Finite Mathematics **Review 1**

1. Assume that $U = \{1,2,3,4,5,6,7,8,9\}$,
and that
$A = \{1,2,3,4,5\}$, $B = \{4,5,6,7\}$, $C = \{5,6,7,8,9\}$,
$D = \{1,3,5,7,9\}$, $E = \{2,4,6,8\}$, $F = \{1,5,7\}$.
Compute the sets $(A \cap D) - B,\ A \oplus B = (A - B) \cup (B - A),\ (B \cap F) \cup (C \cap E)$.

Solution. $A \cap D = \{1,3,5\}$ and therefore $(A \cap D) - B = \{1,3\}$ **;**
$A - B = \{1,2,3\}, \quad B - A = \{6,7\}$ whence $A \oplus B = \{1,2,3,6,7\}$ **;**
$B \cap F = \{5,7\}$, $C \cap E = \{6,8\}$ whence $(B \cap F) \cup (C \cap E) = \{5,6,7,8\}$

2. For the sets defined in Problem 1 compute the
sets $(A \cap B \cap C)', (D \cup E \cup F)', (A - E)'$.
Solution. $A \cap B \cap C = \{5\}$ whence $(A \cap B \cap C)' = \{1,2,3,4,6,7,8,9\}$ **;**
$D \cup E \cup F = \{1,2,3,4,5,6,7,8,9\} = U$ whence $(D \cup E \cup F)' = U' = \varnothing$ **;**
$A - E = \{1,3,5\}$ whence $(A - E)' = \{2,4,6,7,8,9\}$ **.**

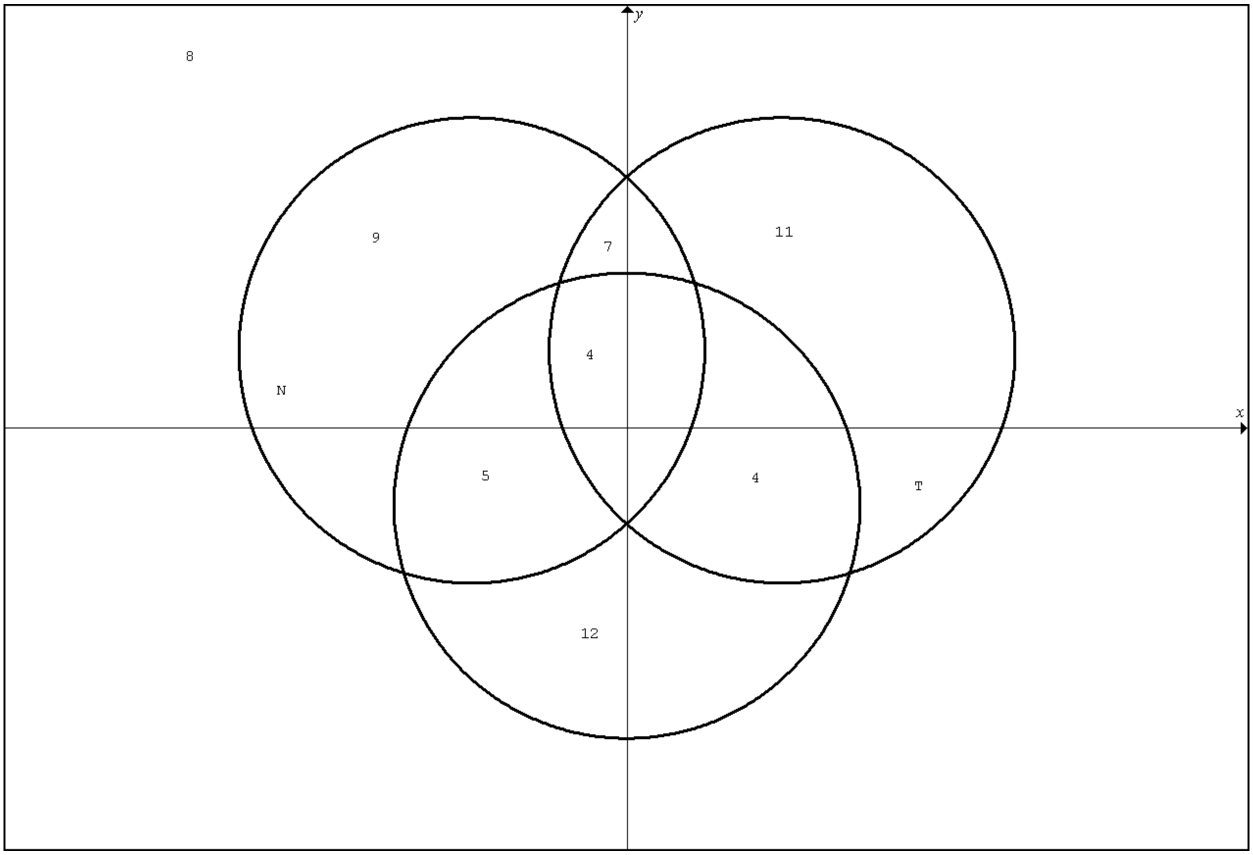
3. In a survey of 60 people it was found that 25 read Newsweek magazine, 26 read Time, and 25 read Fortune. Also 9 read both Newsweek and Fortune, 11 read both Newsweek and Time, 8 read both Time and Fortune, and 8 read none of these three magazines.

a. Find the number of people who read all three magazines.
b. Construct the corresponding Venn diagram.
c. Determine the number of people who read exactly one magazine.

Solution. a. We introduce the set *U* of all surveyed people and its subsets *N*, *T*, and *F* of all people who read Newsweek, Time, and Fortune, respectively. The conditions of the problem then can be written as
$n(U) = 60,$
$n(N) = 25, n(T) = 26, n(F) = 25,$
$n(N \cap F) = 9, n(N \cap T) = 11, n(F \cap T) = 8,$ and
$n(N' \cap T' \cap F') = n[(N \cup T \cup F)'] = 8.$
Now we can use the formula
$$n(N \cup T \cup F) = n(N) + n(T) + n(F) - n(N \cap T) - n(N \cap F) - n(T \cap F) + n(N \cap T \cap F) \quad (*).$$
Notice that $n(N \cup T \cup F) = n(U) - n[(N \cup T \cup F)'] = 60 - 8 = 52$.
After plugging in the numbers we know into the formula (*) we have
$52 = 25 + 26 + 25 - 11 - 9 - 8 + n(N \cap T \cap F)$, whence $n(N \cap T \cap F) = 4$.

Four people read all three magazines.

b.



c. From the Venn diagram we see that the number of people reading exactly one of these two magazines is $9+11+12=32$.

The next problem is similar to the extra credit problem on the test.

4. Prove that for any three sets $A, B,$ and C the following distributive law is valid.

$$A\cup(B\cap C)=(A\cup B)\cap(A\cup C).$$

Solution. We will prove first that

$$A\cup(B\cap C)\subseteq(A\cup B)\cap(A\cup C). \qquad (*)$$

To this end let us consider $x\in A\cup(B\cap C)$. There are two possibilities.

(a) $x\in A$. Then according to the definition of the union of two sets $x\in A\cup B$ and $x\in A\cup C$ whence $x\in(A\cup B)\cap(A\cup C)$.

(b) $x \notin A$. Then, because $x \in A \cup (B \cap C)$, we must have $x \in (B \cap C)$. Therefore $x \in A \cup B$ and $x \in A \cup C$ whence $x \in (A \cup B) \cap (A \cup C)$.

We have proved that every element of the set $A \cup (B \cap C)$ is also an element of the set $(A \cup B) \cap (A \cup C)$. By the definition of a subset it means exactly that

$$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C).$$

Next we have to prove that

$$(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C). \qquad (**)$$

Let $x \in (A \cup B) \cap (A \cup C)$. Again there are two alternatives.

(a) $x \in A$. Then of course $x \in A \cup (B \cap C)$.

(b) $x \notin A$. In this case, because $x \in A \cup B$ we have $x \in B$. Similarly, because $x \in A \cup C$ we have $x \in C$. Therefore $x \in B \cap C$ whence $x \in A \cup (B \cap C)$.

The combination of the inclusions (*) and (**) provides us with the proof that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$