Problem 1. Consider the function \( y = -2 \cos \left( \frac{2\pi}{3} x + \frac{\pi}{3} \right) \).

(a) Find the amplitude, the period, and the shift.

(b) Graph the function.

Solution. (a) We identify the coefficients in this expression as

\[ a = -2, \quad b = \frac{2\pi}{3}, \quad c = -\frac{\pi}{3}. \]

Next we compute the amplitude \( |a| = 2 \), the period \( \frac{2\pi}{b} = 3 \), and the shift

\[ \frac{c}{b} = -\frac{1}{2}. \]

The graph of the function is shown below.
Problem 2 (a) Find the standard restricted domain over which the function \( y \) from Problem 1 is one-to-one.

(b) Find the inverse function to this restriction of \( y \).

(c) Graph the function \( y \) with the restricted domain and the inverse function in the same coordinate system.

Solution (a) the standard interval where the function \( \cos x \) is one-to-one is \([0, \pi]\). Respectively for the function \( y \) we have to consider the interval

\[
0 \leq \frac{2\pi}{3} x + \frac{\pi}{3} \leq \pi
\]

Whence

\[
-\frac{\pi}{3} \leq \frac{2\pi}{3} x \leq \frac{2\pi}{3}, \quad \text{and}
\]

\[
-\frac{1}{2} \leq x \leq 1
\]

(b) We consider the interval above and solve the equation

\[
y = -2 \cos \left( \frac{2\pi}{3} x + \frac{\pi}{3} \right)
\]

for \( x \).

\[
-\frac{y}{2} = \cos \left( \frac{2\pi}{3} x + \frac{\pi}{3} \right),
\]

\[
\frac{2\pi}{3} x + \frac{\pi}{3} = \arccos \left( -\frac{y}{2} \right),
\]

\[
x = \frac{3}{2\pi} \arccos \left( -\frac{y}{2} \right) - \frac{1}{2}.
\]
Therefore the inverse function is defined as

\[ y^{-1}(x) = \frac{3}{2\pi} \arccos\left(-\frac{x}{2}\right) - \frac{1}{2}. \]

(C) The graphs of \( y \) and \( y^{-1} \) are shown below.
Problem 3 Consider the function $y = \tan\left(\frac{1}{3}x - \frac{\pi}{6}\right)$.

(a) Find the period and the shift.

(b) Graph the function.

Solution (a) the period is $\pi \div (1/3) = 3\pi$, the shift is $\pi/6 \div 1/3 = \pi/2$.

(b) The graph is shown below
Problem 4

(a) Find the standard restricted domain over which the function $y$ from Problem 1 is one-to-one.

(b) Find the inverse function to this restriction of $y$.

(c) Graph the function $y$ with the restricted domain and the inverse function in the same coordinate system.

(a) The standard interval where $\tan x$ is one-to-one is $\left(-\pi/2, \pi/2\right)$.

Respectively for $y$ we have $-\frac{\pi}{2} < \frac{x}{3} - \frac{\pi}{6} < \frac{\pi}{2}$, whence $-\pi < x < 2\pi$.

(b) Over this restricted domain we have $\frac{1}{3} x - \frac{\pi}{6} = \arctan y$, whence $x = 3 \arctan y + \pi/2$, and

$$y^{-1}(x) = 3 \arctan x + \pi/2.$$ 

(c) The graphs are shown below.
Problem 5 (a) Find the exact value (no calculators!) of $\csc(\arctan(-7/5))$.

(b) Rewrite $\tan(\arcsin(\sqrt{x}))$ as an equivalent algebraic expression.

Solution (a) First notices that because the functions $\csc$ and $\arctan$ are both odd functions we have

$$\csc(\arctan(-7/5)) = -\csc(\arctan(7/5)) = -\csc(\arctan(7/5)).$$

To find the value of $\csc(\arctan(7/5))$ consider the right triangle with an acute angle $\theta$, opposite side $7$, and adjacent side $5$. Then

$$\csc(\arctan(7/5)) = \csc \theta = 7/\sqrt{5^2 + 7^2} = 7/\sqrt{74},$$

whence the answer to part (a) is $-7/\sqrt{74}$.

(b) Consider the right triangle with an acute angle $\theta$, opposite side $\sqrt{x}$, and hypotenuse $1$. Then

$$\tan(\arcsin(\sqrt{x})) = \tan \theta = \frac{\sqrt{x}}{\sqrt{1-(\sqrt{x})^2}} = \frac{\sqrt{x}}{\sqrt{1-x}} = \frac{x}{\sqrt{1-x}}.$$