

Linear Algebra Quiz

1. Find t if $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 6 & 6 \\ 0 & t & 2 \\ 6 & 6 & 6 \end{bmatrix}$ for matrices in $\mathcal{M}_{3 \times 3}(\mathbb{R})$.

2. What is the dimension of the vector subspace

$$V = \left\{ \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \in \mathcal{M}_{3 \times 3}(\mathbb{R}) : a + e + i = 0 = c + e + g \right\}$$

of $\mathcal{M}_{3 \times 3}(\mathbb{R})$?

3. Let $L : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation such that $\dim \ker L = 2$. Which of the following is always true?

I: L is injective.

II: L is surjective.

III: $\ker L$ is a plane in \mathbb{R}^3 .

IV: $\text{Im}L$ is a line in \mathbb{R}^3 .

4. Matrices $(A, B) \in (\mathcal{M}_{2 \times 2}(\mathbb{R}))^2$ satisfy $\det 2A = 4$ and $\det B^3 = 27$. What $\det(AB)^{-1}$?

5. Find $(\vec{i} - \vec{j} - \vec{k}) \times (\vec{i} + \vec{j} + \vec{k})$.