Vector Calculus Quiz

1. Evaluate $\dfrac{\partial}{\partial x}(x^2y) - \dfrac{\partial}{\partial y}(xy^2)$.

2. Find $\displaystyle\int_0^2 \int_0^1 x^3 y e^{x^2y^2} \mathrm{d}x\mathrm{d}y$.

3. Consider the square S whose vertices are $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$ travelled counter-clockwise in this order, ending and starting at $(0,0)$. Consider also the differential form $\omega = -3x^2y\mathrm{d}x + 3xy^2\mathrm{d}y$. Find $\displaystyle\iint_S (y^2+x^2)\mathrm{d}x\mathrm{d}y$ and $\displaystyle\int_{\partial S} \omega$, where ∂S is the boundary of S.