Finite Mathematics **Review 3**

1. Find the center and the radius of the circle given by the equation

$$x^2 + 4x + y^2 - 8y - 5 = 0.$$

Solution. We will bring the equation to the standard form using completion of squares. We complete the squares using the formula

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 \quad (*).$$

Let us start with $x^2 + 4x$. In this case $b = 4, \frac{b}{2} = 2,$ and $\left(\frac{b}{2}\right)^2 = 4$,

whence by formula (*) $x^2 + 4x = (x+2)^2 - 4$. In a similar way we obtain $y^2 - 8y = (y-4)^2 - 16$. After we plug in these expressions into the original equation we get $(x+2)^2 - 4 + (y-4)^2 - 16 - 5 = 0$, and finally

$$(x+2)^2 + (y-4)^2 = 25.$$

This is the standard equation of a circle with the center at (-2, 4) and the radius 5.

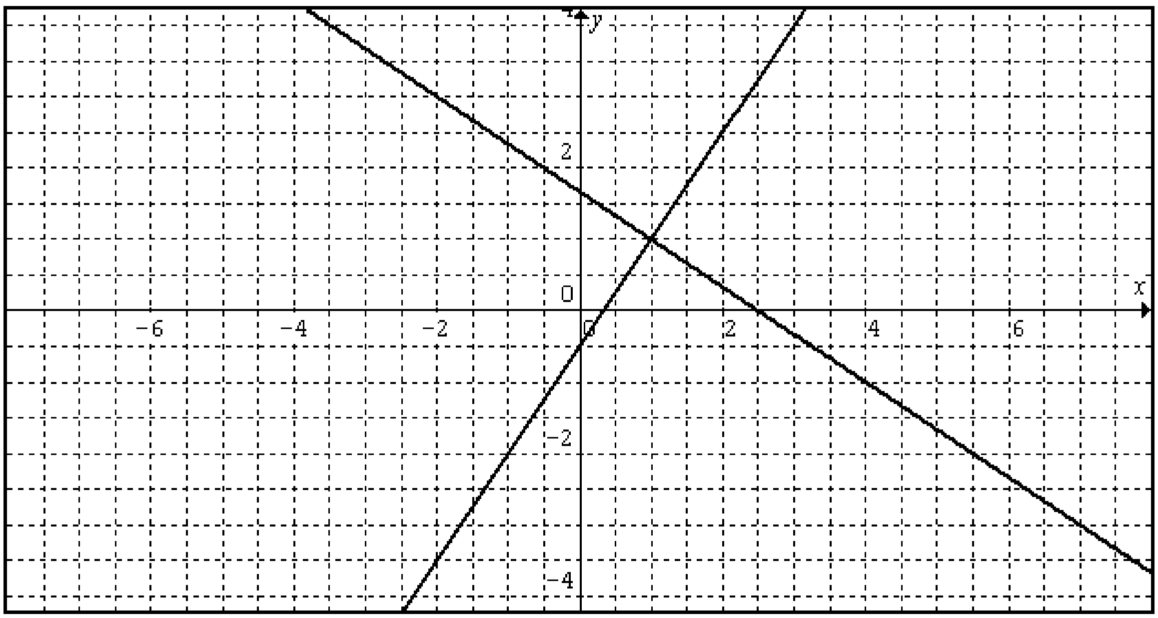
2. Find the slope-intercept equation of the line perpendicular to the line $2x + 3y - 5 = 0$ and containing the point (1, 1).

Solution. First we will find the slope of the given line. To do it we will solve its equation for y. $3y = -2x + 5$ and $y = -\frac{2}{3}x + \frac{5}{3}$. Therefore the slope of the given line is $-\frac{2}{3}$. Because slopes of perpendicular lines are negative reciprocals the slope of the line we are looking for is $\frac{3}{2}$. Next we can write an equation of this line in the point-slope form. $y - 1 = \frac{3}{2}(x-1)$. Finally, solving the last equation for y we get

$$y = \frac{3}{2}x - \frac{1}{2}.$$

3. Graph both lines from problem 2.

Solution. We can use, for example, the *x* and *y*-intercepts of both lines. For the given line plugging in $y = 0$ we find that the *x*-intercept is at $\left(\frac{5}{2}, 0\right)$. The *y*-intercept is at $\left(0, \frac{5}{3}\right)$. For the second line the intercepts are $\left(\frac{1}{3}, 0\right)$ and $\left(0, -\frac{1}{2}\right)$. A computer generated graph is shown below.



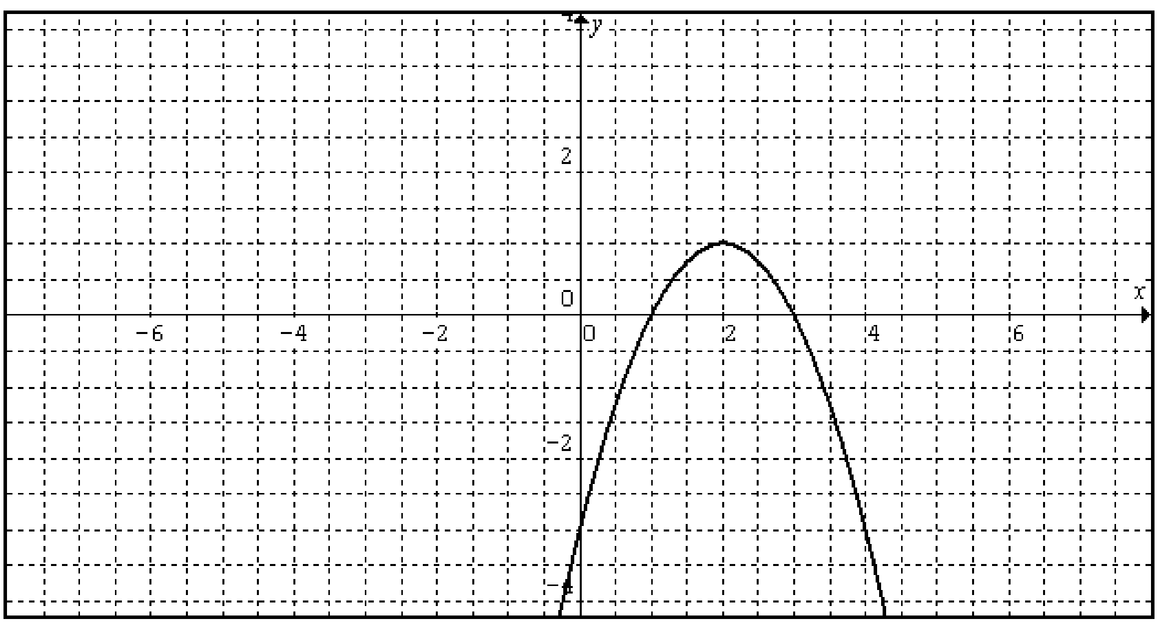
4. For the quadratic function $y=-x^2+4x-3$ find the coordinates of the vertex, the x- intercepts (if any), and graph the function.

Solution. The coefficients of this quadratic function are $a=-1, b=4,$ and $c=-3$.To find the coordinates of the vertex we can use the formulas $x_v=-\frac{b}{2a}=-\frac{4}{2\cdot(-1)}=2$ and $y_v=c-\frac{b^2}{4a}=-3-\frac{4^2}{4\cdot(-1)}=1$. Thus the vertex is at $(2,1)$. Because $a<0$ the vertex will be the highest point of the graph.

To find the x-intercepts we have to solve the quadratic equation $-x^2+4x-3=0$. To do it we can either use the quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, or in this case just factor the left part $-x^2+4x-3=-(x^2-4x+3)=-(x-1)(x-3)$. Anyway the x-intercepts are at (1, 0) and (3, 0).

To graph the function we can plot a couple more points in addition to the vertex and the x-intercepts. It is convenient to notice that for $x=0$ and $x=4$ the function takes the same value -3.

A computer generated graph is shown below.



5. A projectile is fired from the ground level with initial velocity 50 m/sec. When will it reach the maximum height? What is the maximum height?

Solution. Because we use the metric system a formula for the height of the projectile at moment t is $h(t) = -4.9t^2 + v_0 t + h_0 = -4.9t^2 + 50t$. Thus $h(t)$ is a quadratic function of t with coefficients $a = -4.9, b = 50,$ and $c = 0$. Because $a < 0$ this quadratic function attains its greatest value at the vertex. The time when the projectile will reach its maximum height equals to the *t*-coordinate of the vertex, $t_v = -\frac{b}{2a} = -\frac{50}{2(-4.9)} \approx 5.1$ sec. The maximum height equals to the *h*-coordinate of the vertex,
$h_{\max} = h_v = c - \frac{b^2}{4a} = -\frac{50^2}{4(-4.9)} \approx 127.55m.$

6. A helicopter is 200 m high and moving down with the speed 10m/sec. A projectile is fired from the helicopter vertically up with speed 500m/sec.

(a) When will the projectile reach its maximum height?
(b) What is the maximum height?
(c) When it will hit the ground?
(d) What will be the speed of impact?

Solution. We use equations of motion for the metric system.

$$v(t) = v_0 - 9.8t$$
$$h(t) = h_0 + v_0 t - 4.9t^2$$

In our case the initial velocity, v_0, is 500 – 10 = 490 m/sec and the initial height, h_0 is 200 m. Thus we have

$$v(t) = 490 - 9.8t$$
$$h(t) = 200 + 490t - 4.9t^2$$

(a) The height of the projectile is a quadratic function of t with coefficients $a = -4.9, b = 490,$ and $c = 200$. Because $a < 0$ this quadratic function attains its greatest value at the vertex. The time when the projectile will reach its maximum height equals to the *t*-coordinate of the vertex, $t_v = -\frac{b}{2a} = -\frac{490}{2(-4.9)} = 50$ sec.

(b) The maximum height equals to the *h*-coordinate of the vertex,

$$h_{\max} = h_v = c - \frac{b^2}{4a} = 200 - \frac{490^2}{4(-4.9)} = 12450m .$$

(c) At the time when the projectile hits the ground its height equals 0. Therefore we have to solve the quadratic equation $-4.9t^2 + 490t + 200 = 0$. Applying the quadratic formula $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ we obtain $t = \frac{-490 \pm \sqrt{490^2 - 4(-4.9)200}}{2(-4.9)}$. Only the positive solution makes sense and because the denominator is negative we have to take the negative sign in the numerator as well. Therefore the time of impact is

$$t_{impact} = \frac{-490 - \sqrt{490^2 - 4(-4.9)200}}{2(-4.9)} \approx 100.41\sec .$$

(d) The velocity of impact is $v_{impact} = 490 - 9.8t_{impact} \approx -494m/\sec$. The speed of impact is the absolute value of the velocity and thus it is approximately $494m/\sec$.