Finite Mathematics **Review 2**

1. For the numbers 360 and 1035 find

a. Prime factorizations.

Using these prime factorizations find next

b. The greatest common factor.

c. The least common multiple.

Solution. a.

$$360 = 2\times 180 = 2\times 2\times 90 = 2\times 2\times 2\times 45 = 2\times 2\times 2\times 3\times 3\times 5;$$
$$1035 = 3\times 345 = 3\times 3\times 115 = 3\times 3\times 5\times 23.$$

The prime factorizations are $360 = 2^3\times 3^2\times 5$ and $1035 = 3^2\times 5\times 23$.

b. To find the greatest common factor we take each prime factor from factorizations of 360 and 1035 with the smaller of two exponents. Thus $GCF(360,1035) = 3^2\times 5 = 45.$

c. To find the least common multiple we take each prime factor from factorizations of 360 and 1035 with the larger of two exponents.
Thus $LCM(360,1035) = 2^3\times 3^2\times 5\times 23 = 8280$.

2. Find the greatest common factor of the numbers 3464 and 5084 using the Euclidean algorithm.

Solution. The following table shows the sequence of operations.

Dividend	**Divisor**	**Quotient**	**Remainder**
5084	3464	1	1620
3464	1620	2	224

1620	224	7	52
224	52	4	16
52	16	3	4
16	4	4	0

Thus the greatest common factor of 5084 and 3464 is 4.

3. Determine which one of the two infinite decimal fractions

0.12123123412345...

and

0.473121121121....

represents a rational number.

Find the irreducible fraction which corresponds to this rational number.

Solution. The first infinite decimal though has a pattern but it is not a periodic pattern, no group of digits repeats in this pattern. Thus this infinite decimal represents an irrational number.

In the second decimal we have a repeating group of digits – 121 and therefore this decimal represents a rational number.

To find the fraction corresponding to the second decimal we will denote this decimal by x and we will move the decimal point three positions to the right, to the beginning of the first period $1000x = 473.121121121...$ Then we will move the decimal point six positions to the right – to the end of the first period $1000000x = 473121.121121...$ Now we have $1000000x - 1000x = 473121 - 473 = 472648$

whence $x = \frac{472648}{999000}$. It remains to write this fraction in the lowest terms. Clearly we can reduce it by 4, $x = \frac{118162}{249750}$. This fraction we still can reduce by 2, $x = \frac{59081}{124875}$. We do not see at once a simple way to reduce the last fraction so we will use the Euclidean algorithm to find whether its numerator and denominator have a common factor greater than 1. Let us again organize our calculations in the form of a table.

Dividend	**Divisor**	**Quotient**	**Remainder**
124875	59081	2	6713
59081	6713	8	5377
6723	5377	1	1346
5377	1346	3	1339
1346	1339	1	7
1339	7	191	2
7	2	3	1

Therefore the Greatest Common Factor of 124875 and 59081 is 1 and our last fraction is irreducible, $x = \frac{59081}{124875}$.

The next problem is similar to the extra credit problem on the test.

4. Consider the following operation ("arithmetic mean") on the set of all real numbers.

$$a*b=\frac{a+b}{2}$$

(a) Is operation (*) commutative? In other words, is it true that

$$a*b=b*a?$$

Solution. The answer to this question is positive. Indeed, because $a+b=b+a$ we have

$$a*b=\frac{a+b}{2}=\frac{b+a}{2}=b*a.$$

(b) Is operation (*) associative? In other words, is it true that

$$a*(b*c)=(a*b)*c?$$

Solution. In this case the answer is negative. Let us look separately at the left and at the right part of the alleged identity above in more details.

$$a*(b*c)=\frac{a+\frac{b+c}{2}}{2}=\frac{2a+b+c}{4}.$$

Above we have multiplied both the numerator and the denominator of the complex fraction by 2.

On the other hand

$$(a*b)*c=\frac{\frac{a+b}{2}+c}{2}=\frac{a+b+2c}{4}.$$

Clearly, in general $2a+b+c\neq a+b+2c$; take for example $a=1, b=2,$ and $c=3.$ Therefore the operation (*) is not associative.