Community College of Philadelphia Department of Mathematics

Hiring Committee

The following document is provided in order to help candidates interviewing with the Hiring Committee of the Mathematics Department at Community College of Philadelphia. It consists of two parts:

1. non-mathematical questions to the candidates and
2. possible mathematical questions to the candidates.

Candidates are expected to become familiar with the Colleges Mission Statement, as found in
http://www.ccp.edu/site/about/mission.php.
Candidates are normally expected to answer a mathematical question in about ten minutes. The question will be chosen at random by the candidate by choosing a question from a bag containing the questions. The candidate must answer the chosen question on the spot. An extemporaneous mathematical question will also be asked to the candidate. A brief mathematical presentation of about fifteen minutes is also expected from the candidates on a topic of their choosing, but the topic must be related to the Mathematics courses taught at Community College of Philadelphia. For a description of the courses, the candidates are urged to visit http://www.ccp.edu/site/academic/courses/math_crs.php.

**Teaching Philosophy**

The Department of Mathematics at Community College of Philadelphia has a methodologically and philosophically diverse group of instructors. Some believe that computers make a positive contribution in the classroom. Some others do not. Since teaching is the primary service we offer to Community College of Philadelphia, the Hiring Committee would like to know your positions in some teaching-related issues. Please note that we want a candid response, there are no right or wrong answers to this set of questions.

1. What is your idea of what a rigorous two-year mathematics education should accomplish?
2. What mathematical traits, or points of view, do you like to instill in your students?

3. What methods do you use to evaluate your students? Candidates are advised to provide copies of final exams, handouts, computer programes, or any extra material generated by the candidate and utilised in the candidates' classes.

4. Are you aware of the current mathematical-education reform movements in Pre-Calculus and Calculus? If so, what of their traits or emphases do you find attractive or unattractive?

5. Are you familiar with any computer-algebra software? What are your views on utilising computer-algebra software in your classes? Candidates are allowed to make a distinction between usage of technology at elementary (Arithmetic, Algebra) levels, and intermediate (Precalculus, Calculus) levels.

6. Many of our courses are multi-section courses. There is no policy yet as to whether there should be uniform exams for these courses. Some faculty strongly support the idea whilst some others oppose it. What are you r views on uniform exams?

**Curriculum Development**

Community College of Philadelphia serves populations with very different needs: from mature adults who see a need to retrain or to embark upon a new field; to employees of corporations who need to acquire specific skills; to young adults fresh out of High School, many of whom are undecided about what to study. Thus we see a need to reinvent courses, design new ones, simply to adapt in order to better serve our students. Candidates are advised to visit the school website at and read the Mathematics courses descriptions and the description of our Associative Degree in Mathematics.

1. Have you had experience with curriculum development? If so, please describe it.

2. Select a course from our course offerings that you would like to teach. What specific approaches do you offer on teaching this course?

3. Some Mathematics reform literature advocates departure from traditional topics from Mathematics instruction, like operations with algebraic fractions, factorisation of polynomials, algebraically-challenging questions in Calculus, etc. What views do you have on these suggestions?

4. As a two-year-degree granting institution, what strong points do you see in our course offerings for Associate Degree candidates? What weak points? What are your suggestions on how to best remedy these weaknesses?

**Contributions to the Institution**

A great concern for some is what type of contributions a new faculty member can offer in order to improve the Institution, improve the service we give our students, make the Institution visible, or even to attract grants for the Institution. Candidates are advised to visit the school website at www.ccp.edu and familiarise themselves with our Mission Statement.

1. Have you any experience with departmental or extra-departmental committee work? If so, describe the relevant experience.

2. Have you any thoughts on how to make the Mathematics Department offerings attractive to our current students and potential students?

**Compatibility with the Student Body**

Community College of Philadelphia serves a student body that shows enormous variation: in ethnicity, in age, in preparation, in maturity, in physical abilities, and so on. Some of our students feel the need to get constant feedback or tutoring from the instructors. Some of our students ignore what many of us would consider standard classroom protocol.

1. How would you deal with such a varied group? What policies would you observe in your classroom in order to better cater to our group of students?

2. How do you deal with students whose classroom behaviour you deem inappropriate?

3. How do you deal with students needing constant tutoring/educational-support?

**Mathematical Questions**

1. **Arithmetic** Consider the non-terminating decimal $.1010010001..$, in which there are $n$ zeros between the $n$th $1$ and $(n+1)$st $1$, counting from the first $1$ from the tenths place. Find a general formula that can be used to calculate the nth digit. Is this a rational or an irrational number?

2. **Arithmetic** Consider, in base $10$, a reduced proper fraction $\frac{a}{b}$, where $b \neq 0$. If it is known that $\frac{a}{b}$ does not have a terminating decimal expansion, what can be said about $b$? Is it possible to determine the period of the repeating decimal without carrying out the division?

3. **Algebra** It is given that $a$, $b$, $c$, $d \in \mathbb{N}\backslash\{0\}$ and that $\frac{a}{b} < \frac{c}{d} < 1$. Arrange in increasing order the five quantities: $\frac{b}{a}, \frac{d}{c}, \frac{bd}{ac}, \frac{b+d}{a+c}, 1$.

4. **Algebra** Find the lowest degree polynomial relation satisfied by $a$, $b$ and $c$ if it is given that $a^2 - bc = b^2 - ac = c^2 - ab$, and that $a$, $b$ and $c$ are not all equal.

5. **Geometry** A rectangle is inscribed in an isosceles triangle such that the base of the rectangle lies on the base of the triangle while the other two vertices of the rectangle lie on the other two sides. Find the area of the largest rectangle that can be so inscribed in terms of the base and height of the isosceles triangle.

6. **Geometry** Find a geometric interpretation of the harmonic mean of two strictly positive numbers.

7. **Probability** $10000$ CCP students are assigned identity-card numbers from $1$ to $10000$. No two students have the same number. Find the probability that the first student that one encounters will not have an $8$ among the digits of his or her identity-card number.

8. **Probability** A person with $2^{2n}$ units of money makes $2n$ bets, winning exactly $n$ times and losing exactly $n$ times. The wins and losses come in random order, and each wager is for half the remaining money at the time of the wager. If the chance for a win exactly equals the chance for a loss, find the final result. You may assume that a unit of money may be divided at will.

9. **Precalculus** Find the largest subset $D$ of the complex numbers on which the rational expression $\frac{x^2}{1+x^4}$ can be evaluated. Consider the map $\alpha := x \to \frac{x^2}{1+x^4} : D \to \mathbb{C}$. Find the image of $D \cap \mathbb{R}$ under $\alpha$.

10. **Precalculus** If $x + \frac{1}{x} = 1$, find the value of $x^n + \frac{1}{x^n}$.

11. **Discrete Mathematics** A function $\alpha : \mathbb{N} \to \mathbb{N}$ satisfies the relation: $\alpha(3n) := n + \alpha(3n - 3)$, when $n$ is a positive integer bigger than $1$ and $\alpha(3n) := 1$, when $n = 1$. Express $\alpha(3n)$ as a polynomial in $n$.

12. **Discrete mathematics** Each member of a set of six points (of which no three are collinear) are joined by either a blue line or a red line to each of the remaining five. Prove that there is at least one triangle (with three of the six points as vertices) which is either completely blue or completely red.

13. **Calculus** Sum the series:

$$\sum_{n=0}^{+\infty} \arctan\left(\frac{1}{1+n+n^2}\right)$$

.

14. **Calculus** Find the derivative of $\int_{1+x^2}^{1+2x^2} x^x dx$ and the $n$th derivative of $x^x$.

15. **Statistics** Suppose every entry $x_k$ (where $k$ goes from $1$ to $n$) in a data-set consisting of $n$ entries $x_1$ to $x_n$, is changed to $x_k + (-1)^k k$. How are the new mean and new standard deviation related to the old mean and old standard deviation?

16. **Statistics** Give a geometric interpretation of the geometric mean. Define a geometric analogue of the standard deviation and give a geometric interpretation of your definition.

17. **Linear Algebra** Find the matrix of the transposition map:

$$Trns : L \to L^t : Mtrx(\mathbb{R}^n, \mathbb{R}^n) \to Mtrx(\mathbb{R}^n, \mathbb{R}^n)$$

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where $Mtrx(\mathbb{R}^n, \mathbb{R}^n)$ is the set of real-valued n-by-n matrices with respect to the standard (Kronecker) basis of $Mtrx(\mathbb{R}^n, \mathbb{R}^n)$. Also find its determinant and trace.

18. **Linear Algebra** The derivative at a point of a function from $\mathbb{R}^n$ to $\mathbb{R}^m$ may be defined to be a linear map from $\mathbb{R}^n$ to $\mathbb{R}^m$ while at elementary levels, the derivative at a point of a map from $\mathbb{R}$ to $\mathbb{R}$ is defined to be a real number. How does one reconcile these two definitions?

19. **Vector Calculus** A curve from the south-pole $(0, \cdots, 0, -1)$ to the north-pole $(0, \cdots, 0, +1)$ of the n-sphere defined by the equation $x_1^2 + x_2^2 + \cdots + x_n^2 = 1$ in $\mathbb{R}^n$ , turns exactly $n$ times around the straight line joining the south pole to the north pole. Formulate a suitable notion of turning, describe such a curve parametrically, and find its length.

20. **Vector Calculus** Find the extrema of the function $\varphi := (x_1, x_2, \cdots, x_n) \to (x_1)(x_2) \cdots (x_n) : \mathbb{R}^n \to \mathbb{R}$, subject to the constraint $x_1 + x_2 + \ + x_n = 1$ and classify them. What happens if there is a second constraint: $x_1x_2 + x_2x_3 + \ + x_{n-1}x_n + x_nx_1 = 1$, in addition to the first?

21. **Differential Equations** Explain the idea behind the use of Laplace transforms.