1. For the numbers 360 and 1035 find
   a. Prime factorizations.

   Using these prime factorizations find next
   b. The greatest common factor.
   c. The least common multiple.

   **Solution. a.**

   \[
   360 = 2 \times 180 = 2 \times 2 \times 90 = 2 \times 2 \times 2 \times 45 = 2 \times 2 \times 3 \times 3 \times 5; \\
   1035 = 3 \times 345 = 3 \times 3 \times 115 = 3 \times 3 \times 5 \times 23.
   \]

   The prime factorizations are \( 360 = 2^3 \times 3^2 \times 5 \) and \( 1035 = 3^2 \times 5 \times 23 \).

   **b.** To find the greatest common factor we take each prime factor from factorizations of 360 and 1035 with the smaller of two exponents. Thus \( GCF(360,1035) = 3^2 \times 5 = 45 \).

   **c.** To find the least common multiple we take each prime factor from factorizations of 360 and 1035 with the larger of two exponents. Thus \( LCM(360,1035) = 2^3 \times 3^2 \times 5 \times 23 = 8280 \).

2. Find the greatest common factor of the numbers 3464 and 5084 using the Euclidean algorithm.

   **Solution.** The following table shows the sequence of operations.

<table>
<thead>
<tr>
<th>Dividend</th>
<th>Divisor</th>
<th>Quotient</th>
<th>Remainder</th>
</tr>
</thead>
<tbody>
<tr>
<td>5084</td>
<td>3464</td>
<td>1</td>
<td>1620</td>
</tr>
<tr>
<td>3464</td>
<td>1620</td>
<td>2</td>
<td>224</td>
</tr>
</tbody>
</table>
Thus the greatest common factor of 5084 and 3464 is 4.

3. Determine which one of the two infinite decimal fractions

0.12123123412345...

and

0.473121121121121....

represents a rational number.

Find the irreducible fraction which corresponds to this rational number.

**Solution.** The first infinite decimal though has a pattern but it is not a periodic pattern, no group of digits repeats in this pattern. Thus this infinite decimal represents an irrational number.

In the second decimal we have a repeating group of digits – 121 and therefore this decimal represents a rational number.

To find the fraction corresponding to the second decimal we will denote this decimal by $x$ and we will move the decimal point three positions to the right, to the beginning of the first period $1000x = 473.121121121...$ Then we will move the decimal point six positions to the right – to the end of the first period

$1000000x = 473121.121121121...$ Now we have $1000000x – 1000x = 473121 – 473 = 472648$
whence \( x = \frac{472648}{999000} \). It remains to write this fraction in the lowest terms. Clearly we can reduce it by 4, \( x = \frac{118162}{249750} \). This fraction we still can reduce by 2, \( x = \frac{59081}{124875} \). We do not see at once a simple way to reduce the last fraction so we will use the Euclidean algorithm to find whether its numerator and denominator have a common factor greater than 1. Let us again organize our calculations in the form of a table.

<table>
<thead>
<tr>
<th>Dividend</th>
<th>Divisor</th>
<th>Quotient</th>
<th>Remainder</th>
</tr>
</thead>
<tbody>
<tr>
<td>124875</td>
<td>59081</td>
<td>2</td>
<td>6713</td>
</tr>
<tr>
<td>59081</td>
<td>6713</td>
<td>8</td>
<td>5377</td>
</tr>
<tr>
<td>6723</td>
<td>5377</td>
<td>1</td>
<td>1346</td>
</tr>
<tr>
<td>5377</td>
<td>1346</td>
<td>3</td>
<td>1339</td>
</tr>
<tr>
<td>1346</td>
<td>1339</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>1339</td>
<td>7</td>
<td>191</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Therefore the Greatest Common Factor of 124875 and 59081 is 1 and our last fraction is irreducible, \( x = \frac{59081}{124875} \).
The next problem is similar to the extra credit problem on the test.

4. Consider the following operation ("arithmetic mean") on the set of all real numbers.

\[ a \ast b = \frac{a + b}{2} \]

(a) Is operation (\( \ast \)) commutative? In other words, is it true that

\[ a \ast b = b \ast a? \]

**Solution.** The answer to this question is positive. Indeed, because \( a + b = b + a \) we have

\[ a \ast b = \frac{a + b}{2} = \frac{b + a}{2} = b \ast a. \]

(b) Is operation (\( \ast \)) associative? In other words, is it true that

\[ a \ast (b \ast c) = (a \ast b) \ast c? \]

**Solution.** In this case the answer is negative. Let us look separately at the left and at the right part of the alleged identity above in more details.

\[ a \ast (b \ast c) = \frac{a + b + c}{2} = \frac{2a + b + c}{4}. \]

Above we have multiplied both the numerator and the denominator of the complex fraction by 2.

On the other hand

\[ (a \ast b) \ast c = \frac{a + b + c}{2} = \frac{a + b + 2c}{4}. \]

Clearly, in general \( 2a + b + c \neq a + b + 2c \) ; take for example \( a = 1, b = 2, \) and \( c = 3. \) Therefore the operation (\( \ast \)) is not associative.