Math 118 Rules

Names of Special Sets

1. $\mathbb{N}$:= the set of natural numbers (including zero).
2. $\mathbb{P}$:= the set of all strictly positive integers (plus).
3. $\mathbb{M}$:= the set of all strictly negative integers (minus).
4. $\mathbb{Z}$:= the set of all integers.
5. $\mathbb{Q}$:= the set of all rational numbers.
6. $\mathbb{R}$:= the set of all real numbers.
7. $\mathbb{C}$:= the set of all complex numbers.

Logical Symbols

1. $\top$:= True
2. $\bot$:= False
3. $\neg$:= not
4. $\vee$:= or (In Mathematics disjunction is inclusive, i.e. $a \vee b$ means either a, or b, or both.)
5. $\wedge$:= and
6. $\Longrightarrow$:= implies, if ... then
7. $\Longleftrightarrow$:= implies, and is implied by; if and only if
8. $\forall$:= for every; for all (The upside-down A is the A of 'for All')
9. $\exists$:= for some; there exists (The backwards E is the E of 'there Exists')

Set-theoretical Symbols

1. $\varnothing$:= the empty set
2. $\bigcap$:= intersection
3. $\bigcup$:= union
4. $\setminus$:= without
5. $\in$:= in; contained in; element (member of); is in; is contained in; is an element (a member) of
6. $\subset$:= included in; subset of; is included in; is a subset of
7. $()^c$:= complement

Formulas from Logic and Set Theory

1. (a) $p \vee q = q \vee p$
   (b) $A \cup B = B \cup A$
2. (a) $p \wedge q = q \wedge p$
   (b) $A \cap B = B \cap A$
3. (a) $(p \vee q) \vee r = p \vee (q \vee r)$
   (b) $(A \cup B) \cup C = A \cup (B \cup C)$
4. (a) $(p \wedge q) \wedge r = p \wedge (q \wedge r)$
   (b) $(A \cap B) \cap C = A \cap (B \cap C)$
5. (a) $\neg(p \vee q) = (\neg p) \wedge (\neg q)$
   (b) $(A \cup B)^c = A^c \cap B^c$
6. (a) $\neg(p \wedge q) = (\neg p) \vee (\neg q)$
   (b) $(A \cap B)^c = A^c \cup B^c$
7. (a) $(p \wedge q) \vee r = (p \vee r) \wedge (q \vee r)$
   (b) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
8. (a) $(p \vee q) \wedge r = (p \wedge r) \vee (q \wedge r)$
   (b) $(A \cup B) \cup C = (A \cap C) \cup (B \cap C)$
9. (a) $\neg(\neg p) = p$
   (b) $(A^c)^c = A$

   We note that 5, 6 and 9 have 10, 11 and 12 as their respective equivalent forms in which the universe with respect to which complements are being taken, is explicitly mentioned.

10. $\mathscr{U} \setminus (A \cup B) = (\mathscr{U} \setminus A) \cap (\mathscr{U} \setminus B)$
11. $\mathscr{U} \setminus (A \cap B) = (\mathscr{U} \setminus A) \cup (\mathscr{U} \setminus B)$
12. $\mathscr{U} \setminus (\mathscr{U} \setminus A) = A$

Bookkeeping Rules

These rules tell you how parentheses (singular: parenthesis) are to be used.

1. $a = (a)$
2. $a + b = (a) + b = a + (b) = (a) + (b) = (a + b)$
3. $a - b = (a) - b = a - (b) = (a) - (b) = (a - b)$
4. $a \times b = (a) \times b = a \times (b) = (a) \times (b) = (a \times b)$
   $= ab = (a)\, b = a\,(b) = (a)\,(b) = (ab)$
   $= a \cdot b = (a) \cdot b = a \cdot (b) = (a) \cdot (b) = (a \cdot b)$
5. $a \div b = (a) \div b = a \div (b) = (a) \div (b) = (a \div b)$
   $= a/b = (a)/b = a/(b) = (a)/(b) = (a/b)$
   $= \dfrac{a}{b} = \dfrac{(a)}{b} = \dfrac{a}{(b)} = \dfrac{(a)}{(b)} = \left(\dfrac{a}{b}\right)$
6. $a^b = (a)^b = a^{(b)} = (a)^{(b)} = \left(a^b\right)$
   $= a \wedge b = (a) \wedge b = a \wedge (b) = (a) \wedge (b) = (a \wedge b)$

Note that many calculators use the symbol $\wedge$ to denote exponentiation. This usage conflicts with the other use of $\wedge$ to denote conjunction, but this notational conflict is unlikely to cause confusion.

Note that the + sign in $+a$, and the + sign in $a + b$, as also the − sign in $-a$, and the − sign in $a - b$ do not denote the same operation; this is why many calculators use different keys for these different meanings.

Definitions for Subtraction and Division

1. $a - b := a + (-b)$
2. $a \div b := a \times \left(\dfrac{1}{b}\right)$

Rules for Signs

1. $a = +a = +(a)$
2. $-a = -(a) = (-1)\, a$
3. $-(-a) = a$
4. $(+a)\,(+b) = (-a)\,(-b) = ab = +(ab) = +ab$
5. $(+a)\,(-b) = (-a)\,(+b) = -ab = -(ab) = (-1)\, ab$
6. $\dfrac{+a}{+b} = \dfrac{-a}{-b} = \dfrac{a}{b} = +\left(\dfrac{a}{b}\right)$
7. $\dfrac{(+a)}{(-b)} = \dfrac{(-a)}{(+b)} = -\dfrac{a}{b} = -\left(\dfrac{a}{b}\right) = \dfrac{-a}{b} = \dfrac{a}{-b}$

Rules for Fractions

1. $a = \dfrac{a}{1} = \dfrac{+a}{1} = +\left(\dfrac{a}{1}\right)$
2. $\dfrac{a}{b} = \dfrac{1}{b}(a) = (a)\left(\dfrac{1}{b}\right)$
3. $\dfrac{ab}{c} = a\left(\dfrac{b}{c}\right) = \left(\dfrac{a}{c}\right) b = \dfrac{1}{c}(ab)$
4. $\dfrac{ab}{cd} = \left(\dfrac{a}{c}\right)\left(\dfrac{b}{d}\right) = \dfrac{1}{c}(ab)\dfrac{1}{d} = a\left(\dfrac{1}{cd}\right) b$

Rules for Addition and Subtraction of Fractions

1. $\dfrac{a}{c} + \dfrac{b}{c} = \dfrac{a+b}{c}$
2. $\dfrac{a}{c} + \dfrac{b}{d} = \dfrac{ad+bc}{cd}$
3. $\dfrac{a}{c} - \dfrac{b}{c} = \dfrac{a-b}{c}$
4. $\dfrac{a}{c} - \dfrac{b}{d} = \dfrac{ad-bc}{cd}$

Rules for Multiplication and Division

1. $\left(\dfrac{a}{c}\right) \times \left(\dfrac{b}{d}\right) = \dfrac{(a \times b)}{(c \times d)} = \dfrac{ab}{cd}$
2. $\left(\dfrac{a}{c}\right) \div \left(\dfrac{b}{d}\right) = \dfrac{\left(\dfrac{a}{c}\right)}{\left(\dfrac{b}{d}\right)} = \left(\dfrac{a}{c}\right) \times \left(\dfrac{d}{b}\right) = \dfrac{(a \times d)}{(b \times c)} = \dfrac{ad}{bc}$

Rules for Cancellation

In the following four rules, it is assumed that $a \neq 0$, and $c \neq 0$

1. $\dfrac{ab}{ac} = \dfrac{b}{c}$
2. $\dfrac{ab}{ca} = \dfrac{b}{c}$
3. $\dfrac{ba}{ca} = \dfrac{b}{c}$

4. $\dfrac{ba}{ac} = \dfrac{b}{c}$

Switching Rules (Commutative Laws)

1. $a + b = b + a$
2. $a \times b = b \times a$
3. $a - b \neq b - a$, in general.
4. $a \div b \neq b \div a$, in general.
5. $a \wedge b \neq b \wedge a$, in general.

Grouping Rules (Associative Laws)

1. $a + (b + c) = (a + b) + c = a + b + c$

   Because of this rule, we may write expressions like $a + b + c + d$without parentheses; the various different manners of inserting parentheses are all equivalent and the different terms so formed are all equal.

   Therefore, we shall use the compact notation:

2. $\sum_{i=1}^{n} a_i := a_1 + a_2 + a_3 + \cdots + a_{n-2} + a_{n-1} + a_n$,

   where the number of summands, n, is indefinite, that is, n is an unknown finite number.

3. $a \times (b \times c) = (a \times b) \times c = a \times b \times c$

   Similarly, we may write expressions like $a \times b \times c \times d$ without parentheses; the various different manners of inserting parentheses are all equivalent and the different terms so formed are all equal.

   Therefore, we shall use the compact notation:

4. $\prod_{i=1}^{n} a_i := a_1 \times a_2 \times a_3 \times \cdots \times a_{n-2} \times a_{n-1} \times a_n$,

   where the number of multiplicands or factors , n, is indefinite, that is, n is an unknown finite number.

5. $a - b - c := (a - b) - c$
6. $a \div b \div c := (a \div b) \div c$
7. $a \wedge b \wedge c := a \wedge (b \wedge c)$
8. $a - (b - c) \neq (a - b) - c$, in general.
9. $a \div (b \div c) \neq (a \div b) * c$, in general.
10. $a \div (b \div c) \neq (a \div b) \div c$, in general.

Rearranging Rules (Distributive Laws)

1. $ab + ac = a(b + c)$
2. $ba + ca = (b + c)a$
3. $ab - ac = a(b - c)$
4. $ba - ca = (b - c)a$

Rules for Exponentiation

1. $0^a = 0$, provided that $a > 0$
2. $1^a = 1$
3. $a^0 = 1$, in particular $0^0 = 1$
4. $a^1 = a$
5. (a) $a^p a^q = a^{(p+q)}$
   (b) $a^p a^q a^r = a^{(p+q+r)}$
   (c) $a^{p_1} a^{p_2} \cdots a^{p_n} = a^{(p_1+p_2+\cdots p_n)}$
   (d) $\prod_{i=1}^{n} a^{p_i} = a^{\left(\sum_{i=1}^{n} p_i\right)}$
6. (a) $a^p b^p = (ab)^p$
   (b) $a^p b^p c^p = (abc)^p$
   (c) $(a_1)^p (a_2)^p \cdots (a_n)^p = (a_1 a_2 \cdots a_n)^p$
   (d) $\left(\prod_{i=1}^{n} a_i\right)^p = \left(\prod_{i=1}^{n} a_i\right)^p$
7. (a) $\dfrac{a^p}{a^q} = a^{(p-q)}$
   (b) $\dfrac{(a^{p_1})(a^{p_2}) \cdots (a^{p_n})}{(a^{q_1})(a^{q_2}) \cdots (a^{q_n})} = a^{(p_1+p_2+\cdots+p_n)-(q_1+q_2+\cdots+q_n)}$
   (c) $\dfrac{\prod_{i=i}^{n} (a^{p_i})}{\prod_{j=i}^{m} \left(a^{q_j}\right)} = a^{\left(\sum_{i=i}^{n} p_i\right)-\left(\sum_{j=i}^{n} q_j\right)}$
8. $\dfrac{a^p}{b^p} = \left(\dfrac{a}{b}\right)^p = \left(\dfrac{b}{a}\right)^{-p}$, in particular $\left(\dfrac{a}{b}\right)^{-1} = \dfrac{b}{a}$
9. $\dfrac{(a_1)^p (a_2)^p \cdots (a_n)^p}{(b_1)^p (b_2)^p \cdots (b_m)^p} = \left(\dfrac{a_1 a_2 \cdots a_n}{b_1 b_2 \cdots b_m}\right)^p = \left(\dfrac{b_1 b_2 \cdots b_m}{a_1 a_2 \cdots a_n}\right)^{-p}$, in particular

$$\left(\frac{a_1 a_2 \cdots a_n}{b_1 b_2 \cdots b_m}\right)^{-1} = \left(\frac{b_1 b_2 \cdots b_m}{a_1 a_2 \cdots a_n}\right)$$

10. $(a^p)^q = a^{(pq)}$
11. $\left((a^p)^q\right)^r = a^{(pqr)}$

12. $\left(\left(\left(a^{p_1}\right)^{p_2}\right)\right)^{\cdots p_n} = a^{(p_1 p_2 \cdots p_n)}$

13. $\left(\left(\left(a^{p_1}\right)^{p_2}\right)\right)^{\cdots p_n} = a^{\left(\prod_{i=1}^{n} p_i\right)}$

14. $\frac{\left(a^{p_1}\right)^{q_1}\left(a^{p_2}\right)^{q_2}\cdots\left(a^{p_n}\right)^{q_n}}{\left(a^{r_1}\right)^{s_1}\left(a^{r_2}\right)^{s_2}\cdots\left(a^{r_m}\right)^{s_m}} = a^{\left(p_1 q_1 + p_2 q_2 + \cdots + p_n q_n\right) - \left(r_1 s_1 + r_2 s_2 + \cdots + r_m s_m\right)}$

15. $\frac{\prod_{i=i}^{n}\left(\left(a^{p_i}\right)^{q_i}\right)}{\prod_{j=i}^{m}\left(\left(a^{r_j}\right)^{s_j}\right)} = a^{\left(\left(\sum_{i=1}^{n} p_i q_i\right) - \left(\sum_{j=1}^{m} r_j s_j\right)\right)}$

16. $\left(a^{p}\right)^{\left(\frac{1}{q}\right)} = \left(a^{\left(\frac{1}{q}\right)}\right)^{p} = a^{\left(\frac{p}{q}\right)}$, provided $a > 0$

17. $a^{p} = \frac{1}{a^{-p}}$, in particular, $a = a^{1} = \frac{1}{a^{-1}}$, provided $a \neq 0$

18. $a^{-p} = \frac{1}{a^{p}}$, in particular, $a^{-1} = \frac{1}{a}$, provided $a \neq 0$

19. $a^{p^q} = a^{(p^q)}$

Rules for Radicals

1. $\sqrt[p]{a} = a^{\frac{1}{p}}$

   In the above expression, it is to be assumed that, if p is even then a, is assumed to be positive, that is, $a \in [0, \infty[$

2. $\sqrt[p]{0} = 0$

3. $\sqrt[p]{1} = 1$

4. $\sqrt[1]{a} = a^{\frac{1}{1}} = a^{1} = a$

5. $\sqrt[2]{a} = \sqrt{a} = a^{\frac{1}{2}}$

6. $\sqrt[3]{a} = a^{\frac{1}{3}}$

7. $\sqrt[4]{a} = a^{\frac{1}{4}}$

8. $\sqrt[17]{a} = a^{\frac{1}{17}}$

9. $\sqrt[p]{a}\sqrt[p]{b} = \sqrt[p]{ab} = (ab)^{\frac{1}{p}}$

10. $\sqrt[p]{a}\sqrt[p]{b}\sqrt[p]{c} = \sqrt[p]{abc} = (abc)^{\frac{1}{p}}$

11. $\sqrt[p_1]{a}\sqrt[p_2]{a}\cdots\sqrt[p_n]{a} = \sqrt[p_1 p_2 \cdots p_n]{a} = a^{\frac{1}{p_1 p_2 \cdots p_n}}$

    More compactly,

12. $\prod_{i=1}^{n}\left(\sqrt[p_i]{a}\right) = \sqrt[\left(\prod_{i=1}^{n} p_i\right)]{a} = a^{\frac{1}{\prod_{i=1}^{n} p_i}}$

13. $\frac{\sqrt[p]{a}}{\sqrt[p]{a}} = \sqrt[p]{\frac{a}{b}} = \left(\frac{a}{b}\right)^{\frac{1}{p}}$

14. $\sqrt[p]{\sqrt[q]{a}} = \sqrt[pq]{a}$

15. $\sqrt[p]{\sqrt[q]{\sqrt[r]{a}}} = \sqrt[pqr]{a}$

16. $\sqrt[p_1]{\sqrt[p_2]{\cdots\sqrt[p_n]{a}}} = \sqrt[\prod_{i=1}^{n} p_i]{a}$

17. $\sqrt[p]{a}\sqrt[q]{a} = \sqrt[pq]{a^{p+q}} = a^{\frac{p+q}{pq}}$

18. $\sqrt[q]{a^{p}} = \left(\sqrt[q]{a}\right)^{p} = \left(a^{p}\right)^{\left(\frac{1}{q}\right)} = \left(a^{\left(\frac{1}{q}\right)}\right)^{p} = a^{\left(\frac{p}{q}\right)}$

19. $\left(\sqrt[p]{a}\right)^{p} = a$

20. $\sqrt[p]{a^{p}} = \begin{cases} a & \text{if } p \text{ is odd} \\ |a| & \text{if } p \text{ is even} \end{cases}$

Pitfalls

21. $\sqrt[p]{a} + \sqrt[p]{b} \neq \sqrt[p]{a+b}$

22. $\sqrt[p]{a} - \sqrt[p]{b} \neq \sqrt[p]{a-b}$

Fundamental Rules for Rewriting

1. $a = a + 0 = 0 + a$

2. $a = a \times 1 = 1 \times a$

3. $a = a - 0$

4. $a = a \div 1 = a/1 = \frac{a}{1}$

5. $a = a^{1} = a \wedge 1$

6. $a = \sqrt[p]{a^{p}} = \left(\sqrt[p]{a}\right)^{p} = \left(a^{\frac{1}{p}}\right)^{p} = \left(a^{p}\right)^{\frac{1}{p}}$, where suitable restrictions apply, if p is even.

Rules for Factoring

1. (a) $ab + ac = a(b + c)$
   (b) $ab + ac + ad = a(b + c + d)$
   (c) $ab_1 + ab_2 + ab_3 + ab_4 = a(b_1 + b_2 + b_3 + b_4)$
   (d) $ab_1 + ab_2 + \cdots + ab_n = a(b_1 + b_2 + \cdots + b_n)$
   (e) $\left(\sum_{i=1}^{n} ab_i\right) = a\left(\sum_{i=1}^{n} b_i\right)$

2. (a) $ba + ca = (b + c)\,a$
   (b) $ba + ca + da = (b + c + d)\,a$
   (c) $b_1 a + b_2 a + b_3 a + b_4 a = (b_1 + b_2 + b_3 + b_4)\,a$
   (d) $b_1 a + b_2 a + \cdots + b_n a = (b_1 + b_2 + \cdots + b_n)\,a$
   (e) $\sum_{i=1}^{n} b_i a = \left(\sum_{i=1}^{n} b_i\right) a$

In general, if there is an a common to each of the summands, in any position whatsoever, within the summand, it may be extracted from each term, and placed either in front, or at the back, as shown below, in expanded and also in more compact form.

3. $$\begin{aligned} b_1 ac_1 + b_2 ac_2 + \cdots + b_n ac_n &= (b_1 c_1 + b_2 c_2 + \cdots + b_n c_n)\, a \\ &= a\,(b_1 c_1 + b_2 c_2 + \cdots + b_n c_n) \end{aligned}$$

4. $\sum_{i=1}^{n} b_i ac_i = a\left(\sum_{i=1}^{n} b_i c_i\right) = \left(\sum_{i=1}^{n} b_i c_i\right)$

5. $ax + b = a\left(x - \left(\frac{-b}{a}\right)\right)$, where $a \neq 0$

6. $ax - b = a\left(x - \left(\frac{b}{a}\right)\right)$, where $a \neq 0$

7. $a^2 - b^2 = (a-b)(a+b)$

8. $a^2 + b^2 = (a - ib)(a + ib)$ with $i^2 = -1$

9. $a^2 + 2ab + b^2 = (a+b)(a+b) = (a+b)^2$

10. $a^2 - 2ab + b^2 = (a-b)(a-b) = (a-b)^2$

11. $x^2 + (a+b)x + ab = (x+a)(x+b)$

12. $x^2 - (a+b)x + ab = (x-a)(x-b)$

13. $x^2 + (a-b)x - ab = (x+a)(x-b)$

14. $x^2 - (a-b)x - ab = (x-a)(x+b)$

15. $acx^2 + (ad+bc)x + bd = (ax+b)(cx+d)$

16. $acx^2 - (ad+bc)x + bd = (ax-b)(cx-d)$

17. $acx^2 - (ad-bc)x - bd = (ax+b)(cx-d)$

18. $acx^2 + (ad-bc)x - bd = (ax-b)(cx+d)$

19. $ax^2 + bx + c = a\left(x - \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\right)\left(x - \left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)\right)$

20. $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$

21. $a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$

22. $a^3 + 3a^2 b + 3ab^2 + b^3 = (a+b)(a+b)(a+b) = (a+b)^3$

23. $a^3 - 3a^2 b + 3ab^2 - b^3 = (a-b)(a-b)(a-b) = (a-b)^3$

Pitfalls

1. $\frac{a+b}{a+c} \neq \frac{b}{c}$, in general.

2. $\frac{a+b}{b} \neq a$, in general.

3. $\frac{c}{a} + \frac{c}{b} \neq \frac{c}{a+b}$, in general.

4. $\frac{a}{c} + \frac{b}{d} \neq \frac{a+b}{c+d}$, in general.

5. $(a+b)^p \neq a^p + b^p$, in general

Fundamental Property of Real Numbers

1. Law of Trichotomy: $\forall a, b \in \mathbb{R}$, $(a < b) \vee (a = b) \vee (a > b)$

Abbreviations Involving Equality and Inequalities

1. $a \neq b :\Longleftrightarrow \neg\,(a = b) \Longleftrightarrow (a < b) \vee (a > b)$

2. $a \leq b :\Longleftrightarrow (a = b) \vee (a < b)$

3. $a \geq b :\Longleftrightarrow (a = b) \vee (a > b)$

4. $(a \leq b) \wedge (a \geq b) :\Longleftrightarrow (a = b)$

5. $(a = b = c) :\Longleftrightarrow (a = b) \wedge (b = c)$

6. $(a < b < c) :\Longleftrightarrow (a < b) \wedge (b < c)$

7. $(a \leq b \leq c) :\Longleftrightarrow (a \leq b) \wedge (b \leq c)$

8. $(a > b > c) :\Longleftrightarrow (a > b) \wedge (b > c)$

9. $(a \geq b \geq c) :\Longleftrightarrow (a \geq b) \wedge (b \geq c)$

10. $\neg\,(a < b) \Longleftrightarrow (a \geq b)$

11. $\neg\,(a \leq b) \Longleftrightarrow (a > b)$

12. $\neg\,(a > b) \Longleftrightarrow (a \leq b)$

13. $\neg\,(a \leq b) \Longleftrightarrow (a > b)$

Intervals

1. $[a,b] \quad := \{x \in \mathbb{R} \mid a \leq x \leq b\}$
2. $[a,b[\quad := \{x \in \mathbb{R} \mid a \leq x < b\}$
3. $]a,b] \quad := \{x \in \mathbb{R} \mid a < x \leq b\}$
4. $]a,b[\quad := \{x \in \mathbb{R} \mid a < x < b\}$
5. $[a,+\infty[\quad := \{x \in \mathbb{R} \mid a \geq x\}$
6. $]a,+\infty[\quad := \{x \in \mathbb{R} \mid a > x\}$
7. $]-\infty,b] \quad := \{x \in \mathbb{R} \mid x \leq b\}$
8. $]-\infty,b[\quad := \{x \in \mathbb{R} \mid x < b\}$
9. $]-\infty,+\infty[\quad := \{x \in \mathbb{R} \mid -\infty < x < +\infty\} = \{x \in \mathbb{R} \mid \top\} = \mathbb{R}$

Rules of Inference for Equations and Inequalities

$\top$

1. $\dfrac{}{a = a}$

$a = b$

2. $\dfrac{}{b = a}$

$(a = b) \wedge (b = c)$

3. $\dfrac{}{a = c}$

$\top$

4. $\dfrac{}{a \leq a}$

$\top$

5. $\dfrac{}{a \geq a}$

$a < b$

6. $\dfrac{}{b > a}$

$a > b$

7. $\dfrac{}{b < a}$

$a \leq b$

8. $\dfrac{}{b \geq a}$

$a \geq b$

9. $\dfrac{}{b \leq a}$

$(a < b) \wedge (b < c)$

10. $\dfrac{}{a < c}$

$(a \leq b) \wedge (b \leq c)$

11. $\dfrac{}{a \leq c}$

$(a > b) \wedge (b > c)$

12. _____

$a > c$

$(a \geq b) \wedge (b \geq c)$

13. _____

$a > c$

$(a \geq b) \wedge (b \geq a)$

14. _____

$a = b$

15. Adding equals preserves equalities and inequalities:

$a = b$	$a < b$	$a \leq b$	$a > b$	$a \geq b$
$+c = +c$	$+c = +c$	$+c = +c$	$+c = +c$	$+c = +c$
_____	_____	_____	_____	_____
$a+c = b+c$	$a+c < b+c$	$a+c \leq b+c$	$a+c > b+c$	$a+c \geq b+c$

16. Subtracting equals preserves equalities and inequalities:

$a = b$	$a < b$	$a \leq b$	$a > b$	$a \geq b$
$-c = -c$	$-c = -c$	$-c = -c$	$-c = -c$	$-c = -c$
_____	_____	_____	_____	_____
$a-c = b-c$	$a-c < b-c$	$a-c \leq b-c$	$a-c > b-c$	$a-c \geq b-c$

17. Multiplying by strictly positive equals preserves equalities and inequalities:

$a = b$	$a < b$	$a \leq b$	$a > b$	$a \geq b$
$\times c = \times c$	$\times c = \times c$	$\times c = \times c$	$\times c = \times c$	$\times c = \times c$
_____	_____	_____	_____	_____
$a \times c = b \times c$	$a \times c < b \times c$	$a \times c \leq b \times c$	$a \times c > b \times c$	$a \times c \geq b \times c$

18. Multiplying by strictly negative equals reverses inequalities ($c < 0$):

$$\begin{array}{ccccc} a = b & a < b & a \leq b & a > b & a \geq b \\ \times c = \times c & \times c = \times c & \times c = \times c & \times c = \times c & \times c = \times c \\ \hline a \times c = b \times c & a \times c > b \times c & a \times c \geq b \times c & a \times c < b \times c & a \times c \leq b \times c \end{array}$$

Here $c < 0$; hence, the sense of the inequality is reversed.

19. Dividing by strictly positive equals preserves equalities and inequalities:

$$\begin{array}{ccccc} a = b & a < b & a \leq b & a > b & a \geq b \\ \div c = \div c & \div c = \div c & \div c = \div c & \div c = \div c & \div c = \div c \\ \hline a \div c = b \div c & a \div c < b \div c & a \div c \leq b \div c & a \div c > b \div c & a \div c \geq b \div c \end{array}$$

20. Dividing by strictly negative equals reverses inequalities $c < 0$:

$$\begin{array}{ccccc} a = b & a < b & a \leq b & a > b & a \geq b \\ \div c = \div c & \div c = \div c & \div c = \div c & \div c = \div c & \div c = \div c \\ \hline a \div c = b \div c & a \div c > b \div c & a \div c \geq b \div c & a \div c < b \div c & a \div c \leq b \div c \end{array}$$

Here $c < 0$; hence, the sense of the inequality is reversed.

21. Equals substituted into equals yield equals:

$$\frac{\begin{array}{c} e_1(x) = e_2(x) \\ f_1 = f_2 \end{array}}{e_1\langle x \longleftarrow f_1 \rangle = e_2\langle x \longleftarrow f_2 \rangle},$$

where $e_1(x)$ and $e_2(x)$ are equal expressions with the free variable x, and, f_1 and f_2 are equal expressions that are substituted in the place of x, in $e_1(x)$ and $e_2(x)$, respectively.

Definition of, Properties of and Rules of Inference Involving Absolute Values

1. $|x| := \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$
2. $\forall\, x \in \mathbb{R}, \quad |x| \geq 0$
3. $|a+b| \leq |a| + |b|$
4. $\big|\, |a| - |b| \,\big| \leq |a-b|$

5. $|ab| = |a||b|$
6. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
7. $|a^b| = |a|^b$

$$|x| = a$$

8. _____

$$(x = -a) \vee (x = a)$$

$$|x| < a$$

9. _____

$$(-a < x) \wedge (x < a)$$

$$|x| \leq a$$

10. _____

$$(-a \leq x) \wedge (x \leq a)$$

$$|x| > a$$

11. _____

$$(-a > x) \vee (x > a)$$

$$|x| \geq a$$

12. _____

$$(-a \geq x) \vee (x \geq a)$$

$$|a| < |b|$$

13. _____

$$(a < b) \vee (a > b) \vee (a < -b) \vee (a > -b)$$

$$|a| \leq |b|$$

14. _____

$$(a \leq b) \vee (a \geq b) \vee (a \leq -b) \vee (a \geq -b)$$

More Rules of Inference For Equations

$$ax + b = 0$$

1. _____

$$x = \frac{-b}{a}$$

$$ab = 0$$

2. _____

$$(a = 0) \vee (b = 0)$$

$$abc = 0$$

3. _____

$$(a = 0) \vee (b = 0) \vee (c = 0)$$

$$a_1 a_2 \cdots a_n = 0$$

4. _____

$$(a_1 = 0) \vee (a_2 = 0) \vee \cdots (a_n = 0)$$

$$x^2 = a^2$$

5. _____

$$(x = a) \vee (x = -a)$$

$$\left(x^{-1} = \frac{1}{x} = a\right) \wedge (a \neq 0)$$

6. _____

$$\left(x = a^{-1} = \frac{1}{a}\right)$$

7. **Quadratic Formula**

$$ax^2 + bx + c = 0$$

_____

$$\left(x = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\right) \vee \left(x = \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)\right)$$

8. **Cramer's Rule:**

$$ax + by = e$$

$$cx + dy = f$$

$$\left(x = \frac{ed - bf}{ad - bc}\right) \wedge \left(y = \frac{af - ce}{ad - bc}\right)$$

Math 118 Homework

1 Homework

For each of the following left-sides and right-sides draw the graphs of the sets $G_=$, $G_<$, $G_\leq$, $G_>$, $G_\geq$, where these sets are defined below.

- $G_= := \{(x,y) \in \mathbb{R}^2 | LS = RS\}$
- $G_< := \{(x,y) \in \mathbb{R}^2 | LS < RS\}$
- $G_\leq := \{(x,y) \in \mathbb{R}^2 | LS \leq RS\}$
- $G_> := \{(x,y) \in \mathbb{R}^2 | LS > RS\}$
- $G_\geq := \{(x,y) \in \mathbb{R}^2 | LS \geq RS\}$

Use the numbers $-5,-4,-3,-2,-1,0,1,2,3,4,5$ for x.

1. $LS := x, \quad RS := y$
2. $LS := y, \quad RS := x$
3. $LS := x+1, \quad RS := y+2$
4. $LS := x-1, \quad RS := y+1$
5. $LS := 3x-2, \quad RS := y$
6. $LS := 2x+1, \quad RS := y$
7. $LS := x^2, \quad RS := y$
8. $LS := x, \quad RS := y^2$
9. $LS := x^2+y^2, \quad RS := 1$
10. $LS := xy, \quad RS := 1$
11. $LS := xy, \quad RS := 0$
12. $LS := y, \quad RS := 1+x+x^2$
13. $LS := y, \quad RS := 1-x$
14. $LS := x+y+1, \quad RS := 2x-3y+1$
15. $LS := x^3+y^3, \quad RS := 1$

2 Homework

Compute the following expressions, and draw pictures

1. $\{1,2,3,4\} \cup \{1,-2,3,-4,5\}$
2. $\{1,2,3,4\} \cap \{1,-2,3,-4,5\}$
3. $\{1,2,3,4\} \times \{1,-2,3,-4,5\}$
4. $\{1,2\} \times \{2,3\} \times \{3,4\}$
5. $(\{1,2\} \cap \{2,3\}) \cup \{3,4\}$
6. $\{1,2\} \cap (\{2,3\} \cup \{3,4\})$
7. $\{1,2\} \cup \{2,3\} \cup \{3,4\}$
8. $\{1,2\} \cap \{2,3\} \cap \{3,4\}$
9. $\{1,2\} \cap (\{2,3\} \times \{3,4\})$
10. $\{1,2\} \times (\{2,3\} \cap \{3,4\})$
11. $\{1,2\} \cap (\{2,3\} \cup \{3,4\})$
12. $[1,2] \cup \{1,2,3\}$
13. $[1,2] \cap \{1,2,3\}$
14. $[1,2[\cup \{1,2,3\}$
15. $[1,2[\cap \{1,2,3\}$
16. $\{(1,2)\} \cup \{1,2,3\}$
17. $\{(1,2)\} \cap \{1,2,3\}$
18. $\{(1,2)\} \cup \{1,2,3\}$
19. $[1,2] \cup [0,3[$

20. $[1,2[\cup[0,3[$

21. $]1,2]\cup]0,3[$

22. $[1,2]\cap[0,3[\cap\{1,2\}$

3 Homework

1. Find two examples with actual numbers for a, b, and c, for one of which the assertion is true, and for the other of which, the assertion is false.

   These examples are intended to show that assertions are not true in general.

   (a) $\forall a,b,c \in \mathbb{N}\ ((a \div b) \div c = a \div (b \div c))$

   (b) $\forall a,b,c \in \mathbb{N}\ ((a \wedge b) \wedge c = a \wedge (b \wedge c))$

2. Find *all* the formation-trees for the expression $a+b+c+d+e$; you are not allowed to change the order of the letters.

3. Find

   (a) a formation-tree for each of the following expressions, and

   (b) calculate the *value* of the expression, *using the formation-tree* that you found in part 3a.

   (a) $2+2-2\times2\div2\wedge2$
   (b) $2\wedge2+2-2\times2\div2$
   (c) $2\div2\wedge2+2-2\times2$
   (d) $2\times2\div2\wedge2+2-2$
   (e) $2-2\times2\div2\wedge2+2$
   (f) $2+2-2-2+2-2+2+2+2$
   (g) $2-2-2-2\div2\div2\div2$
   (h) $2-2-2+2-2\wedge2\wedge2\wedge2$
   (i) $2+2-2\times2\div2\wedge2$
   (j) $2+2-2\times2\wedge2\div2\wedge2$
   (k) $2+(2+(2\times2)\div2)\wedge2$
   (l) $(2+2+(2\times2)\div2)\wedge2$
   (m) $2+(2+2\times(2\div2\wedge2))$
   (n) $-2+2-2+2-2+2$
   (o) $-(2+2-2+2-2+2)$
   (p) $(2-2)-2\div(2\div2\wedge2)\wedge2$

4. Find a formation tree for each of the following expressions.

   (a) $a \wedge b \wedge c \wedge d \div e \div f \div g$
   (b) $(a \wedge b \wedge c) \wedge \big(d \div e \div f\big) \div g$
   (c) $a \wedge \big(b \wedge c \wedge \big(d \div e \div f \div g\big)\big)$
   (d) $(a \wedge b) \wedge (c \wedge d \div e) \div f \div g$
   (e) $a \wedge b \wedge \big(c \wedge \big(d \div e \div f\big) \div g\big)$
   (f) $a+b+c-d-e\times f\times g\div i\div j\wedge k\wedge l$
   (g) $a \wedge b \div c \times d + e - f \wedge g$
   (h) $a \div b \times c \times d \wedge e \wedge f \wedge g$
   (i) $a \wedge \big(b \div (c \wedge d \wedge e) \div f\big) \wedge g$
   (j) $(a-b-c) \wedge \big(d-e-f\big) \wedge g$

4 Homework

For each of the following expressions:

- find a formation-tree,
- calculate the value by hand,
- calculate the value, using a calculator (your work should include the sequence of keystrokes used)
- see that the answers to parts (b) and (c) agree, and
- find a situational meaning (that is: describe a situation and ask a question about the situation, to which the given expression is an answer).

1. $1+2-3-4$
2. $-2-3$
3. -2^3-3^2
4. $-2(-3)+(-2)(-3)$
5. -2^3+1
6. $(-2)^3+1$
7. $\dfrac{-2-3}{4}$
8. $\dfrac{2}{3+5}$
9. 2^2-3
10. 2^{3-2}
11. $\dfrac{2^3}{3^2+5^2}$
12. $\dfrac{1}{2-3}+4$
13. $\dfrac{1}{2}+\dfrac{2}{3}$
14. $\left(\dfrac{-1}{2}\right)-\left(\dfrac{2}{-3}\right)-\dfrac{3}{5}$
15. $\left(\dfrac{-1}{2}\right)\times\left(\dfrac{2}{-3}\right)\times\dfrac{3}{5}$
16. $\left(\dfrac{-1}{2}\right)\div\left(\dfrac{2}{-3}\right)\times\dfrac{3}{5}$
17. $\left(\dfrac{-1}{2}\right)\times\left(\dfrac{2}{-3}\right)\div\dfrac{3}{5}$
18. $\left(\dfrac{-1}{2}\right)^2-\left(\dfrac{2}{-3}\right)^2\div\dfrac{3}{5}$
19. $2\left(1+\dfrac{1}{2}\right)\left(1+\dfrac{1}{2}+\dfrac{1}{3}\right)$
20. $\left(\dfrac{1}{-2}\right)\left(\dfrac{-1}{3}\right)+\left(\dfrac{1}{-2}\right)\left(\dfrac{-1}{3}\right)\left(\dfrac{-2}{1}\right)$
21. 2^{1^2}
22. 2^{2^1}
23. $(-1)^{(1+1^2)^{(1+(-1)^4)}}$
24. $\left(\dfrac{2}{-3}\right)-\left(\dfrac{-3}{2}\right)$
25. $\left(\dfrac{2+3}{3}\right)-\left(\dfrac{3}{2+3}\right)$
26. $\left(\dfrac{2}{-3}\right)+\left(\dfrac{-3}{5}\right)+\left(\dfrac{5}{2}\right)$
27. $\dfrac{\left(\dfrac{1+2}{2}\right)-\left(\dfrac{1}{1+2}\right)}{\left(\dfrac{2+1}{2}\right)+\left(\dfrac{1}{2+1}\right)}$
28. $\dfrac{1}{2}+\dfrac{1}{\dfrac{1}{2}+\dfrac{1}{\dfrac{1}{2}+\dfrac{1}{\dfrac{1}{2}+\dfrac{1}{2}}}}$
29. $1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1}}}}$
30. $\dfrac{1+\dfrac{1-\dfrac{1}{2}}{1+\dfrac{1}{2}}}{1-\dfrac{1-\dfrac{1}{2}}{1+\dfrac{1}{2}}}$
31. $\dfrac{\dfrac{1}{2}-\dfrac{1}{3}}{\dfrac{1}{2}+\dfrac{1}{3}}$
32. $\dfrac{\dfrac{\dfrac{1}{2}-\dfrac{1}{3}}{\dfrac{1}{2}+\dfrac{1}{3}}+\dfrac{\dfrac{1}{2}+\dfrac{1}{3}}{\dfrac{1}{2}-\dfrac{1}{3}}}{\dfrac{\dfrac{1}{2}-\dfrac{1}{3}}{\dfrac{1}{2}+\dfrac{1}{3}}-\dfrac{\dfrac{1}{2}+\dfrac{1}{3}}{\dfrac{1}{2}-\dfrac{1}{3}}}$
33. $\dfrac{1}{\left(\dfrac{1}{1+2}\right)-\left(\dfrac{-1}{1-2}\right)}$

5 Homework

Find normal forms for the following expressions:

1. $a(1+a)(1+a+b)$
2. $ab+abc$
3. a^{b^c}
4. $a^{(1+a^2)^{(1+a^4)}}$
5. $\left(\dfrac{a}{b}\right)-\left(\dfrac{b}{a}\right)$
6. $\left(\dfrac{a+b}{b}\right)-\left(\dfrac{b}{a+b}\right)$
7. $\dfrac{a}{b}+\dfrac{b}{c}+\left(\dfrac{c}{a}\right)$
8. $\dfrac{2a}{b}-\dfrac{3b}{c}+\left(\dfrac{4c}{a}\right)-1$
9. $\left(\dfrac{x+1}{x}\right)-x$
10. $a^2-\dfrac{b}{a-b}$
11. $(ab-1)-\dfrac{1}{1+ab}$
12. $\left(\dfrac{2a+1}{a}\right)-2$
13. $\dfrac{\left(\dfrac{a+b}{b}\right)-\left(\dfrac{b}{a+b}\right)}{\left(\dfrac{a+b}{b}\right)+\left(\dfrac{b}{a+b}\right)}$
14. $\left(\dfrac{a}{1+bc}\right)+\left(\dfrac{b}{1+ca}\right)+\left(\dfrac{c}{1+ab}\right)$
15. $x+\dfrac{1}{x+\dfrac{1}{x+\dfrac{1}{x+\dfrac{1}{x}}}}$

6 Homework

Find situational meanings for the following expressions (that is, describe a situation, and ask a question about the situation to which the given expression is an answer). In other words, *make up your own* word problem which has the given expression as as an answer:

1. $2+3$
2. $3+2$
3. $2-3$
4. $3-2$
5. $(1+a)(1+b)(1+c)$
6. 2×3
7. 3×2
8. $2\div 3$
9. $3\div 2$
10. 2^3
11. 3^2
12. $(2+3)+5$
13. $(2+3)+5$
14. $2+3+5$
15. $(2-3)-5$
16. $2-(3-5)$
17. $2-3-5$
18. $2\times 3-5$
19. $2-3\times 5$
20. $(2-3)\times 5$
21. $2\div(3+5)$
22. $\dfrac{\left(\frac{2}{3}\right)}{5}$
23. $\dfrac{2}{\left(\frac{3}{5}\right)}$
24. $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{2}-\frac{1}{3}}$
25. $\left(\frac{1}{2}\right)\div 2$
26. $\left(\frac{1}{2}\right)\div\left(\frac{2}{3}\right)$
27. 2^3+4
28. $2-4^3$
29. 2^3+3^2
30. 2^3-3^2
31. $2x+3$
32. $2x-3$
33. $2(x+3)$
34. $2(x-3)$
35. $x+y+z$
36. $x+xy$
37. $x(1+a)$
38. $a-b-c$
39. $3x+2y-1$
40. $1-a-ab$
41. $\dfrac{1}{x}+\dfrac{1}{y}$
42. $\dfrac{x-y}{x+y}$
43. x^2+x^3
44. $1+x^2-x^3$
45. $\dfrac{1}{x}+\dfrac{1}{x^2}+\dfrac{1}{x^3}$
46. $(x+y)(x-y)$
47. ax^2+bx+c

7 Homework

Find normal forms for the following expressions:

1. $(-1)\,ab\,(-1)\,ac\,(-1)\,ca$
2. $(-2)\,ab^2c^3b^2a\,(-2)$
3. $ab-2bc+3ca$
4. ab^2-ba^2+3ca
5. $(1+a)(1+b)(1+c)$
6. $(1+a+b)(1+a+b)$
7. $-(a+b)(-1-a+b)(2-3a+4b)$
8. $(a-ab)2$
9. $ab(1+ab)$
10. $3ab+2+abc-4ab+3abc-7$
11. $\left(ab^2-abcd\right)^2 d$
12. $ab-2ab(b-c)$
13. $ab(a+b)+bc(b+c)+ca(c+a)$
14. $(a+b)^2+(a-b)^2$
15. $(a+b)^2-(a-b)^2$

16. $(a+b)^3+(a-b)^3$
17. $(a+b)^3-(a-b)^3$
18. $(a+b)^4+(a-b)^4$
19. $(a+b)^4-(a-b)^4$
20. $(a+b+c)^2+(a-b+c)^2$
21. $(a+b+c)^2-(a-b+c)^2$
22. $(a+b+c)^2+(a-b-c)^2$
23. $(a+b+c)^2-(a-b-c)^2$
24. $(-a+b-c)^2+(a-b+c)^2$
25. $-(a-b-c)^2+(a-b+c)^2$
26. $(a+b)^2-a^2-b^2+4ab-1$

8 Homework

Carry out the following substitutions:

1. $\left(a+\frac{1}{a}\right)\langle\ a\longleftarrow 2\ \rangle$
2. $\left(a+\frac{1}{a}\right)\langle\ a\longleftarrow \frac{2}{a}\ \rangle$
3. $\left(a^2+ab+b^2\right)\left\langle \begin{array}{c} a\longleftarrow b \\ \\ b\longleftarrow a \end{array} \right\rangle$
4. $(a+ab+abc)\left\langle \begin{array}{c} a\longleftarrow (x+y) \\ \\ b\longleftarrow (x-y) \end{array} \right\rangle$
5. $\left(\left(a^2+ab+b^2\right) \left\langle \begin{array}{c} a\longleftarrow x \\ \\ b\longleftarrow y \end{array} \right\rangle \right) \left\langle \begin{array}{c} x\longleftarrow a \\ \\ y\longleftarrow b \end{array} \right\rangle$
6. $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{ab}\right)\left\langle \begin{array}{c} a\longleftarrow \frac{x}{y} \\ \\ a\longleftarrow \frac{y}{x} \end{array} \right\rangle$
7. $\left(\frac{a+\frac{1}{b}}{b+\frac{1}{a}}\right)\left\langle \begin{array}{c} a\longleftarrow b+a^2 \\ \\ b\longleftarrow a+b^2 \end{array} \right\rangle$
8. $\left(a^{x+y}\right)\left\langle \begin{array}{c} a\longleftarrow 1 \\ \\ x\longleftarrow \frac{1}{2} \end{array} \right\rangle$
9. $\left(\frac{1^x}{1+\frac{x}{2}}\right)\left\langle \begin{array}{c} x\longleftarrow \frac{1}{2} \\ \\ y\longleftarrow \frac{1}{2} \end{array} \right\rangle$
10. $\left(\frac{1}{x}+y+\frac{1}{z}\right)\left\langle \begin{array}{c} x\longleftarrow (x+y) \\ \\ y\longleftarrow -(x+y) \\ \\ a\longleftarrow b \end{array} \right\rangle$
11. $(x+y+z)\left\langle \begin{array}{c} x+y\longleftarrow a \\ \\ y+z\longleftarrow b \end{array} \right\rangle$
12. $(x+y+z)\left\langle \begin{array}{ccc} x & \longleftarrow & 0 \\ \\ y+z & \longleftarrow & x+y \end{array} \right\rangle$

9 Homework

1. Use 'Rules for Factoring' to match each of the following expressions with the left or right side of a rule, and then rewrite the expression to match the other side, that is the other side of the same rule that is did not match.

   (a) $2x+3x$
   (b) $xa+xy$
   (c) a^2-2^2
   (d) $a^2-(a-b)^2$
   (e) $(a+b)^2+(a-b)^2$
   (f) $(a+b)^2-(a-b)^2$

(g) $(a+b)^3+(a-b)^3$

(h) $(a+b)^3-(a-b)^3$

(i) $(a+b)^4+(a-b)^4$

(j) $(a+b)^4-(a-b)^4$

(k) $(a+b+c)^2+(a-b+c)^2$

(l) $(a+b+c)^2-(a-b+c)^2$

(m) $(a+b+c)^2+(a-b-c)^2$

(n) $(a+b+c)^2-(a-b-c)^2$

(o) $(-a+b-c)^2+(a-b+c)^2$

(p) $-(a-b-c)^2+(a-b+c)^2$

(q) $(a+b)^2-a^2-b^2+4ab-1$

(r) $\left(\left(\frac{1}{a+b}\right)+3\right)^2$

(s) $\left(2-\frac{3}{x}\right)^3$

(t) $\left(2^4-\left(\frac{1}{2}\right)^4\right)$

(u) $\left(\frac{x-1}{x+1}\right)^n-\left(\frac{x+1}{x-1}\right)^n$

(v) $ab+ca$

(w) $2^4-\left(\frac{1}{2}\right)^4$

(x) $\left(\left(\frac{1}{a+b}\right)+3\right)^4$

2. Match each of the following expressions with the left or right side of any equation of your choice, after rewriting it in a way that you consider suitable.

(a) a^2+b^2

(b) $100-a^2$

(c) $125+(a+b)^3$

(d) -1000

(e) $\left(2^4-\left(\frac{1}{2}\right)^4\right)$

(f) $-(a-b-c)^2+(a-b+c)^2$

(g) $9a^2-16b^2$

(h) $2a^2-3b^2$

(i) $27x^3-1$

(j) $a^2b^2-c^2d^2$

3. Rewrite each of the following expressions in 10 different ways so that the resulting expression is of length at least 20. The same symbol may not be used more than twice:

(a) 1

(b) 2

(c) $1+2$

(d) x

(e) $x+y$

(f) xy

(g) $x-y$

(h) x^2

(i) $(1+x)(1-x)$

(j) $1+x+x^2$

(k) $1-x+y-xy$

10 Homework

For each of the following equations

- write down the machine and geometric interpretations,
- solve for the unknown exactly, by playing a game,
- check that your solution works,
- find an approximate solution by drawing a graph,

- find an approximate solution, using your calculator, and
- check that all the solutions agree approximately.

1. $3x+4=7$
2. $3x-4=7$
3. $-3x+4=7$
4. $-3x-4=7$
5. $3x+4=-7$
6. $3x-4=-7$
7. $-3x+4=-7$
8. $-3x-4=-7$
9. $\sqrt{2}(x)+\sqrt{3}=\sqrt{7}$
10. $\frac{1}{2}x+\frac{3}{4}=\frac{2}{3}$
11. $\frac{1}{2}x+\sqrt{2}=2.5$
12. $3\left(\frac{1}{x}\right)+4=7$
13. $3\left(\frac{3}{x+2}\right)+4=7$
14. $3\left(\frac{x-1}{x+1}\right)+4=7$
15. $3\sqrt{x}+4=7$
16. $3\sqrt[3]{x}+4=7$

11 Homework

For each of the following inequalities:

- write down the machine and geometric interpretations,
- solve for the unknown exactly, by playing a game,
- check that your solution works,
- find an approximate solution by drawing a graph,
- find an approximate solution, using your calculator, and
- check that all the solutions agree approximately.

1. $3x+4<7$
2. $3x-4\leq 7$
3. $-3x+4>7$
4. $-3x-4\geq 7$
5. $3x+4<-7$
6. $3x-4\leq -7$
7. $-3x+4>-7$
8. $-3x-4\geq -7$
9. $\sqrt{2}(x)+\sqrt{3}<\sqrt{7}$
10. $\frac{1}{2}x+\frac{3}{4}\leq\frac{2}{3}$
11. $\frac{1}{2}x+\sqrt{2}>2.5$
12. $3\left(\frac{1}{x}\right)+4\geq 7$
13. $3\left(\frac{3}{x+2}\right)+4<7$
14. $3\left(\frac{x-1}{x+1}\right)+4\leq 7$
15. $3\sqrt{x}+4>7$
16. $3\sqrt[3]{x}+4\geq 7$

12 Homework

For each of the following inequalities:

- write down the machine and geometric interpretations,
- solve for the unknown exactly, by playing a game,
- check that your solution works,

- find an approximate solution by drawing a graph,
- find an approximate solution, using your calculator, and
- check that all the solutions agree approximately.

1. $1 < 3x + 4 < 7$
2. $-1 \leq 3x - 4 \leq 7$
3. $11 \geq -3x + 4 > 7$
4. $11 > -3x - 4 \geq 7$
5. $-1 \leq 3x + 4 < -7$
6. $-8 < 3x - 4 \leq -7$
7. $2 \geq -3x + 4 > -7$
8. $2 > -3x - 4 \geq -7$
9. $\sqrt{2} < \sqrt{2}(x) + \sqrt{3} < \sqrt{7}$
10. $\frac{3}{4} \geq \frac{1}{2}x + \frac{3}{4} \geq \frac{2}{3}$
11. $1.5 \geq \frac{1}{2}x + \sqrt{2} > 2.5$
12. $10 > 3\left(\frac{1}{x}\right) + 4 \geq 7$
13. $1 \leq 3\left(\frac{3}{x+2}\right) + 4 < 7$
14. $0 \leq \left(\frac{x-1}{x+1}\right) + 1 \leq 2$
15. $11 > 3\sqrt{x} + 4 > 7$
16. $11 \geq 3\sqrt[3]{x} + 4 \geq 7$

13 Homework

For each of the following equations:

- write down the machine and geometric interpretations,
- solve for the unknown exactly, by playing a game,
- check that your solution works,
- find an approximate solution by drawing a graph,
- find an approximate solution, using your calculator, and
- check that all the solutions agree approximately.

1. $|3x + 4| = 7$
2. $|3x - 4| = 7$
3. $|-3x + 4| = 7$
4. $|-3x - 4| = 7$
5. $|3x + 4| = -7$
6. $|3x - 4| = -7$
7. $|-3x + 4| = -7$
8. $|-3x - 4| = -7$
9. $|\sqrt{2}(x) + \sqrt{3}| = \sqrt{7}$
10. $\left|\frac{1}{2}x + \frac{3}{4}\right| = \frac{2}{3}$
11. $\left|\frac{1}{2}x + \sqrt{2}\right| = 2.5$
12. $\left|3\left(\frac{1}{x}\right) + 4\right| = 7$
13. $\left|3\left(\frac{3}{x+2}\right) + 4\right| = 7$
14. $\left|3\left(\frac{x-1}{x+1}\right) + 4\right| = 7$
15. $\left|3\sqrt{x} + 4\right| = 7$
16. $\left|3\sqrt[3]{x} + 4\right| = 7$

14 Homework

For each of the following equations:

- write down the machine and geometric interpretations,
- solve for the unknown exactly, by playing a game,

- check that your solution works,
- find an approximate solution by drawing a graph,
- find an approximate solution, using your calculator, and
- check that all the solutions agree approximately.

1. $3|x|+4=7$
2. $3|x|-4=7$
3. $-3|x|+4=7$
4. $-3|x|-4=7$
5. $3|x|+4=7$
6. $3|x|-4=7$
7. $-3|x|+4=-7$
8. $-3|x|-4=-7$
9. $\sqrt{2}|x|+\sqrt{3}=\sqrt{7}$
10. $\frac{1}{2}|x|+\frac{3}{4}=\frac{2}{3}$
11. $\frac{1}{2}|x|+\sqrt{2}=2.5$
12. $3\left(\frac{1}{|x|}\right)+4=7$
13. $3\frac{3}{|x+2|}+4=7$
14. $3\left|\frac{x-1}{x+1}\right|+4=7$
15. $|x|+x+1=7$
16. $2|x|+3|x+1|=7$

15 Homework

For each of the following inequalities:

- write down the machine and geometric interpretations,
- solve for the unknown exactly, by playing a game,
- check that your solution works,
- find an approximate solution by drawing a graph,
- find an approximate solution, using your calculator, and
- check that all the solutions agree approximately.

1. $|3x+4|<7$
2. $|3x-4|\leq 7$
3. $|-3x+4|>7$
4. $|-3x-4|\geq 7$
5. $|3x+4|<-7$
6. $|3x-4|\leq -7$
7. $|-3x+4|>-7$
8. $|-3x-4|\geq -7$
9. $|\sqrt{2}(x)+\sqrt{3}|<\sqrt{7}$
10. $\left|\frac{1}{2}x+\frac{3}{4}\right|\leq\frac{2}{3}$
11. $\left|\frac{1}{2}x+\sqrt{2}\right|>2.5$
12. $\left|3\left(\frac{1}{x}\right)+4\right|\geq 7$
13. $\left|3\left(\frac{3}{x+2}\right)+4\right|<7$
14. $\left|3\left(\frac{x-1}{x+1}\right)+4\right|\leq 7$
15. $\left|3\sqrt{x}+4\right|>7$
16. $\left|3\sqrt[3]{x}+4\right|\geq 7$

16 Homework

For each of the following equations

- write down the machine and geometric interpretations,

- solve for the unknown exactly, by playing a game,
- check that your solution works,
- find an approximate solution by drawing a graph,
- find an approximate solution, using your calculator, and
- check that all the solutions agree approximately.

1. $2(3(x-1)+4)+1=7$
2. $2\left(3\left(\frac{1}{x}-1\right)+4\right)+1=7$
3. $\frac{1}{2}\left(\frac{2}{3}(2x-3)+\frac{3}{2}\right)=\frac{1}{2}$
4. $2\left(\frac{x-1}{x+1}\right)+3=5$
5. $2\left(\frac{\left(\frac{x-1}{x+1}\right)-\left(\frac{x+1}{x-1}\right)}{\left(\frac{x-1}{x+1}\right)+\left(\frac{x+1}{x-1}\right)}\right)+3=5$
6. $2(3|x-1|+4)+1=7$
7. $2\left(3\left|\frac{1}{x}-1\right|+4\right)+1=7$
8. $\frac{1}{2}\left(\frac{2}{3}|2x-3|+\frac{3}{2}\right)=\frac{1}{2}$
9. $2\left(\frac{|x|-1}{|x|+1}\right)+3=5$
10. $2\left|\frac{x-1}{x+1}\right|+3=5$
11. $2\left|\frac{|x|-1}{|x|+1}\right|+3=5$
12. $3|x+1|=2|x-1|$
13. $3|x+1|-2|x-1|=1$
14. $(|x+1|)(|x-1|)=2$
15. $\frac{|x+1|-1}{|x-1|+1}+1=2$
16. $\left|\frac{|x+1|-1}{|x-1|+1}\right|+1=2$

17 Homework

For each of the following equations

- write down the machine and geometric interpretations,
- solve for the unknown exactly, by playing a game,
- check that your solution works,
- find an approximate solution by drawing a graph,
- find an approximate solution, using your calculator, and
- check that all the solutions agree approximately.

1. $2(3(x-1)+4)+1<7$
2. $2\left(3\left(\frac{1}{x}-1\right)+4\right)+1\leq 7$
3. $\frac{1}{2}\left(\frac{2}{3}(2x-3)+\frac{3}{2}\right)>\frac{1}{2}$
4. $2\left(\frac{x-1}{x+1}\right)+3\geq 5$
5. $2\left(\frac{\left(\frac{x-1}{x+1}\right)-\left(\frac{x+1}{x-1}\right)}{\left(\frac{x-1}{x+1}\right)+\left(\frac{x+1}{x-1}\right)}\right)+3<5$
6. $2(3|x-1|+4)+1\leq 7$
7. $2\left(3\left|\frac{1}{x}-1\right|+4\right)+1>7$
8. $\frac{1}{2}\left(\frac{2}{3}|2x-3|+\frac{3}{2}\right)\geq\frac{1}{2}$
9. $2\left(\frac{|x|-1}{|x|+1}\right)+3<5$
10. $2\left|\frac{x-1}{x+1}\right|+3\leq 5$
11. $2\left|\frac{|x|-1}{|x|+1}\right|+3>5$
12. $3|x+1|=2|x-1|$
13. $3|x+1|-2|x-1|\geq 1$
14. $(|x+1|)(|x-1|)<2$
15. $\frac{|x+1|-1}{|x-1|+1}+1\leq 2$
16. $\left|\frac{|x+1|-1}{|x-1|+1}\right|+1>2$

18 Homework

Factor the following expressions:

1. $ax+bx$
2. $xa+xb+xc$
3. $4a^2-2^2$
4. $2a^2-b^2$
5. $(a+b)^2+\left(\frac{(a-b)^2}{9}\right)$
6. $\left(\frac{(a+b)^2}{a^4}\right)-3(a-b)^2$
7. a^4+b^4
8. a^3-b^6
9. $(a+b)^8+(a-b)^8$
10. $a^4(a+b)^4-b^8(a-b)^4$
11. $(a+b+c)^2+(a-b+c)^2$
12. x^2+5x+6
13. x^2-5x+6
14. x^2-x-6
15. x^2+x-6
16. x^2+4x+4
17. x^2-4x+4
18. $4x^2+4x+1$
19. $4x^2-4x+1$
20. $(x+1)^2+5(x+1)+6$
21. $(2x-3)^2-5(2x-3)+6$
22. $(ax-b)^2-5(ax-b)+6$
23. x^4+5x^2+6
24. x^6-5x^3+6
25. $\left(x^4-\left(\frac{1}{2}\right)^4\right)$
26. $\left(\frac{x-1}{x+1}\right)^4-\left(\frac{x+1}{x-1}\right)^4$
27. $x^2+10x+21$
28. $x^2-3x+70$
29. $x^2-\left(1+\sqrt{2}\right)x+\sqrt{2}$
30. $x^2-\left(\sqrt{a}+\sqrt{b}\right)x+\sqrt{ab}$
31. $abx^2-(a+b)x+1$
32. $ax^2-(a+b)x+b$
33. $x^2-3x+70$
34. $6x^2+13x+6$
35. $6x^2-13x+6$
36. $1+x+y+xy$
37. $1-x+y-xy$
38. $1+x-y-xy$
39. $1-x-y+xy$
40. $x^2-\left(\sqrt{2}+\sqrt{3}\right)x+\sqrt{6}$
41. $\left(\frac{1}{4}\right)x^2-x+\left(\frac{1}{4}\right)$
42. $6x^2+5x-6$
43. $6x^2-5x-6$

19 Homework

Show that the following are true:

1. $\left(\sqrt{2}+\sqrt{3}\right)^2+=5+2\sqrt{6}$
2. $\left(\sqrt{a}+\sqrt{b}\right)^2=a+b+2\sqrt{ab}$
3. $\left(\sqrt{a}-\sqrt{b}\right)^2=a+b-2\sqrt{ab}$
4. $\left(\sqrt{2}+\sqrt[4]{2}\right)^2=2+\sqrt{2}+\sqrt[4]{2^7}$
5. $\left(a+\frac{1}{a}\right)^2=a^2+2+\left(\frac{1}{a}\right)^2$
6. $(a+b)^2+(a-b)^2=2\left(a^2+b^2\right)$
7. $(a+b)^2-(a-b)^2=4ab$
8. $\left(a+\frac{1}{a}\right)^2-\left(a-\frac{1}{a}\right)^2=4,\quad(a\neq 0)$
9. $a^2+ab+b^2=\frac{(a+b)^2}{2}+\frac{\left(a^2+b^2\right)}{2}$
10. $\left(a-\frac{1}{a}\right)\left(b+\frac{1}{b}\right)+\left(a+\frac{1}{a}\right)\left(b-\frac{1}{b}\right)=2\left(ab-\frac{1}{ab}\right)$
11. $\left(a-\frac{1}{a}\right)\left(b+\frac{1}{b}\right)-\left(a+\frac{1}{a}\right)\left(b-\frac{1}{b}\right)=2\left(\frac{a}{b}-\frac{b}{a}\right)$
12. $\left(a+\frac{1}{a}\right)\left(b+\frac{1}{b}\right)-\left(a-\frac{1}{a}\right)\left(b-\frac{1}{b}\right)=2\left(ab+\frac{1}{ab}\right)$
13. $\left(a+\frac{1}{a}\right)\left(b+\frac{1}{b}\right)+\left(a-\frac{1}{a}\right)\left(b-\frac{1}{b}\right)=2\left(\frac{a}{b}+\frac{b}{a}\right)$
14. $\left(a^2+b^2\right)\left(c^2+d^2\right)=(ad+bc)^2+(ac-bd)^2$
15. $\left(a^2+b^2+c^2+d^2\right)\left(p^2+q^2+r^2+s^2\right)=\left(\alpha^2+\beta^2+\gamma^2+\delta^2\right)$,

where

$$\begin{aligned}\alpha &= ap-bq-cr-ds\\ \beta &= aq+bp+cs-dr\\ \gamma &= as-br+cp+dq\\ \delta &= ar+bs-cq+dp\end{aligned}$$

16. $\left(\frac{1-t^2}{1+t^2}\right)^2+\left(\frac{2t}{1+t^2}\right)^2=1$

17. $\left(\frac{t^2+1}{t^2-1}\right)^2-\left(\frac{2t}{t^2-1}\right)^2=1$

20 Homework

Show that the following are true:

1. $\left(x+\frac{1}{x}=3\right)\Longrightarrow\left(x^2+\frac{1}{x^2}=7\right)$
2. $\left(x+\frac{1}{x}=a\right)\Longrightarrow\left(x^2+\frac{1}{x^2}=a^2-2\right)$
3. $\left(x+\frac{1}{x}=a\right)\Longrightarrow\left(x^2+\frac{1}{x^2}=a^3-3a\right)$
4. $\left(x+\frac{1}{x}=a\right)\Longrightarrow\left(x^4+\frac{1}{x^4}=a^4-4a^2-3\right)$
5. $(a+b+c=0)\Longrightarrow\left(\left(a^2+ab+b^2\right)=\left(b^2+bc+c^2\right)=\left(a^2+ac+c^2\right)\right)$
6. $(a+b+c=0)\Longrightarrow\left(a^3+b^3+c^3=3abc\right)$
7. $(a+b+c=2s)\Longrightarrow\left(s^2+(s-a)^2+(s-b)^2+(s-c)^2=a^2+b^2+c^2\right)$
8. $(ab+bc+ca=0)\Longrightarrow\left(a^2+b^2+c^2=(a+b+c)^2\right)$

21 Homework

Solve the following equations for the designated unknown, by factoring:

1. $ax+bx=1,\quad x$
2. $x^2-4=0,\quad x$
3. $4a^2-2^2=0,\quad a$
4. $2a^2-b^2=0,\quad b$
5. $(a+b)^2+\left(\frac{(a-b)^2}{9}\right)=0,\quad b$
6. $\left(\frac{(a+b)^2}{a^4}\right)-3(a-b)^2=0,\quad a$
7. $3x^2-7=0,\quad x$
8. $(x)^3+10=0,\quad x$
9. $8(x)^3-1=0,\quad x$
10. $a^4(a+b)^4-b^8(a-b)^4=0,\quad b$
11. $(a+b+c)^2-(a-b+c)^2=0,\quad c$
12. $x^2+5x+6=0,\quad x$
13. $x^2-5x+6=0,\quad x$
14. $x^2-x-6=0,\quad x$
15. $x^2+x-6=0,\quad x$
16. $x^2+4x+4=0,\quad x$
17. $x^2-4x+4=0,\quad x$
18. $4x^2+4x+10=0,\quad x$
19. $4x^2-4x+10=0,\quad x$
20. $(x+1)^2+5(x+1)+6=0,\quad x$
21. $(2x-3)^2-5(2x-3)+6=0,\quad x$
22. $(ax-b)^2-5(ax-b)+6=0,\quad x$
23. $x^4+5x^2+6=0,\quad x$
24. $x^6-5x^3+6=0,\quad x$
25. $\left(x^4-\left(\frac{1}{2}\right)^4\right)=0,\quad x$
26. $\left(\frac{x-1}{x+1}\right)^4-\left(\frac{x+1}{x-1}\right)^4=0,\quad x$
27. $x^2+10x+21=0,\quad x$
28. $x^2-3x+70=0,\quad x$
29. $x^2-\left(1+\sqrt{2}\right)x+\sqrt{2}=0,\quad x$
30. $x^2-\left(\sqrt{a}+\sqrt{b}\right)x+\sqrt{ab}=0,\quad x$
31. $abx^2-(a+b)x+1=0,\quad x$
32. $ax^2-(a+b)x+b=0,\quad x$
33. $x^2-3x+70=0,\quad x$
34. $6x^2+13x+6=0,\quad x$
35. $6x^2-13x+6=0,\quad x$

36. $1+x+y+xy=0, \quad x,\ y$

37. $1-x+y-xy=0, \quad x,\ y$

38. $1+x-y-xy=0, \quad x,\ y$

39. $1-x-y+xy=0, \quad x,\ y$

40. $x^2-\left(\sqrt{2}+\sqrt{3}\right)x+\sqrt{6}=0, \quad x$

41. $\left(\frac{1}{4}\right)x^2-x+\left(\frac{1}{4}\right)=0, \quad x$

42. $6x^2+5x-6=0, \quad x$

43. $6x^2-5x-6=0, \quad x$

22 Homework

Solve the following quadratic equations by:

- the quadratic formula, and draw a graph indicating the roots on the graph,
- factoring,
- completing the square.

1. $3x^2-7=0, \quad x$
2. $x^2+5x+6=0, \quad x$
3. $x^2-5x+6=0, \quad x$
4. $x^2-x-6=0, \quad x$
5. $x^2+x-6=0, \quad x$
6. $x^2+4x+4=0, \quad x$
7. $x^2-4x+4=0, \quad x$
8. $4x^2+4x+10=0, \quad x$
9. $4x^2-4x+10=0, \quad x$
10. $(x+1)^2+5(x+1)+6=0, \quad x$
11. $(2x-3)^2-5(2x-3)+6=0, \quad x$
12. $(ax-b)^2-5(ax-b)+6=0, \quad x$
13. $x^4+5x^2+6=0, \quad x$
14. $x^6-5x^3+6=0, \quad x$
15. $x^2+10x+21=0, \quad x$
16. $x^2-3x+70=0, \quad x$
17. $x^2-\left(1+\sqrt{2}\right)x+\sqrt{2}=0, \quad x$
18. $x^2-\left(\sqrt{a}+\sqrt{b}\right)x+\sqrt{ab}=0, \quad x$
19. $abx^2-(a+b)x+1=0, \quad x$
20. $ax^2-(a+b)x+b=0, \quad x$
21. $x^2-3x+70=0, \quad x$
22. $6x^2+13x+6=0, \quad x$
23. $6x^2-13x+6=0, \quad x$
24. $x^2-\left(\sqrt{2}+\sqrt{3}\right)x+\sqrt{6}=0, \quad x$
25. $\left(\frac{1}{4}\right)x^2-x+\left(\frac{1}{4}\right)=0, \quad x$
26. $6x^2+5x-6=0, \quad x$
27. $6x^2-5x-6=0, \quad x$

23 Homework

Solve the following systems of equations for x and y by:

- the formula,
- playing a game,
- drawing a graph, and
- find a situational meaning.

1. $$\begin{aligned} x+y&=1\\ x-y&=0 \end{aligned}$$
2. $$\begin{aligned} 2x+3y&=1\\ 2x-3y&=0 \end{aligned}$$
3. $$\begin{aligned} \sqrt{2}x+\sqrt{3}y&=1\\ \sqrt{2}x-\sqrt{3}y&=0 \end{aligned}$$

4. $\frac{1}{2}x + y = 1$

$-x + \frac{1}{3}y = 2$

5. $\frac{1}{2}x - 2y = -1$

$2x - y = 1$

6. $ax - by = 1$

$bx + ay = 1$

7. $ax - y = 1$

$x - by = 1$

8. $\frac{1}{2}x + \frac{1}{3}y = 1$

$\frac{1}{3}x - \frac{1}{2}y = 1$

9. $-x + y = 1$

$x - y = 1$

10. $x + y = 1$

$2x + 2y = 2$