Finite Mathematics 102 Review 4

1. How many seven–card hands with 3 diamonds, 2 hearts, and 2 black cards can be dealt from a standard deck of 52 cards?

Solution: we can take 3 diamonds from 13 in $C(13,3) = 286$ ways, 2 hearts in $C(13,2) = 78$ ways, and 2 black cards in $C(26,2) = 325$ ways. By multiplication principle the answer is $C(13,3) \times C(13,2) \times C(26,2) = 286 \times 78 \times 325 = 7250100$.

2. In how many ways 5 history books, 7 biology books, and 8 sociology books can be arranged if the books on the same subject should stand together?

Solution: history books can be arranged in 5! = 120 ways, biology books in 7! = 5040 ways, and sociology books in 8! = 40320 ways. The subjects can be arranged in 3! = 6 ways. By multiplication principle the answer is $3! \times 5! \times 7! \times 8! = 6 \times 120 \times 5040 \times 40320 = 146313216000$.

3. How many six-letter "words" exist if we use all letters of English alphabet and repetitions are allowed?

Solution: 26 possibilities for the first letter, 26 for the second, and so on. By multiplication principle the answer is $26^6 = 308915776$.

4. The same question as in Problem 3 if repetitions are not allowed.

Solution: we choose 6 different objects from 26; the answer is $P(26,6) = 165765600$.

5. The same question as in Problem 3 if the string must contain exactly two letters “a”.

Solution: We can position two letters “a” in $C(6,2)$ ways. Remaining positions can be occupied by any of 25 letters (“a” excluded). Therefore the answer is $C(6,2)\times 25^4 = 15\times 25^4 = 5859375$.

6. How many different 9-letter strings one can make from the letters in the word COMMITTEE?

Solution: If all the letters were different the answer would be 9! Because we have three pairs of identical letters (MM, TT, and EE) the answer is $\frac{9!}{2!2!2!} = 45360$.

7. Write the binomial expansion of the expression $(3z - 1)^5$.

Solution: we use line five of Pascal’s triangle to write

$$[3z+(-1)]^5 = (3z)^5 + 5(3z)^4(-1) + 10(3z)^3(-1)^2 + 10(3z)^2(-1)^3 + 5(3z)(-1)^4 + (-1)^5 =$$
$$= 243z^5 - 405z^4 + 270z^3 - 90z^2 + 15z - 1.$$