

$$\begin{pmatrix} 5 & -4 & 2 & 4 \\ 5 & 3 & -1 & 17 \\ 15 & -5 & 3 & 24 \end{pmatrix} \quad \overrightarrow{R_2 - R_1} \begin{pmatrix} 5 & -4 & 2 & 4 \\ 0 & 7 & -3 & 13 \\ 15 & -5 & 3 & 24 \end{pmatrix} \quad \overrightarrow{R_3 - 3R_1} \begin{pmatrix} 5 & -4 & 2 & 4 \\ 0 & 7 & -3 & 13 \\ 0 & 7 & -3 & 12 \end{pmatrix}.$$

$$\overrightarrow{R_3 - R_2} \begin{pmatrix} 5 & -4 & 2 & 4 \\ 0 & 7 & -3 & 13 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The last equation is a contradiction: 0 = 1. Therefore the system has no solutions.

3. Show that the following system has infinitely many solutions and express its solutions in parametric form.

$$\begin{aligned} x - y + z &= 3 \\ 2x + 3y - 5z &= 5 \\ 8x + 7y - 13z &= 21 \end{aligned}$$

Solution.

$$\begin{pmatrix} 1 & -1 & 1 & 3 \\ 2 & 3 & -5 & 5 \\ 8 & 7 & -13 & 21 \end{pmatrix} \quad \overrightarrow{R_2 - 2R_1} \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 5 & -7 & -1 \\ 8 & 7 & -13 & 21 \end{pmatrix} \quad \overrightarrow{R_3 - 8R_1} \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 5 & -7 & -1 \\ 0 & 15 & -21 & -3 \end{pmatrix}$$

$$\overrightarrow{R_3 - 3R_2} \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 5 & -7 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The last equation is a tautology: 0 = 0, and we are left with two equations

$$\begin{aligned} x - y + z &= 3 \\ 5y - 7z &= -1 \end{aligned}.$$

Let us make z the parameter, $z = t$, where t can be any real number. Then

$5y - 7t = -1, 5y = 7t - 1, y = \frac{7t-1}{5}$

and $x - \frac{7t-1}{5} + t = 3, x = 3 - t + \frac{7t-1}{5} = \frac{15 - 5t + 7t - 1}{5} = \frac{2t+14}{5}$. The solutions in parametric form are

$$\begin{aligned} x &= \frac{2t+14}{5} \\ y &= \frac{7t-1}{5} \\ z &= t \end{aligned}$$

In Problems 4 – 6 let

$$A=\begin{bmatrix}1 & 2 & 5 & -1\\ 3 & 0 & 2 & -4\end{bmatrix} \quad B=\begin{bmatrix}8 & 10 & -5 & 3\\ -2 & -1 & 0 & 0\end{bmatrix}.$$

4. Compute $3A-2B$.

Solution.

$$3A-2B=3\begin{bmatrix}1 & 2 & 5 & -1\\ 3 & 0 & 2 & -4\end{bmatrix}-2\begin{bmatrix}8 & 10 & -5 & 3\\ -2 & -1 & 0 & 0\end{bmatrix}=\begin{bmatrix}3 & 6 & 15 & -3\\ 9 & 0 & 6 & -12\end{bmatrix}-\begin{bmatrix}16 & 20 & -10 & 6\\ -4 & -2 & 0 & 0\end{bmatrix}.$$

$$=\begin{bmatrix}-13 & -14 & 25 & -9\\ 13 & 2 & 6 & -12\end{bmatrix}$$

5. Compute AB^T.

Solution.

$$AB^T=\begin{bmatrix}1 & 2 & 5 & -1\\ 3 & 0 & 2 & -4\end{bmatrix}\begin{bmatrix}8 & -2\\ 10 & -1\\ -5 & 0\\ 3 & 0\end{bmatrix}=$$

$$=\begin{bmatrix}1\cdot 8+2\cdot 10+5(-5)+(-1)3 & 1(-2)+2(-1)+5\cdot 0+(-1)0\\ 3\cdot 8+0\cdot 10+2(-5)+(-4)3 & 3(-2)+0(-1)+2\cdot 0+(-4)0\end{bmatrix}=\begin{bmatrix}0 & -4\\ 2 & -6\end{bmatrix}.$$

6. Compute $A^T B$.

Solution.

$$\begin{bmatrix}1 & 3\\ 2 & 0\\ 5 & 2\\ -1 & -4\end{bmatrix}\begin{bmatrix}8 & 10 & -5 & 3\\ -2 & -1 & 0 & 0\end{bmatrix}=\begin{bmatrix}1\cdot 8+3(-2) & 1\cdot 10+3(-1) & 1(-5)+3\cdot 0 & 1\cdot 3+3\cdot 0\\ 2\cdot 8+0(-2) & 2\cdot 10+0(-1) & 2(-5)+0\cdot 0 & 2\cdot 3+0\cdot 0\\ 5\cdot 8+2(-2) & 5\cdot 10+2(-1) & 5(-5)+2\cdot 0 & 5\cdot 3+2\cdot 0\\ (-1)8+(-4)(-2) & (-1)10+(-4)(-1) & (-1)(-5)+(-4)0 & (-1)3+(-4)0\end{bmatrix}=$$

$$=\begin{bmatrix}2 & 7 & -5 & 3\\ 16 & 20 & -10 & 6\\ 36 & 48 & -25 & 15\\ 0 & -6 & 5 & -3\end{bmatrix}.$$

7. Let $A=\begin{bmatrix}1 & 2\\ -1 & 2\end{bmatrix}$. Compute the inverse matrix A^{-1}.

Solution. Recall that if $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ and $\det A = ad-bc\neq 0$ then the matrix A is invertible and $A^{-1}=\frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$. In our case $ad-bc=1\cdot 2-2(-1)=4$ and
$$A^{-1}=\frac{1}{4}\begin{bmatrix}2 & -2\\ 1 & 1\end{bmatrix}=\begin{bmatrix}1/2 & -1/2\\ 1/4 & 1/4\end{bmatrix}.$$

8. Let $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 4 & 9\end{bmatrix}$. Compute A^{-1}.

Solution. We will solve the problem in two ways – by Gauss – Jordan elimination and using cofactors (when solving a similar problem on the test you can solve it the way you like).

(a) Gauss – Jordan elimination.

$$\left[\begin{matrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 4 & 9\end{matrix}\right.\left|\left.\begin{matrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{matrix}\right]\right. \overrightarrow{R_2-R_1} \left[\begin{matrix}1 & 1 & 1\\ 0 & 1 & 2\\ 1 & 4 & 9\end{matrix}\right.\left|\left.\begin{matrix}1 & 0 & 0\\ -1 & 1 & 0\\ 0 & 0 & 1\end{matrix}\right]\right. \overrightarrow{R_3-R_1}$$

$$\left[\begin{matrix}1 & 1 & 1\\ 0 & 1 & 2\\ 0 & 3 & 8\end{matrix}\right.\left|\left.\begin{matrix}1 & 0 & 0\\ -1 & 1 & 0\\ -1 & 0 & 1\end{matrix}\right]\right. \overrightarrow{R_3-3R_2} \left[\begin{matrix}1 & 1 & 1\\ 0 & 1 & 2\\ 0 & 0 & 2\end{matrix}\right.\left|\left.\begin{matrix}1 & 0 & 0\\ -1 & 1 & 0\\ 2 & -3 & 1\end{matrix}\right]\right. \overrightarrow{R_3/2}$$

$$\left[\begin{matrix}1 & 1 & 1\\ 0 & 1 & 2\\ 0 & 0 & 1\end{matrix}\right.\left|\left.\begin{matrix}1 & 0 & 0\\ -1 & 1 & 0\\ 1 & -3/2 & 1/2\end{matrix}\right]\right. \overrightarrow{R_2-2R_3} \left[\begin{matrix}1 & 1 & 1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{matrix}\right.\left|\left.\begin{matrix}1 & 0 & 0\\ -3 & 4 & -1\\ 1 & -3/2 & 1/2\end{matrix}\right]\right. \overrightarrow{R_1-R_3}$$

$$\left[\begin{matrix}1 & 1 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{matrix}\right.\left|\left.\begin{matrix}0 & 3/2 & -1/2\\ -3 & 4 & -1\\ 1 & -3/2 & 1/2\end{matrix}\right]\right. \overrightarrow{R_1-R_2} \left[\begin{matrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{matrix}\right.\left|\left.\begin{matrix}3 & -5/2 & 1/2\\ -3 & 4 & -1\\ 1 & -3/2 & 1/2\end{matrix}\right]\right..$$

We got that the inverse matrix is
$$A^{-1}=\begin{bmatrix}3 & -5/2 & 1/2\\ -3 & 4 & -1\\ 1 & -3/2 & 1/2\end{bmatrix}=\frac{1}{2}\begin{bmatrix}6 & -5 & 1\\ -6 & 8 & -2\\ 2 & -3 & 1\end{bmatrix}.$$

It is a good idea to check our answer.

$$A^{-1}A=\frac{1}{2}\begin{bmatrix}6&-5&1\\-6&8&-2\\2&-3&1\end{bmatrix}\begin{bmatrix}1&1&1\\1&2&3\\1&4&9\end{bmatrix}=$$

$$=\frac{1}{2}\begin{bmatrix}6\cdot1+(-5)1+1\cdot1 & 6\cdot1+(-5)2+1\cdot4 & 6\cdot1+(-5)3+1\cdot9\\(-6)1+8\cdot1+(-2)\cdot1 & (-6)1+8\cdot2+(-2)4 & (-6)1+8\cdot3+(-2)9\\2\cdot1+(-3)1+1\cdot1 & 2\cdot1+(-3)2+1\cdot4 & 2\cdot1+(-3)3+1\cdot9\end{bmatrix}=$$

$$=\frac{1}{2}\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.$$

(b) Method of cofactors. We will use the notation $\begin{vmatrix}a&b\\c&d\end{vmatrix}=ad-bc$. Recall also the rule of signs for cofactors $\begin{vmatrix}+&-&+\\-&+&-\\+&-&+\end{vmatrix}$. The matrix of cofactors, A^C , can be computed in the following way.

$$A^C=\begin{bmatrix}\begin{vmatrix}2&3\\4&9\end{vmatrix} & -\begin{vmatrix}1&3\\1&9\end{vmatrix} & \begin{vmatrix}1&2\\1&4\end{vmatrix}\\ -\begin{vmatrix}1&1\\4&9\end{vmatrix} & \begin{vmatrix}1&1\\1&9\end{vmatrix} & -\begin{vmatrix}1&1\\1&4\end{vmatrix}\\ \begin{vmatrix}1&1\\2&3\end{vmatrix} & -\begin{vmatrix}1&1\\1&3\end{vmatrix} & \begin{vmatrix}1&1\\1&2\end{vmatrix}\end{bmatrix}=$$

$$=\begin{bmatrix}2\cdot9-3\cdot4 & -(1\cdot9-3\cdot1) & 1\cdot4-2\cdot1\\-(1\cdot9-1\cdot4) & 1\cdot9-1\cdot1 & -(1\cdot4-1\cdot1)\\1\cdot3-1\cdot2 & -(1\cdot3-1\cdot1) & 1\cdot2-1\cdot1\end{bmatrix}=\begin{bmatrix}6&-6&2\\-5&8&-3\\1&-2&1\end{bmatrix}.$$

Next we compute the matrix $\left(A^C\right)^T=\begin{bmatrix}6&-5&1\\-6&8&-2\\2&-3&1\end{bmatrix}$.This matrix is proportional to the matrix A^{-1}. The product $\left(A^C\right)^T A=\begin{bmatrix}6&-5&1\\-6&8&-2\\2&-3&1\end{bmatrix}\begin{bmatrix}1&1&1\\1&2&3\\1&4&9\end{bmatrix}=\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$. Therefore

$$A^{-1}=\frac{1}{2}\left(A^C\right)^T=\begin{bmatrix}3&-5/2&1/2\\-3&4&-1\\1&-3/2&1/2\end{bmatrix}.$$

9. **AN INPUT-OUTPUT MODEL FOR A THREE-SECTOR ECONOMY**. Consider the economy consisting of three sectors: agriculture *(A)*, manufacturing (*M*), and transportation (*T*), with an input-output matrix given by

$$\begin{array}{c|c} & \begin{array}{ccc} A & M & T \end{array} \\ \begin{array}{c} A \\ M \\ T \end{array} & \begin{bmatrix} 0.4 & 0.1 & 0.1 \\ 0.1 & 0.4 & 0.3 \\ 0.2 & 0.2 & 0.2 \end{bmatrix} \end{array}$$

a. Find the gross output of goods needed to satisfy a consumer demand for $200 million worth of agricultural products, $100 million worth of manufactured products, and $60 million worth of transportation.
b. Find the value of goods and transportation consumed in the internal process of production in order to meet this gross output.

Solution. (a) The demand matrix, *D*, is $D = \begin{bmatrix} 200 \\ 100 \\ 60 \end{bmatrix}$. The matrix, *X*, of the gross output required to satisfy this demand is given by the formula $X = (I - A)^{-1} D$. To find *X* we will first compute $I - A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.4 & 0.1 & 0.1 \\ 0.1 & 0.4 & 0.3 \\ 0.2 & 0.2 & 0.2 \end{bmatrix} = \begin{bmatrix} 0.6 & -0.1 & -0.1 \\ -0.1 & 0.6 & -0.3 \\ -0.2 & -0.2 & 0.8 \end{bmatrix}$.

To find $(I - A)^{-1}$ it is convenient to write $I - A = 0.1B$ where $B = \begin{bmatrix} 6 & -1 & -1 \\ -1 & 6 & -3 \\ -2 & -2 & 8 \end{bmatrix}$.

Recall that $(0.1B)^{-1} = (0.1)^{-1} B^{-1} = 10B^{-1}$. Therefore we will first find the matrix B^{-1}. We will use the method of cofactors. Let us compute the matrix of cofactors, B^C.

$$B^C = \begin{bmatrix} \begin{vmatrix} 6 & -3 \\ -2 & 8 \end{vmatrix} & -\begin{vmatrix} -1 & -3 \\ -2 & 8 \end{vmatrix} & \begin{vmatrix} -1 & 6 \\ -2 & -2 \end{vmatrix} \\ -\begin{vmatrix} -1 & -1 \\ -2 & 8 \end{vmatrix} & \begin{vmatrix} 6 & -1 \\ -2 & 8 \end{vmatrix} & -\begin{vmatrix} 6 & -1 \\ -2 & -2 \end{vmatrix} \\ \begin{vmatrix} -1 & -1 \\ 6 & -3 \end{vmatrix} & -\begin{vmatrix} 6 & -1 \\ -1 & -3 \end{vmatrix} & \begin{vmatrix} 6 & -1 \\ -1 & 6 \end{vmatrix} \end{bmatrix} =$$

$$= \begin{bmatrix} 6 \cdot 8 - (-3)(-2) & -[(-1)8 - (-3)(-2)] & (-1)(-2) - 6(-2) \\ -[(-1)8 - (-1)(-2)] & 6 \cdot 8 - (-1)(-2) & -[6(-2) - (-1)(-2)] \\ (-1)(-3) - (-1)6 & -[6(-3) - (-1)(-1)] & 6 \cdot 6 - (-1)(-1) \end{bmatrix} = \begin{bmatrix} 42 & 14 & 14 \\ 10 & 46 & 14 \\ 9 & 19 & 35 \end{bmatrix}.$$

The transpose to this matrix is $\left(B^C\right)^T = \begin{bmatrix} 42 & 10 & 9 \\ 14 & 46 & 19 \\ 14 & 14 & 35 \end{bmatrix}$. Next

$$B\left(B^C\right)^T = \begin{bmatrix} 6 & -1 & -1 \\ -1 & 6 & -3 \\ -2 & -2 & 8 \end{bmatrix}\begin{bmatrix} 42 & 10 & 9 \\ 14 & 46 & 19 \\ 14 & 14 & 35 \end{bmatrix} =$$

$$= \begin{bmatrix} 6\cdot 42 - 1\cdot 14 - 1\cdot 14 & 6\cdot 10 - 1\cdot 46 - 1\cdot 14 & 6\cdot 9 - 1\cdot 19 - 1\cdot 35 \\ -1\cdot 42 + 6\cdot 14 - 3\cdot 14 & -1\cdot 10 + 6\cdot 46 - 3\cdot 14 & -1\cdot 9 + 6\cdot 19 - 3\cdot 35 \\ -2\cdot 42 - 2\cdot 14 + 8\cdot 14 & -2\cdot 10 - 2\cdot 46 + 8\cdot 14 & -2\cdot 9 - 2\cdot 19 + 8\cdot 35 \end{bmatrix} = \begin{bmatrix} 224 & 0 & 0 \\ 0 & 224 & 0 \\ 0 & 0 & 224 \end{bmatrix}$$

Therefore

$$B^{-1} = \frac{1}{224}\left(B^C\right)^T = \frac{1}{224}\begin{bmatrix} 42 & 10 & 9 \\ 14 & 46 & 19 \\ 14 & 14 & 35 \end{bmatrix} \text{ and}$$

$$(I - A)^{-1} = 10B^{-1} = \frac{10}{224}\begin{bmatrix} 42 & 10 & 9 \\ 14 & 46 & 19 \\ 14 & 14 & 35 \end{bmatrix} = \frac{5}{112}\begin{bmatrix} 42 & 10 & 9 \\ 14 & 46 & 19 \\ 14 & 14 & 35 \end{bmatrix}.$$

Finally,

$$X = (I - A)^{-1}D = \frac{5}{112}\begin{bmatrix} 42 & 10 & 9 \\ 14 & 46 & 19 \\ 14 & 14 & 35 \end{bmatrix}\begin{bmatrix} 200 \\ 100 \\ 60 \end{bmatrix} = \frac{5}{112}\begin{bmatrix} 42\cdot 200 + 10\cdot 100 + 9\cdot 60 \\ 14\cdot 200 + 46\cdot 100 + 19\cdot 60 \\ 14\cdot 200 + 14\cdot 100 + 35\cdot 60 \end{bmatrix} =$$

$$= \frac{5}{112}\begin{bmatrix} 9940 \\ 8540 \\ 6300 \end{bmatrix} = \begin{bmatrix} 443.75 \\ 381.25 \\ 281.25 \end{bmatrix}.$$

It means that in order to satisfy the demand the gross product in agriculture should be \$443.75 million, in manufacturing - \$381.25 million, and in transportation - \$281.25 million.

(b) The answer to part b is given by the formula

$X - D = \begin{bmatrix} 443.75 \\ 381.25 \\ 281.25 \end{bmatrix} - \begin{bmatrix} 200 \\ 100 \\ 60 \end{bmatrix} = \begin{bmatrix} 243.75 \\ 281.25 \\ 221.25 \end{bmatrix}$ whence the value of the goods internally consumed is: in agriculture – \$243.75 million, in manufacturing – \$281.25 million, and in transportation – \$221.25 million.

10. Solve Problem 1 on page 8 in the supplement.
Display the scatter plot and find the regression line for the points (3, 5), (4, 6), (5, 8), (6, 10), and (7, 9). Display the graph of the regression line on the scatter plot.

Solution. We are looking for an equation of the regression line in the form $y = mx + b$. If we could find the line going exactly through all the five points above then *m* and *b* would satisfy the following system of linear equations.

$$\begin{aligned} 3m + b &= 5 \\ 4m + b &= 6 \\ 5m + b &= 8 \\ 6m + b &= 10 \\ 7m + b &= 9 \end{aligned}$$

In the matrix form this system can be written as $AX = B$ where

$$X = \begin{bmatrix} m \\ b \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 6 \\ 8 \\ 10 \\ 9 \end{bmatrix}, \quad \text{and } A = \begin{bmatrix} 3 & 1 \\ 4 & 1 \\ 5 & 1 \\ 6 & 1 \\ 7 & 1 \end{bmatrix}.$$

But the points are not on the same straight line and the system $AX = B$ has no exact solutions. We will get the best **approximate** solution if we solve the system $A^T AX = A^T B$. Let us compute the matrices involved in this system.

$$A^T = \begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \quad A^T A = \begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 4 & 1 \\ 5 & 1 \\ 6 & 1 \\ 7 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 3^2 + 4^2 + 5^2 + 6^2 + 7^2 & 3+4+5+6+7 \\ 3+4+5+6+7 & 1+1+1+1+1 \end{bmatrix} = \begin{bmatrix} 135 & 25 \\ 25 & 5 \end{bmatrix},$$

$$A^T B = \begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 8 \\ 10 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \cdot 5 + 4 \cdot 6 + 5 \cdot 8 + 6 \cdot 10 + 7 \cdot 9 \\ 5 + 6 + 8 + 10 + 9 \end{bmatrix} = \begin{bmatrix} 202 \\ 38 \end{bmatrix}.$$

To solve the system $A^T AX = A^T B$ notice that the inverse matrix $\left(A^T A\right)^{-1}$ can be computed as $\left(A^T A\right)^{-1} = \dfrac{1}{135 \cdot 5 - 25 \cdot 25} \begin{bmatrix} 5 & -25 \\ -25 & 135 \end{bmatrix} = \dfrac{1}{50} \begin{bmatrix} 5 & -25 \\ -25 & 135 \end{bmatrix} = \begin{bmatrix} 0.1 & -0.5 \\ -0.5 & 2.7 \end{bmatrix}.$

Therefore $X = \left(A^T A\right)^{-1}(A^T B) = \begin{bmatrix} 0.1 & -0.5 \\ -0.5 & 2.7 \end{bmatrix}\begin{bmatrix} 202 \\ 38 \end{bmatrix} = \begin{bmatrix} 20.2-19 \\ -101+102.6 \end{bmatrix} = \begin{bmatrix} 1.2 \\ 1.6 \end{bmatrix}$.

We get $m = 1.2, b = 1.6$, and the slope-intercept equation of the regression line is $y = 1.2x + 1.6$.

The computer generated graph below shows the points and the regression line on the same screen.