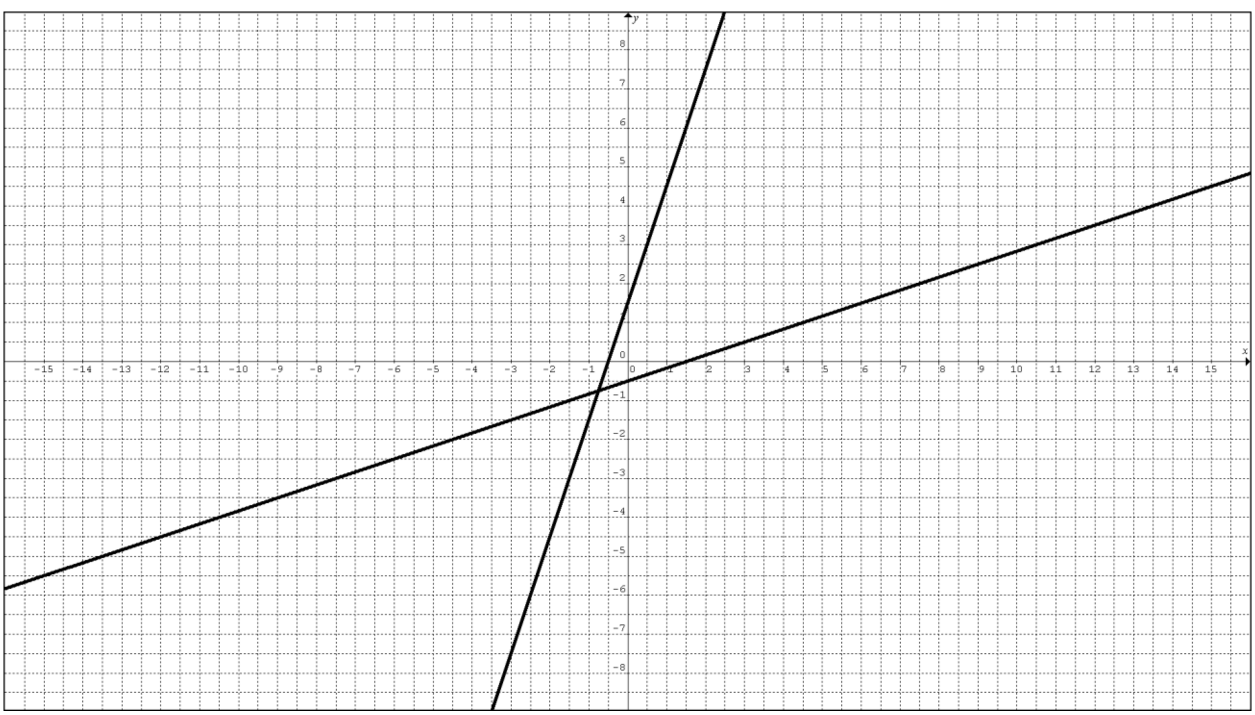
Problem 1 Let $y(x)=\frac{1}{3}x-\frac{1}{2}$. Find the inverse function $y^{-1}(x)$ and graph both functions in the same coordinate system.

Solution First we solve the equation $y=\frac{1}{3}x-\frac{1}{2}$ for x. Multiplying both parts of it by 3 we get $3y=x-\frac{3}{2}$ whence $x=3y+\frac{3}{2}$. Interchanging the input x and the output y in the last formula we get

$$y^{-1}(x)=3x+\frac{3}{2}.$$

The graphs of the functions $y(x)$ and $y^{-1}(x)$ are shown below.



In problems 2 – 4 consider the following quadratic function

$$y = -3x^2 + 7x + 1.$$

Problem 2 Write the function in the standard form

$$y = a(x - x_v)^2 + y_v.$$

Find the coordinates of the vertex. Find the x **-intercepts of the graph of** y **(if any). Graph the function.**

Solution We first factor out the coefficient by x^2**,**

$$y = -3\left(x^2 - \frac{7}{3}x - \frac{1}{3}\right).$$

Next we complete the square inside the parentheses using the formula

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2.$$

In our case we have $x^2 - \frac{1}{3}x = \left(x - \frac{7}{6}\right)^2 - \frac{49}{36}$**. After we plug in this expression into the formula for** y **we get**

$$y = -3\left(x - \frac{7}{6}\right)^2 + \frac{61}{12}$$

This is the standard form of the function $y = -3x^2 + 7x + 1.$

From the standard form we see that the coordinates of the vertex are

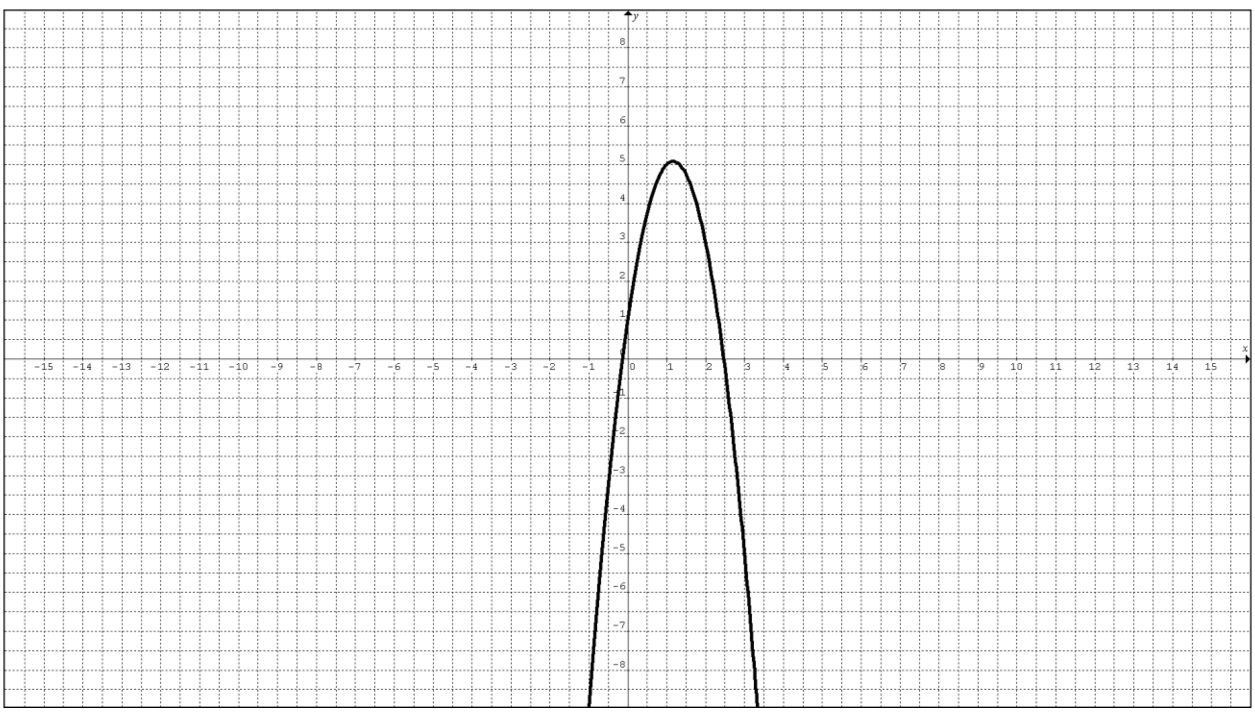
$$x_v = \frac{7}{6}, \quad y_v = \frac{61}{12}.$$

The x-intercepts can be found by quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{7^2 - 4 \cdot (-3) \cdot 1}}{2 \cdot (-3)} = \frac{7 \pm \sqrt{61}}{-6}$$

The x-intercepts are located approximately at -0.13 and 2.47.

The graph of the function is shown below.



Problem 3 Consider function y over the restricted domain $[x_v, \infty)$. Find the inverse function for this restriction of y. State the domain and the range of the inverse function.

Solution The restricted domain of y is the interval $\left[\frac{7}{6}, \infty\right)$. To find an expression for the inverse function we have to solve the equation $y = -3x^2 + 7x + 1$ for x. Writing it as $3x^2 - 7x + y - 1 = 0$ and applying the quadratic formula we get

$$x = \frac{7 \pm \sqrt{49 - 4 \cdot 3 \cdot (y-1)}}{6} = \frac{7 \pm \sqrt{61 - 12y}}{6}$$

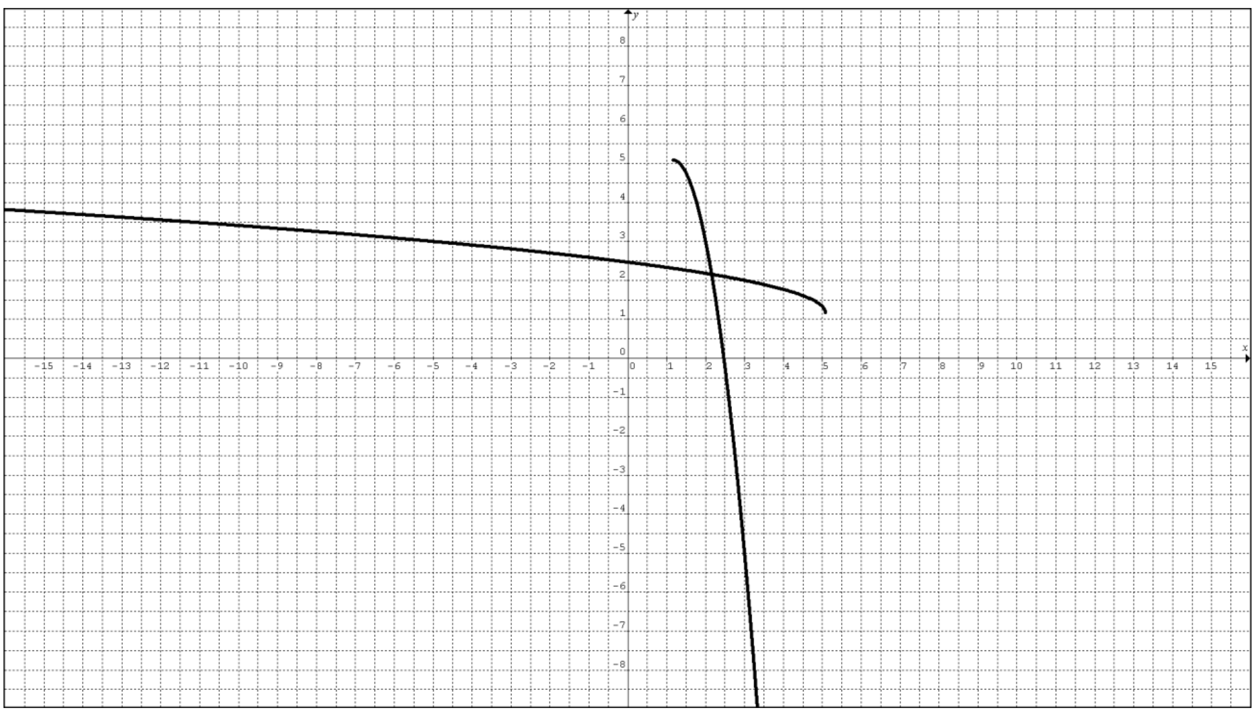
Because according to our choice of the restricted domain $x \geq \frac{7}{6}$ the correct sign in the above formula is +. Interchanging the input x and the output y we get the inverse function in the form

$$y = \frac{7 + \sqrt{61 - 12x}}{6}$$

Notice now that that the original function has the domain $\left[\frac{7}{6}, \infty\right)$ and the range $\left(-\infty, \frac{61}{12}\right]$. Because for the inverse function we have to interchange the domain and the range we see that the domain of the inverse function is $\left(-\infty, \frac{61}{12}\right]$ and its range is $\left[\frac{7}{6}, \infty\right)$.

Problem 4. Graph the function y with the restricted domain and its inverse in the same coordinate system.

Solution Notice that the graph of y is the part of the graph from problem 2to the right of the vertex. The graph of the inverse function can be obtained by interchanging the x and y coordinates, in other words by reflecting the graph of y about the line $y = x$. The graphs are shown below.



Problem 5 A penny is thrown up from the observation deck of the Empire State building (1050ft) with the velocity 50 ft/sec. Answer the following questions

(a) At what moment will the penny reach its maximum height?
(b) What is the maximum height?
(c) At what moment will the penny hit the ground?

Solution. In a problem like this the height of the object after t **seconds is given by the formula**

$$h(t) = -16t^2 + v_0 t + h_0$$

where v_0 **is the initial velocity of the object in ft/sec and** h_0 **is its initial height. In our case** $v_0 = 50$ **and** $h_0 = 1050$ **whence**

$$h(t) = -16t^2 + 50t + 1050$$

Because the leading coefficient of the quadratic function h **is negative the function takes it greatest value at the vertex.**

(a) The t **coordinate of the vertex is given by the formula**

$$t_v = -\frac{b}{2a} = -\frac{50}{2 \cdot (-16)} = 1.5625\, s$$

This answers question (a).

(b) The h **coordinate of the vertex is given by**

$$h_v = c - \frac{b^2}{4a} = 1050 - \frac{50^2}{4(-16)} = 1089.0625\ ft$$

This answers question (b).

To answer question (c) we have to find the positive solution of the equation

$$h(t) = -16t^2 + 50t + 1050$$

$$-16t^2 + 50t + 1050 = 0$$

By the quadratic formula

$$t = \frac{-50 - \sqrt{50^2 - 4(-16)1050}}{2(-16)} \approx 9.812\ s$$