Problem 1. Consider the function $y = -2\cos\left(\frac{2\pi}{3}x + \frac{\pi}{3}\right)$**.**

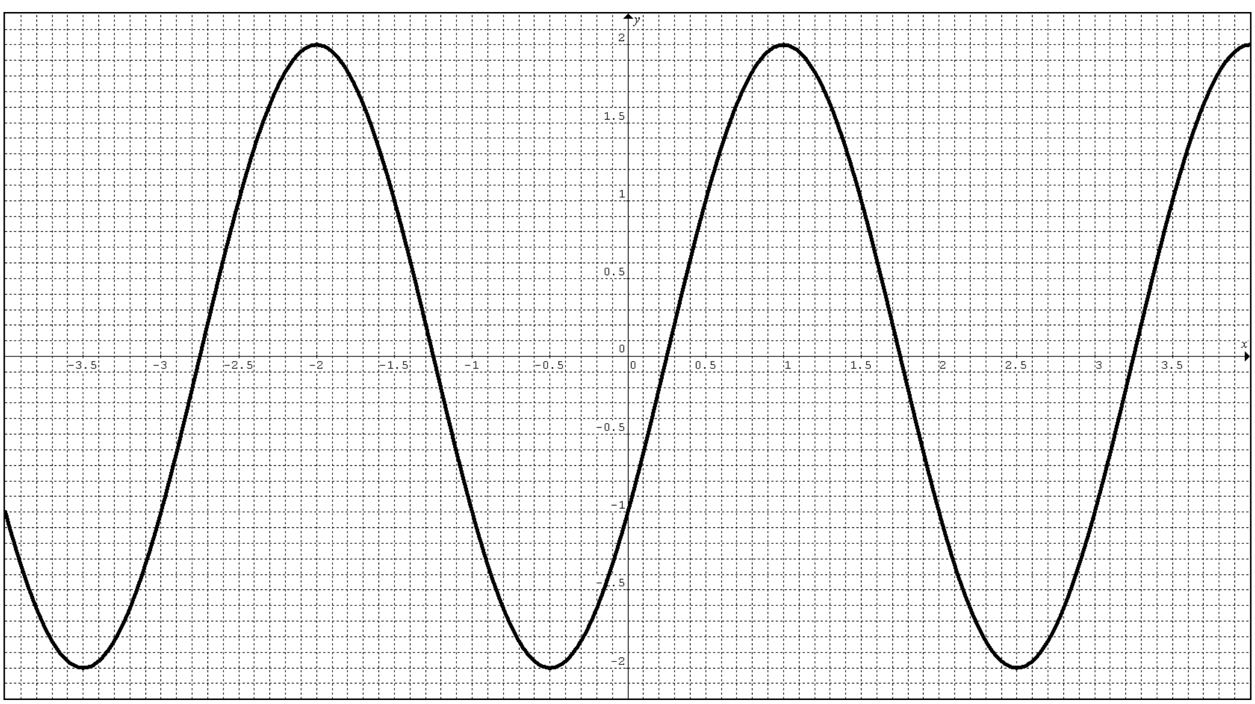
(a) Find the amplitude, the period, and the shift.

(b) Graph the function.

Solution. (a) We identify the coefficients in this expression as

$$a = -2, b = \frac{2\pi}{3}, c = -\frac{\pi}{3}.$$

Next we compute the amplitude $|a| = 2$**, the period** $\frac{2\pi}{b} = 3$**, and the shift** $\frac{c}{b} = -\frac{1}{2}$**. The graph of the function is shown below.**



Problem 2 (a) Find the standard restricted domain over which the function y from Problem 1 is one-to-one.

(b) Find the inverse function to this restriction of y.

(c) Graph the function y with the restricted domain and the inverse function in the same coordinate system.

Solution (a) the standard interval where the function $\cos x$ is one-to-one is $[0, \pi]$. Respectively for the function y we have to consider the interval

$$0 \le \frac{2\pi}{3}x + \frac{\pi}{3} \le \pi$$

Whence $-\frac{\pi}{3} \le \frac{2\pi}{3}x \le \frac{2\pi}{3}$, **and**

$$-\frac{1}{2} \le x \le 1$$

(b) We consider the interval above and solve the equation

$y = -2\cos\left(\frac{2\pi}{3}x + \frac{\pi}{3}\right)$ **for** x.

$$-\frac{y}{2} = \cos\left(\frac{2\pi}{3}x + \frac{\pi}{3}\right),$$

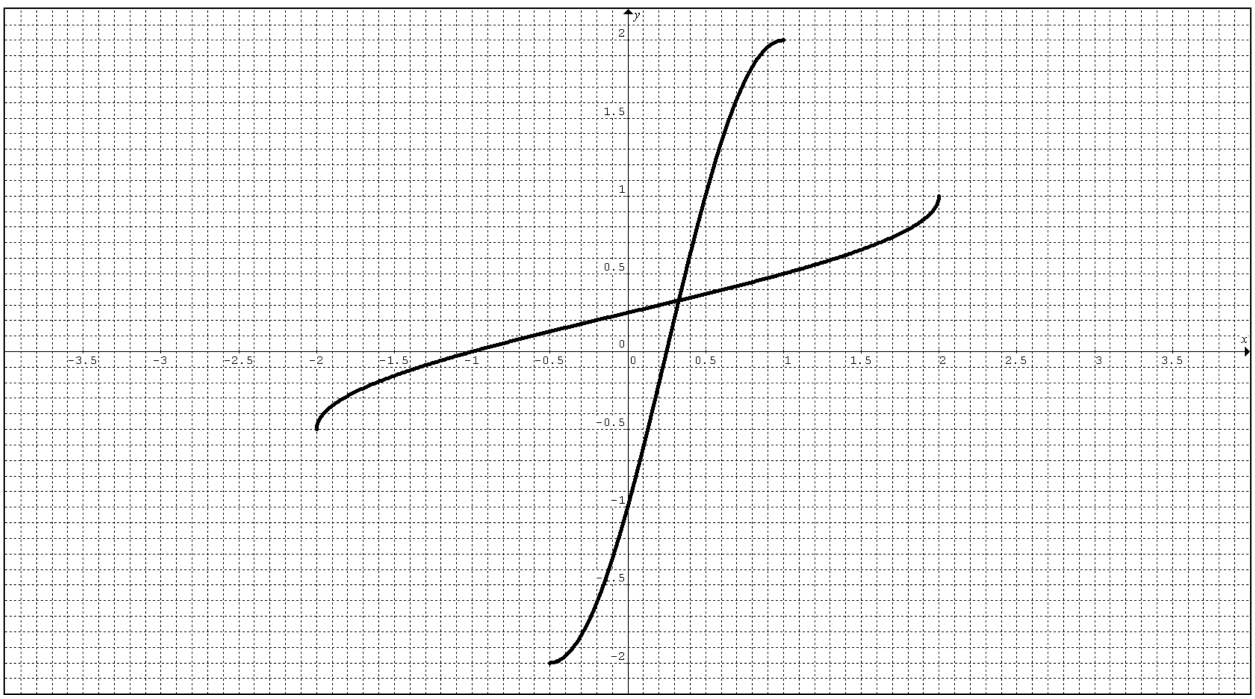
$$\frac{2\pi}{3}x + \frac{\pi}{3} = \arccos\left(-\frac{y}{2}\right),$$

$$x = \frac{3}{2\pi}\arccos\left(-\frac{y}{2}\right) - \frac{1}{2}$$

Therefore the inverse function is defined as

$$y^{-1}(x) = \frac{3}{2\pi}\arccos\left(-\frac{x}{2}\right) - \frac{1}{2}.$$

(C) The graphs of y and y^{-1} are shown below.



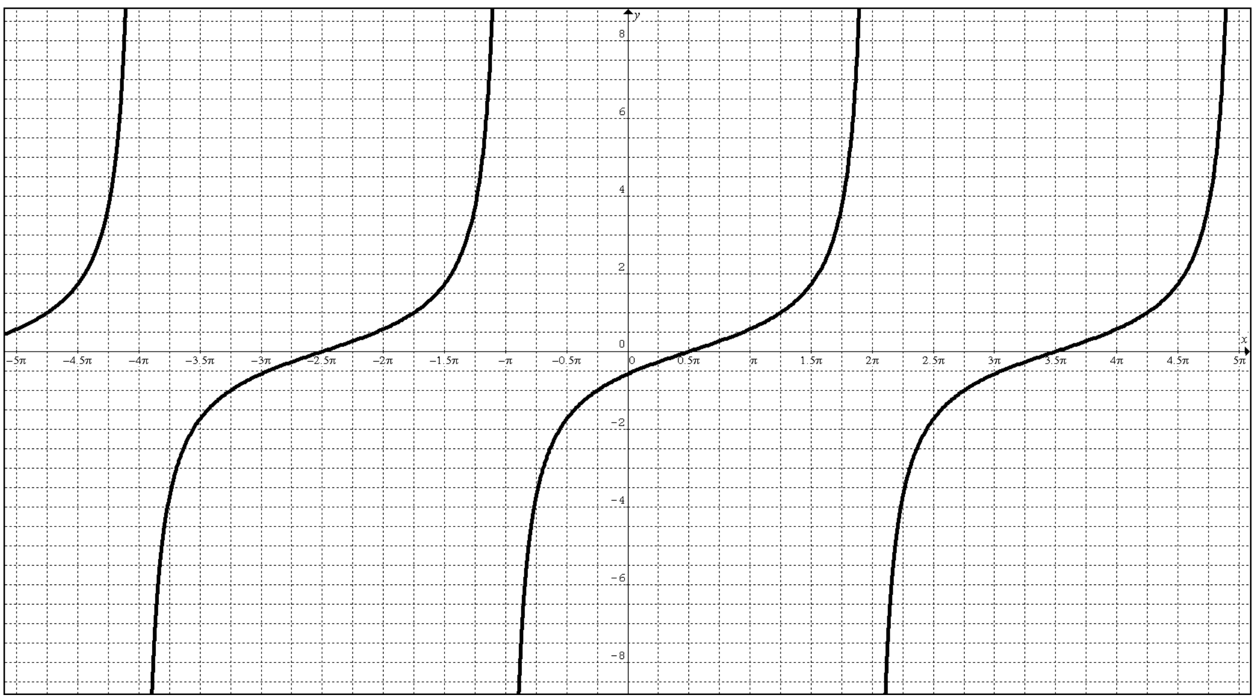
Problem 3 Consider the function $y = \tan\left(\frac{1}{3}x - \frac{\pi}{6}\right)$.

(a) Find the period and the shift.

(b) Graph the function.

Solution (a) the period is $\pi \div (1/3) = 3\pi$**, the shift is** $\pi/6 \div 1/3 = \pi/2$.

(b) The graph is shown below



Problem 4(a) Find the standard restricted domain over which the function y from Problem 1 is one-to-one.

(b) Find the inverse function to this restriction of y.

(c) Graph the function y with the restricted domain and the inverse function in the same coordinate system.

(a) The standard interval where $\tan x$ **is one-to-one is** $(-\pi/2, \pi/2)$.

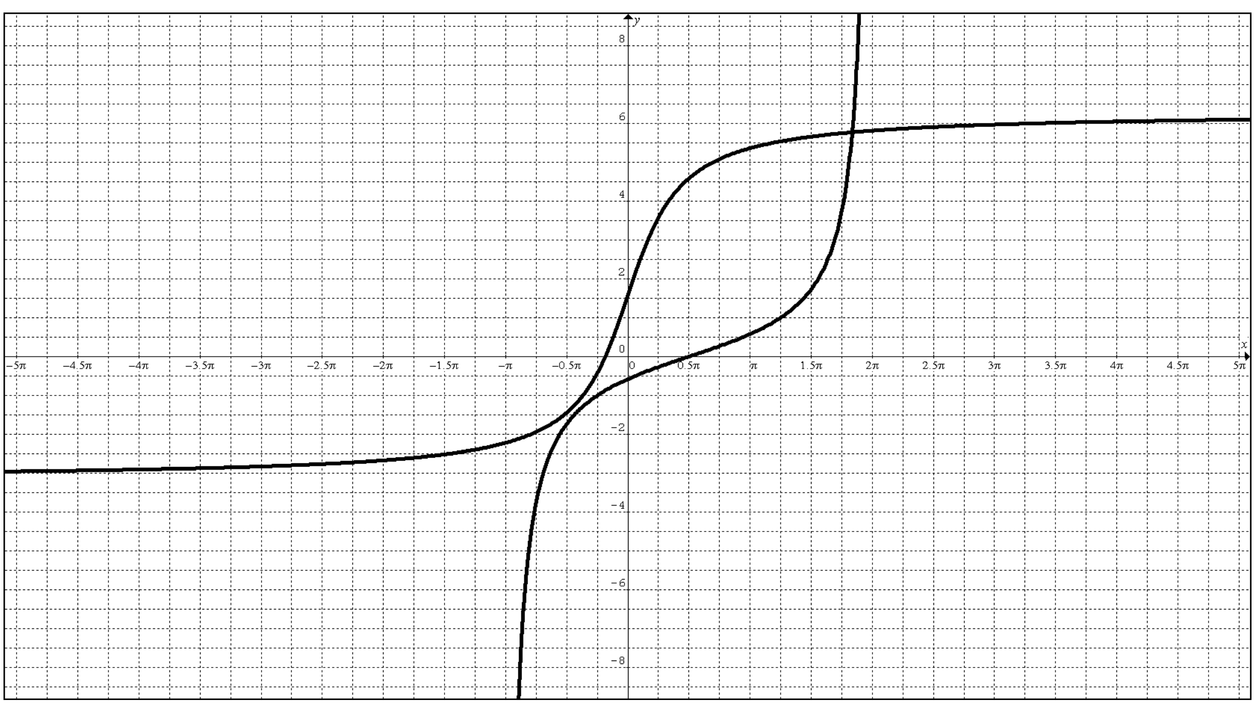
Respectively for y **we have** $-\dfrac{\pi}{2} < \dfrac{1}{3}x - \dfrac{\pi}{6} < \dfrac{\pi}{2}$, **whence** $-\pi < x < 2\pi$.

(b) Over this restricted domain we have $\dfrac{1}{3}x - \dfrac{\pi}{6} = \arctan y$, **whence**

$x = 3\arctan y + \pi/2$, **and**

$$y^{-1}(x) = 3\arctan x + \pi/2.$$

(c) The graphs are shown below.



Problem 5 (a) Find the exact value (no calculators!) of $\csc(\arctan(-7/5))$.

(b) Rewrite $\tan(\arcsin(\sqrt{x}))$ **as an equivalent algebraic expression.**

Solution (a) First notices that because the functions $\csc$ **and** $\arctan$ **are both odd functions we have**

$$\csc(\arctan(-7/5)) = \csc(-\arctan(7/5)) = -\csc(\arctan(7/5)).$$

To find the value of $\csc(\arctan(7/5))$ **consider the right triangle with an acute angle** θ**, opposite side** 7**, and adjacent side** 5**. Then**

$\csc(\arctan(-7/5)) = \csc\theta = 7\big/\sqrt{5^2+7^2} = 7\big/\sqrt{74}$**, whence the answer to part (a) is** $-7\big/\sqrt{74}$**.**

(b) Consider the right triangle with an acute angle θ**, opposite side** $\sqrt{x}$**, and hypotenuse** 1**. Then**

$$\tan(\arcsin(\sqrt{x})) = \tan\theta = \frac{\sqrt{x}}{\sqrt{1-\left(\sqrt{x}\right)^2}} = \frac{\sqrt{x}}{\sqrt{1-x}} = \sqrt{\frac{x}{1-x}}.$$