Review 4

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the minimum sample size you should use to assure that your estimate of $\hat{p}$ will be within the required margin of error around the population p.

1) Margin of error: 0.002; confidence level: 93%; $\hat{p}$ and $\hat{q}$ unknown 1) _____
A) 410 B) 409 C) 204,757 D) 204,756

2) Margin of error: 0.07; confidence level: 90%; from a prior study, $\hat{p}$ is estimated by 0.19. 2) _____
A) 75 B) 85 C) 255 D) 6

Use the given degree of confidence and sample data to construct a confidence interval for the population proportion p.

3) Of 346 items tested, 12 are found to be defective. Construct the 98% confidence interval for the proportion of all such items that are defective. 3) _____
A) $0.0345 < p < 0.0349$ B) $0.0154 < p < 0.0540$
C) $0.0118 < p < 0.0576$ D) $0.0110 < p < 0.0584$

Use the confidence level and sample data to find a confidence interval for estimating the population μ.

4) A random sample of 79 light bulbs had a mean life of $\bar{x} = 400$ hours with a standard deviation of $\sigma = 28$ hours. Construct a 90 percent confidence interval for the mean life, μ, of all light bulbs of this type. 4) _____
A) $393 < \mu < 407$ B) $394 < \mu < 406$ C) $395 < \mu < 405$ D) $392 < \mu < 408$

Use the margin of error, confidence level, and standard deviation σ to find the minimum sample size required to estimate an unknown population mean μ.

5) Margin of error: \$121, confidence level: 95%, σ = \$528 5) _____
A) 4 B) 64 C) 2 D) 74

Use the given degree of confidence and sample data to construct a confidence interval for the population mean μ. Assume that the population has a normal distribution.

6) A laboratory tested twelve chicken eggs and found that the mean amount of cholesterol was 193 milligrams with s = 15.4 milligrams. Construct a 95 percent confidence interval for the true mean cholesterol content of all such eggs. 6) _____
A) $185.0 < \mu < 201.0$ B) $183.2 < \mu < 202.8$
C) $183.1 < \mu < 202.9$ D) $183.3 < \mu < 202.7$

Find the appropriate minimum sample size.

7) You want to be 95% confident that the sample variance is within 30% of the population variance. 7) _____
A) 346 B) 723 C) 97 D) 130

Use the given degree of confidence and sample data to find a confidence interval for the population standard deviation σ. Assume that the population has a normal distribution.

8) The amounts (in ounces) of juice in eight randomly selected juice bottles are: 8) _____
15.7 15.6 15.3 15.3
15.1 15.6 15.3 15.4
Find a 98 percent confidence interval for the population standard deviation σ.
A) (0.17, 0.66) B) (0.12, 0.42) C) (0.13, 0.42) D) (0.13, 0.48)

Solutions

Problem 1

We use formula 7-3 on page 328

$$n = \frac{[z_{\alpha/2}]^2 \cdot 0.25}{E^2}.$$

In our case $E = 0.002, \quad \alpha = 1 - 0.93 = 0.07, \quad \alpha/2 = 0.035.$ **From Table A-2 we find**

$$z_{0.035} = 1.81$$

Plugging in the numbers into the formula above and rounding to the nearesr larger integer we get n **= 204757. The correct answer is "C".**

Problem 2

We use formula 7-2 on page 328

$$n = \frac{[z_{\alpha/2}]^2 \, \hat{p}(1-\hat{p})}{E^2}.$$

In this problem

$$E = 0.07, \quad \hat{p} = 0.19, \quad 1 - \hat{p} = 0.81, \quad \alpha = 0.10,$$

$$z_{\alpha/2} = z_{0.05} = 1.645.$$

Therefore $n = 85$**. The correct answer is "B".**

Problem 3

We use the formula on page 326 for the confidence interval

$$\hat{p} - E < p < \hat{p} + E \text{ where } E = z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

Into this formula we plug in the numbers

$$\hat{p} = \frac{12}{346} \approx 0.0347, \quad 1 - \hat{p} \approx 0.9653, \quad n = 346,$$

$$z_{\alpha/2} = z_{0.01} = 2.33,$$

and compute $E \approx 0.0229$**, whence the confidence interval is**

$$0.0118 < p < .0576.$$

The correct answer is "C".

Problem 4

We use the formula from Page 340 for the confidence interval for the population mean

$$\overline{x} - E < \mu < \overline{x} + E, \text{ where } E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}.$$

Plugging in the numbers

$$n = 79, \quad \overline{x} = 400, \quad \sigma = 28, \quad z_{\alpha/2} = z_{0.05} = 1.645$$

we find that $E = 1.645 \cdot \frac{28}{\sqrt{79}} \approx 5.18$**. Rounding to the nearest larger integer we take** $E = 6$ **whence the confidence interval is** $394 < \overline{x} < 406$**. The correct answer is "B".**

Problem 5

We use formula 7-5 on page 343

$$n = \left\lceil \frac{z_{\alpha/2}\sigma}{E} \right\rceil^2$$

where $E = 121, \quad \sigma = 528, \quad z_{\alpha/2} = z_{0.025} = 1.96$**. Therefore**

$$n = \left\lceil \frac{1.96 \cdot 528}{121} \right\rceil^2 \approx 73.15.$$

Rounding upward we get n **= 74. The correct answer is "D".**

Problem 6

We use formula on page 351

$$\overline{x} - E < \mu < \overline{x} + E, \text{ where } E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

and t **is the Student distribution with** n-1 **degrees of freedom. In our case** $n = 12$ **and the number of degrees of freedom is**11**. From Table A-3 on age 774 we find** $t_{0.025} = 2.201$ **whence** $E = 2.201 \cdot \frac{15.4}{\sqrt{12}} \approx 9.8$ **and the confidence interval is**

$$183.2 < \mu < 202.8.$$

The correct answer is "B".

Problem 7

From Table 7-2 on page 371 we find that the sample size should be 97. The correct answer is "C".

Problem 8

We use the formula on page 367

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}.$$

From formula 3-5 on page 94 we see that

$$(n-1)s^2 = \sum x^2 - \left(\sum x\right)^2 / n = 1900.65 - 123.3^2/8 = 0.28875.$$

Looking at Table A-4 and noticing that the number of degrees of freedom is 7, area to the right of χ_L^2 **is 0.99, and area to the right of** χ_R^2 **is 0.01, we see that** $\chi_L^2 = 1.239$ **and** $\chi_R^2 = 18.475$**. Therefore the confidence interval is**

$$\sqrt{\frac{0.28875}{18.475}} < \sigma < \sqrt{\frac{0.28875}{1.239}}.$$

Or after rounding the left bound downward and the right one upward we obtain

$$0.12 < \sigma < 0.49.$$

The closest answer to the one we got is "D".