PROPOSAL CONCERNING

ELEMENTARY MATHEMATICS

AND

ITS EFFECTS ON THE CURRICULUM

Mathematics Department of Community College of Philadelphia
May 2005

The underlying ideas and principal features of this proposal were overwhelmingly approved by the Mathematics Department in April 2005.

Members of the Mathematics Department are planning to submit a proposal presenting the ideas developed in this document for the National Association of Developmental Education (NADE) 30th Annual Conference. The conference will take place in Philadelphia in February 2006.

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Overview:

There has been broad agreement among members of the Mathematics Department about the need for changes to, and the nature of the problems in the remedial mathematics program. Several groups had been involved in tackling the perceived problems. When the administration exhibited an interest in seeing some change with respect to student-outcomes in elementary mathematics courses the Committee of Elementary Education and its Effect on the Curriculum (CEMEC) was formed. Based on discussions on the forum of the CCP Mathematics Department, conversations with students, available statistical data, research and acquaintance with remedial programs at other institutions, the Committee concluded that the CCP program would be enhanced by changes in the following areas: pedagogical aspects of instruction, content of curriculum of remedial classes and the depth at which the material is covered, better utilization of technology, improvement of the placement test (misplacement of students is quite common), insufficient homogeneity of students in sections of remedial math classes, lack of uniform standards across the program, insufficient dissemination of information about the program among students, and the lack of a forum for discussion for instructors.

The goal of the Committee was to find highly recommended, *known-to-work* practices that could be adopted at our college. This proposal is the result of the work of the Committee and presents a comprehensive and coherent plan of changes that should be implemented in order to benefit and improve the CCP remedial program of mathematics.

Background for recommendations:

The recommendations presented in this proposal were based on the three items listed below:

1. There are three important national organizations dedicated to mathematical issues (including education) and the Mathematics Department of CCP is strongly influenced by all three of them. These are:

AMATYC   American Mathematical Association of Two-year Colleges (http://www.amatyc.org). This organization, as the name suggests, deals with issues of primary concern with institutions like CCP and is

the only organization exclusively devoted to providing a national forum for the improvement of the instruction of the mathematics in the first two years of college.

MAA   Mathematical Association of America (http://www.maa.org). It is the largest professional organization that focuses on undergraduate mathematics education, supports learning in the mathematical sciences by encouraging effective curriculum, teaching, and assessment at all levels.

AMS   American Mathematical Society (http://www.ams.org). This organization is the most important learned body in the profession of mathematics. It has organs devoted to, and articulations with, every aspect of mathematics as it concerns mathematicians, users of mathematics, teachers of mathematics, and society-at-large.

It is also extremely important to realize that CCP is embedded in the area between K-12 and 4-year institutions. Elementary courses offered at the community college are essentially in response to deficiencies in K-12 instruction because its student body is a direct product of the K-12 curriculum, available locally. There is therefore the need to understand the nature of this problem within the perspective presented by national studies.

Thus, we felt that in our work, we should pay very close attention to any recommendations or publications issued by the above organizations, as well as any reports on K-12 education. As a result, the proposed changes and improvements presented in this proposal follow closely and are in agreement with ideas presented in the following documents:

[1]. Crossroad in Mathematics: Standards for Introductory College Mathematics before Calculus – document issued in September 1995 and its revision 'Beyond Crossroad Draft Version 6.0' issued in October 2004, (available at http://www.amatyc.org/Crossroads/xroads.html). Both documents were issued and recommended by AMATYC (supported by the National Science Foundation). Participating in the effort were representatives from the American Mathematical Society, the Mathematical Association of America, the National Association of Developmental Education, and the National Council of Teachers of Mathematics. The documents establish standards for the mathematics programs that bridge the gap between high school mathematics and college mathematics and that satisfy the needs of students whose educational plans do not include calculus.

[2]. Undergraduate Programs and Courses in the Mathematical Sciences: Committee on the Undergraduate Program in Mathematics (CUMP) Curriculum

Guide, (available at http://www.maa.org/cump/) – issued in 2004 by the Mathematical Association of America (supported by the National Science Foundation). The document makes recommendations to guide mathematical departments in designing curricula for all students, even those who take just one course.

[3]. Thomas B. Fordham Foundation: The State of State Math Standards 2005 – adopted by American Mathematical Society (available at http://www.edexcellence.net/foundation/publication/publication.cfm?id=338).
The Thomas B. Fordham Foundation supports research, publications, and action projects of national significance in elementary/secondary education reform. This document reviews existing mathematical standards in American school system, identifies the existing problems and creates a list of recommendations for mathematical educators.

We also examined a lot of research carried out outside the auspices of these organizations (see Bibliography).

2. As a source of possible solutions that are already in use elsewhere and that could be adopted in CCP, we acquainted ourselves with the remedial mathematical programs of the following colleges, all twelve of which, were selected by the League for Innovation in the Community Colleges as vanguard colleges in the nation:

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Kirkwood Community College,
Sinclair Community College,
Cascadia Community College,
Community College of Denver,
Valencia Community College,
Richland Community College,
Palomar College,
Community College of Baltimore County,
Humber College,
Lane Community College,
Madison Area Technical College,
Moraine Valley Community College,
as well as
Camden County Community College,
Miami Dade College,
Borough of Manhattan Community College,
Cauyahoga Community College,
Alleghany County Community College,

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3. Finally, in addition to all these findings, we have drawn a lot from our own expertise. The Math Department of Community College of Philadelphia consists of members with a variety of backgrounds and areas of expertise. There are professors familiar not only with American but also Chinese, French, Icelandic, Indian, Polish, Russian, Spanish, Venezuelan and other educational systems. In light of the statement from American Federation of Teachers "As a first step [to improving mathematical performance of students], we should dig deeper into data such as those provided by the Program for International Students Assessment (PISA) and examine and learn from the educational structures of other nations" [4], we believe that the Department should make full use of the diversity of its faculty to follow these recommendations.

Major Areas of Attention and Recommendations:

I.   Teaching issues
    1. Curriculum and pedagogical aspects
    2. Integration of technology
II.   Faculty: preparation, cooperation, and responsibilities
III. Students' issues
    1. Students' support system
    2. Placement issues
IV.   Structure of remedial math program
    1. Uniformity of the program
    2. Flexibility of the program

Rationale of the Proposal:

I. Teaching issues

1. Curriculum and pedagogical aspects

Consistently poor performance of American students on international assessments of math knowledge and problem-solving skills is a cause for serious concern for the mathematical community ([5], [6], [7]).

This problem of identifying the reasons for this situation and how to address them has been analyzed by numerous authors ([3], [8], [9], [10], [11]). The body of research consistently points in the direction of school math curricula and the way math is taught as one of the central problems. It uniformly recommends creating curricula that would place stronger emphasis on understanding the underlying abstract concepts as opposed to mechanical repetitions.

We examined our curriculum and the way it is taught and concluded that:

It is quite often presented in a very repetitive manner. A teacher presents a problem and the same type of problem, where the variation does not explore the complexities inherent in the recursive scheme of the algorithm, is solved by students several times, with no additional gain in insight beyond the trivialities of the variation.

The textbooks that are commonly used (and, in some way, adopted by the Department) reinforce that type of teaching.
For example, in Elementary Algebra Math 017 by Marvin L. Bittinger, at present commonly used textbook for Math 017, a problem "Multiply and simplify: $2^4 \cdot 2^3$" is repeated 24 times in a row. Exponents and base are, of course, different each time, but one might wonder if after 4 or 5 exercises the student stops thinking, does not even bother to check if the bases are the same, blindly adds exponents, and after ten minutes does not remember anything.
There are a limited number of questions that require any type of analysis or reflection; almost everything amounts to a kind of blind repetition. Although repetition and drill constitute an essential part of learning, if such drill and repetition ignore the underlying complexities of the recursive schemes, thereby not involving any kind of thinking, the process becomes meaningless.

Better understanding comes from solving a lot of seemingly different problems, problems that present the same concepts, but in *a different light*. Instead, what is usually done is a repetition of a large number of identical questions in a row (not even interchanging them with any other types of problems). Even at a very elementary level some questions could be asked to increase the understanding of the concepts by students.

The following simple examples illustrate what we mean:

> 1. If an improper fraction represents a number greater than or equal to 2, and the denominator of that fraction is 5, what can be said about its numerator?

Of course, the above type of question is just an example; it is not intended that routine problems on fractions should be eliminated. Both kinds should be a part of class work.

> 2. Combine, if possible: $xy + yx$.

None of the textbooks that we have examined have such problems in the section on combining like terms. The variables in all terms are always in the same order. One should not be surprised that a lot of students have problems with recognizing that $xy = yx$?

The fact that there are two different chapters for "US Customary Units of Measurements" and "The metric system of Measurements" while the methods used in both of those chapters, from a mathematical point of view, are identical, creates an impression that these are two separate topics. We should make our students realize that we are actually using the same procedure, and even more, that the same procedure can be used in many other situations.

Changing this state of affairs is essential for improving our remedial program and thus we strongly recommend the **revision of the material taught in Math 016 and Math 017 and the way it is taught.**

In our revision we were mostly guided by a report made in 2004 by the Thomas B. Fordham Foundation approved by American Mathematical Society [3], Curriculum Guide 2004 issued by the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUMP) [2], Crossroads in Mathematics: Standards for Introductory College Mathematics issued by American Mathematical Association of Two-Year Colleges [1]. We also relied on research presented in [12], [13], [14].

The main directions of proposed changes are:

- Putting more emphasis on standard arithmetic as the foundation of future knowledge. As the Fordham Foundation report states "In general, too little attention is paid to the coherent development of fractions" and it recommends to "develop coherent arithmetic standards that emphasize both conceptual understanding and computational fluency", [3].
- Exposing students to large diversity of problems that "present key ideas and concepts from a variety of perspectives", as CUMP recommends in Recommendations for Departments, Programs, and all courses in Mathematical Sciences, [2]. As an integral part of the revision of our curriculum, we plan to **collect a set of non-routine problems**, appropriate for the student population at hand, problems developing analytical and critical reasoning and helping students to acquire mathematical habits of mind.
- Promote the importance of teaching students the precise use of mathematical language. It is simply not possible to *know math* without knowing the vocabulary and symbols of math. As a recommendation of CUMP says: "Every course should ... help all students progress in reading mathematics with understanding and communicate ideas with clarity and coherence through writing and speaking", [2]. Thus, students should be taught the syntax of symbols (such as "=") and how to present mathematical work. The ability of clear and accurate communication of ideas is absolutely essential and it will be used by students far beyond mathematics classes. Experience with computers and the internet has made students aware of the rigidity of syntax intelligible to a machine, and this makes the task both easier (and more vital) than before.
- Exposing students to some fundamentals of logic (conjunction, disjunction, negation, implication) as early as Math 017 ("Courses at this level should not simply be repeats of those offered in high school. Their goal is to prepare college students to study additional mathematics, thus expanding their educational and career options", [1]). Apart from providing students with a basis for mathematical thinking, logical constructions are present in many aspects of daily life. Here is an example:

  "You can deduct the actual expenses of running your car **or** truck **or** take the standard mileage rate. You must use actual expenses **if** you used your vehicle for hire (such as a taxicab) **or** you used more than one vehicle simultaneously in your business (such as in fleet operations). You cannot use actual expenses for a lease vehicle **if** you previously used the standard mileage rate for that vehicle" (instructions for federal tax form).

This change is in line with the recommendation that "logical thinking or logical reasoning as a method should permeate the entire curriculum", [16].

- Change of policy regarding the use of calculators in our classrooms. We examined the literature about the use of calculators. We found out that there is research supporting both: use and ban of calculators in classrooms ([3], [17], [18]). The most recent trends, however, seem to suggest that while calculators play a positive role in higher level classes, their extensive use in elementary level classes should not be advised: "One of the most debilitating trends in current state math standards is their excessive emphasis on calculators. Calculators enable students to do arithmetic quickly, without thinking about the numbers involved in a calculation. But for elementary math students, the main goal of math education is to get them to think about numbers and to learn arithmetic. Calculators defeat that purpose", [3] or as it was plainly put by Liping Ma (*): "use of calculators kills the understanding of numbers. If you subtract 7 from 53 mentally you practice the concept of 'decomposing' 1 ten into 10 units in order to subtract 7", [19].

Finally, the following recommendation was issued by AMATYC, [1]:
"Traditional development courses try to cover every possible skill that students might need in subsequent courses. This coverage is likely to be too shallow to equip students for later study or for applying mathematics outside the mathematics classroom. Instead faculty should include fewer topics but cover them in greater depth, with greater understanding, and with more flexibility."
We examined the curriculum of Math 016 and Math 017 from the point of view of the amount of the material covered.
We indeed found that what is now supposed to be covered in Math 016 and Math 017 is unrealistic, i.e. the classes cover too much, at too rapid a pace. A commonly used textbook in Math 016, Basic Mathematics, Math 016 by Marvin L. Bittinger, contains 55 sections, each of them covering a different topic. A textbook used in Math 017, Elementary Algebra, Math 017, also by Marvin L. Bittinger, lists 38 sections. Given the level of preparation of our students, and the fact that there are

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(*) Liping Ma – author of the book, *Knowing and Teaching Elementary Mathematics*, is quoted on all sides of discussions about how to teach mathematics in elementary school in the United States. She has a Ph.D. from Stanford University, and earned a master's degree in education from East China Normal University. She is the recipient of the Arthur R. and Pearl Butler Scholarship at Michigan State, a Spencer Foundation Dissertation Fellowship at Stanford, and a McDonnell Foundation Postdoctoral Fellowship in Cognitive Studies for Educational Practice at the University of California, Berkeley.

approximately 39 hours of instruction in the semester (including time for tests, reviews, and events like "snow days"), teaching this amount of material, with any hope for good understanding it by students, is not feasible. The additional difficulty comes from the fact that students in our remedial classes often not only lack mathematical preparation, but they also lack skills, understanding, and attitudes necessary to be successful in college environment.

The truth is that their learning skills are poor, working habits not developed, and expectations unrealistic. These students have to be taught how to be a student, they have to be gradually immersed in an academic atmosphere. And this cannot be done by covering the material in a rush and sending them home to study on their own. This requires time, extra time with students in a classroom. We have checked what is done in this respect in other colleges, and we found out that quite a lot of colleges have more than 3 hours of instructions per week in their remedial classes. For example most of twelve colleges selected by the League for Innovation in the Community Colleges as vanguard colleges in the nation offer sequences of courses that give students much more time than what CCP gives to its students to prepare for college algebra classes. A student taking Math 016 and Math 017 is expected to be ready for a college level class after 3+3=6 credit hours of instruction (The only exception is a subset of CAP 016 students who receive 4+3= 7hours of instructions. The fourth hour of Math 016 takes place in the Learning Lab):

Cascadia Community College (quarter system):
Math 075 Pre Algebra	4 credit hours
Math 085 Elements of Algebra	4 credit hours
Math 095 Intermediate Algebra	4 credit hours

(Although the name of Math 095 might suggest it is a college level class, the content of it coincides with CCP Math 017. Math 095 is listed as a remedial class, and students are not allowed to apply any of its credits to college degree or certificate).

Community College of Denver (semester system):
MAT 030 Fundamentals of Mathematics	2 credit hours
MAT 060 Pre-Algebra	3 credit hours
MAT 090 Introductory Algebra	4 credit hours

Kirkwood Community College (semester system):
PT070D Personal Achievement Math	1-2 credit hours
PT080D Personal Achievement Pre-Algebra	3 credit hours
MT010D Beginning Algebra	3 credit hours

Palomar College (semester system):
Math 10 Basic Arithmetic	3 lectures+ 16 supplementary hours
Math 15 Pre Algebra	3 lectures+ 16 supplementary hours
Math 50 Beginning Algebra	4 lectures

Richland Community College (semester system):
DMAT 0060 Basic Mathematics I   1 credit hour
DMAT 0061 Basic Mathematics II   1 credit hour
DMAT 0090 Pre-Algebra Mathematics   3 credit hours
DMAT 0091 Elementary Algebra   3 credit hours

Sinclair Community College (quarter system):
DEV 084 Basic Mathematics I   4 credit hours
DEV 085 Basic Mathematics II   4 credit hours
DEV 108 Introduction to Algebra   4 credit hours

Valencia Community College (semester system):
Math 0012C Pre-Algebra   4 credit hours
Math 0024C Beginning Algebra   4 credit hours

We should point out that the comparison of other college programs to the CCP program can be only done with some degree of approximation, since the exact material covered in arithmetic or basic algebra classes often differs a little bit from institution to institution (for example Humber College, the only Canadian college on the list of vanguard colleges, has a program incomparable to the CCP one).

There are also other colleges that implemented the idea of having more instructional time with students in remedial classes. For example:

Borough of Manhattan Community College (semester system):
MAT010 Basic Mathematics I (Arithmetic)   6 credit hours
MAT 051 Elementary Algebra   4 credit hours

Cuyahoga Community College (semester system):
Math 0800 Developmental Special Topics in Mathematics   1-3 credit hours
Math 0910 Basic Arithmetic and Pre-Algebra   3 credit hours
Math 0950 Beginning Algebra I   4 credit hours
Math 0960 Beginning Algebra II   4 credit hours

Miami Dade College:
MAT 0002 College Preparatory Arithmetic   6 contact hours lecture/lab
MAT 0024 College Preparatory Algebra   6 contact hours lecture/lab

We propose to **increase the number of instruction hours in Math 016 and Math 017 from 3 to 4 per week for all students.** We engaged in a lot of discussion with all members of the Department and there is broad agreement among faculty that an extra hour of instruction in Math 016 and Math 017 is needed.

2. Integration of technology

There is almost uniform agreement in educational literature that technology should be integrated into any successful curriculum.

"Mathematical faculty will model the use of appropriate technology in the teaching of mathematics so that students can benefit from the opportunities it presents as a medium of instruction", [1].

In 2000 a major collaborative work was carried out among five community colleges in the VCCS—Virginia Community College System—to observe experienced instructors, gather data and extract the most effective ideas and teaching methods being utilized in the elementary mathematics classrooms, [20]. One of their findings is that using computer software or just lecture as the only mode of teaching does not produce adequate success rates. Their data suggest that both modes of instruction should be used for elementary mathematics. This is in with agreement with departmental experience at CCP. In years 1997- 2001, several sections of remedial mathematics were taught using Academic Systems Software. The use of this software was later abandoned, because of lack of satisfactory outcomes (students failed to devote the necessary time required to keep up with their work). Thus, we propose to **incorporate the use of computers in our classrooms.** Ideally**,** each remedial class would meet once a week in a computer lab. However, given space limitations, other options of utilizing computers can be explored (learning lab, portal access).

A vast data base of problems will be gathered and made available to students. Students will have the opportunity to work on those problems under the supervision of instructors in a classroom. Following the examples provided by computer–based testing at Virginia Tech, [21] and The Institute for Mathematics Learning (IML) in West Virginia University, we eventually plan to extend the opportunity to work on computer problems outside the classroom. We plan to prepare **computerized weekly homework-assignments** for each elementary mathematics class.

Since each class meets in a computer room once a week, it should not be difficult to show students how to use the system. A student accessing homework from a computer will be issued a random sub-selection of problems prepared. Thus, all students will have a similar but non-identical homework. The following are advantages of using computerized homework:

> ➢ students will get immediate feedback if their answer is incorrect, and they will be prompted to find their mistakes
> ➢ since all students get slightly different homework, they cannot copy the answers from each other
> ➢ students will get regularly graded homework

Implementation of this idea will depend on technical ability of software available to the Math Department. A further consideration about different types of software is included in section "Details of the Proposal" (page 30 and 35).

II. Faculty: preparation, cooperation, and responsibilities

We, as a Department, are constantly engaged in various discussions about improving educational services to our students. However, most of these discussions take place in a rather private setting: in our offices or via e-mails (often sent to all faculty members but not always reaching our part-time faculty). We believe that it will be extremely beneficial if this discussion could be extended beyond that setting and was available to everybody who would like to participate and take advantage of it.   To this end, we propose to **create a web page, and discussion-board seminars/workshops, with links to other sources concerning teaching of elementary mathematics** and where interested instructors could discuss all teaching issues. Currently several instructors have web pages with their material posted, and some share their tests, quizzes and worksheets. The web page will be a method of organizing and centralizing this, as well as providing a forum for discussion.

Special care should be taken of each instructor who is to teach a remedial class at Community College of Philadelphia for the first time. According to AMATYC recommendations "**each beginning instructor, full- or part-time, should be assigned a mentor** who is a full-time mathematics department member. The member should be available to assist beginning faculty in resolving problems and in meeting responsibilities." [1]. We propose to adopt this policy.

Finally, we believe that in order for all instructors to be well informed about issues specific to our Department and related to teaching elementary mathematics courses, **written materials should be developed describing policies, procedures and recommendations adopted for remedial courses** by the Department. We plan to prepare an "**Information Sheet for Instructors**" with all helpful information as well as recommendations. The "Information Sheet for Instructors" will be given to all instructors, including those who just have started their appointments at Community College of Philadelphia. It will be also available on the web page devoted to remedial mathematics teaching (see above).

We would also like to point out that upon the introduction of all changes recommended in this proposal, several seminars should be organized to acquaint the faculty as well as advisers with all new procedures and practices.

III. Students' issues

1. Students' support system

"Institutions should provide tutorial services for students who need such assistance. Tutors should either hold credentials in mathematics or, in case of peer tutoring, have mathematics faculty recommendations. Tutoring facilities should be staffed, scheduled, and located so that their services are available to all students", [1].

The existence of support services is extremely important. According to a study <u>What Works in Student Retention? Two-Year Public Colleges</u>, one of three retention practices responsible for the contribution to retention in two-year public colleges is "Learning support: including a comprehensive learning assistance center/lab; required remedial/developmental coursework; tutoring program; and math, writing, and reading centers/labs" , [22].

CCP already has a properly functioning Learning Lab, which organizes and facilitates individual tutoring as well as various types of help sessions for students. In order to ensure that all students are aware of available help, we propose to create an **"Information Sheet for Students"**. All instructors will be asked to distribute it to all students during the first class meeting thus making sure that all students are informed about existing facilities well in advance. It will also provide some advice on how to study mathematics and how to be successful in college. This type of information is especially important for remedial students who often need a lot of help in adapting to an academic environment.

Lane Community College (a college selected by the League for Innovation in the Community College as one of twelve vanguard colleges in the nation) has a very informative site on its web page, giving students a lot of tips on how to study as well as link to other sites with similar information. We plan to make use of this site when developing "Information Sheet for Students" (also, on our Department page we already have some links for students, among others: how to deal with math anxiety).

2. Placement issues

Another practice listed as contributing highly to retention in two-year public colleges is "Assessment: including mandated course placement testing", [22].

Mathematics is vertically structured knowledge, and thus the recommendation issued by AMATYC "students should be admitted into mathematics classes only if they meet the prerequisites", [1], should be taken very seriously.

Arithmetic is the foundation of all mathematics. It is absolutely essential for students at any level to know arithmetic and yet the study carried out by the Department at the beginning of the Spring Semester 2004 shows that a majority of students in Math 017 and Math 118 had not mastered operations on numbers:
A test in arithmetic was administered to several sections of Math 118 and Math 017. Of the students who took the test there were:
- 125 students in Math 017, and
- 175 students in Math 118
Therefore, the collected data are statistically meaningful.
The tests:
- were not multiple-choice
- prohibited the use of calculators, and
- were administered during the second week of classes, so that the students, very likely, had already had some kind of a review.
We enclose a copy of both exams (with the Math 017 exam being a subset of the Math 118 exam, see Appendix C and D) and all detailed statistics.
To summarize the results in brief:

If we assume an acceptable level of competence is 60% and above:
  8% of students of Math 017 are competent
 16% of students of Math 118 are competent

The results are quite appalling. Based on the results presented here (see also Appendix C), and based on the experience we have all had in our classes - experience that, unfortunately, does not contradict the findings - we conclude that the problem owes its origin partly to the placement of students. We hope to **prevent some of our students' failures by placing our students in appropriate classes.**

We believe that it is essential for a student wishing to proceed with his mathematical study, to have a sound knowledge of basic arithmetic and algebra. It is simply not fair to students or to instructors to let them enter higher-level courses without carefully checking if they have the necessary knowledge of the prerequisites. This is almost always true, and especially true in math: not being ready for a class means failure in the class.

We examined the CCP math Placement Test:
Currently there are two types of placement tests that are used: paper and pencil and computerized Compass Test.
The Paper and Pencil Test includes only a very limited number of arithmetic and basic algebra questions and their weights are not significant. It has not been revised since 1996, and it definitely needs revision.

We also analyzed the Compass Test. We have several major concerns about the test:

- It does a very poor job distinguishing between Math 016 and Math 017 students: The following experiment was performed. A Compass Test was taken 15 times. Each time all answers were given randomly. Eight out of these 15 times Compass Test determined the placement in Math 017 and seven in Math 016. These are, of course, unacceptable results.

- The test is adaptive i.e. number of questions and their level of difficulty depends on previous answers of a student. So, for example, if a student is not able to answer arithmetic questions then questions from higher level material are not even asked and the student is placed based only on lower level questions. Unfortunately we found out that a student can be placed after only 5 questions (*). After asking only five questions, the Compass Test was able to determine that a student is at a level of Math 017, but not of Math 016 or Math 118. It might be especially unfair to our population of students. There are a lot of students who were out of school for years and who are suddenly faced with an already stressful situation of taking an exam. Even if their first five answers were wrong (it should not be a case, if eventually, student was placed in Math 017, and this took place several times after 5 questions), one can easily imagine a situation, that after "getting used to the exam", next 20 answers would be answered correctly.

- The material covered by a test was not very comprehensive; in particular, arithmetic was poorly represented. In addition, a calculator was available all the time, making some of those questions a matter of pushing a few buttons. Our assessment is that the Compass Test does not adequately evaluate arithmetic skills of a student.

Based on the above observations, we conclude that some kind of a revision of both tests leading to an improvement of the current situation should be made as soon as possible. Eventually, however, we plan to acquire a placement test that would satisfy the following requirements (our requirements are consistent with, and follow the recommendations about how the testing should be done, delineated by Quinn in [21]):

- Test should be fully computerized

(*) The Compass Test was taken 15 times inputting randomly selected answers, the number of questions asked before a respondent was placed were as follows: 9, 7, 7, 7, 7, 7, 6, 6, 6, 5, 5, 5, 5, 5, 5.

- Test should include an option of asking non-multiple-choice answers, thus it should allow entering and being able to recognize algebraic expressions. The current trend is to replace multiple choice questions by open-ended (for example, for the first time this year, SAT introduced non multiple choice math section).
- Test should consist of a large number of a variety of problems, comprehensively covering tested material. For each student a random subset of these problems should be generated. An individual test will not be comprehensive (limited number of questions on such a test does not allow completely full representation of the material) but since essentially everything appears eventually, only mastering entire material (not answers to several specific questions) guarantees success.
- Test should be accompanied by materials allowing students to refresh the material and prepare for the test.
- The Department should have full control of the content of the test. It should be easy to add, or remove problems, in response to feedback from teachers or changes in standards.
- A desirable feature of such test would be that any time a student answers the question incorrectly, he is informed that he made a mistake and is allowed to correct it (on the placement test once, otherwise until the corrected answer is obtained).

Apart from technical requirements for the Placement Test presented above, we propose as an ultimate goal of the remedial part of the Placement Test, **to significantly increase the number of questions on arithmetic and basic algebra**. As was said, arithmetic and basic algebra are skills that are absolutely essential for any success in a study of mathematics. The only way to determine if a student has acquired these skills is to ask questions about it. Neither of the current placement tests does that in a sufficient way. Throughout the rest of this proposal, the section of the Placement Test containing questions on arithmetic and basic algebra will be referred to as **the Gateway Test part on the Placement Test**. Successful completion of the Gateway Test will allow a student to be placed in a math class according to the results of the other part of the Placement Test. Failure will result in placing students in one of remedial classes. The important and difficult issue of how to determine in which remedial class a student should be placed will be given serious consideration and cannot be decided at this moment. It will depend on a lot of factors. For example if the idea of informing a student any time he makes a mistake on the Placement Test and allowing him to make a correction is implemented, one can expect a little bit more from a student than if any, even the smallest mistake counts. We realize that before setting any cut-offs for the Gateway Test a lot of experimentation has to take place. In general, the approach will be to start with a

moderately low expectation and once all aspects of the proposed plan are in place, gradually increase expectation to a desirable level.

Since some students need only a quick review or a few hours of study to recall algebra and arithmetic, we propose that students be informed of the existence of the Gateway Test, of its content and format, and be given an opportunity to prepare themselves for the exam. (students should be encouraged to prepare for the placement test, as done by a lot of colleges, Valencia Community College or Cascadia Community College – to name a few).

We also propose that since exam-taking always involves an element of randomness; to make things a little bit easier for students and to decrease the amount of stress, students should be permitted to repeat the exam. The practice of **allowing students to retake the Placement Exam** is quite commonly used. For example Cuyahoga Community College in Cleveland, Ohio or Valencia Community College uses this policy.

On the other hand**, placement scores** for students who take a placement test but do not enroll for the current semester should not be valid indefinitely. We propose that it will be **valid for one calendar year** from the date the exam was taken (again, it is not uncommon practice, for example, Cuyahoga Community College).

Finally we propose to address the following issue related to the placement of students:

There are a significant number of students who have problems performing operations on whole numbers and who are unable to follow simple directions. Such students do not have any real chance of succeeding in any of our classes, much less in the for-credit classes. The college experience is frustrating for them. Their reaction, whether in the form of questions that are not relevant, displays of exasperation, or disruptive behavior, detracts from the classroom environment. A significant amount of classroom time is devoted to such issues, to the detriment of other students.

Following the practices used by the English Department at CCP, we propose **the introduction of some minimum requirements for students entering Math 016**. Thus, if a student scores below a certain level on the Gateway Part of the Placement Test (how low to be determined through gathering appropriate statistics), the student will be asked to take a Threshold Test.

We tentatively plan to base the requirements on the Threshold Test on the math curriculum of elementary schools. It seems desirable to us that the same knowledge be required of our students. It is hard to believe that a person without elementary school knowledge can succeed and benefit from college, even after taking a year of remedial instruction.

Successful completion of the Threshold Test will allow a student to be placed in Math 016. Students who are not ready for Math 016 will be directed to Adult Basic Education.

IV. Structure of remedial math program

1. Uniformity of the program

We plan to adopt the following recommendations of AMATYC :

"Department should take necessary steps to insure homogeneity between different sections of a given course without infringing on faculty members' academic freedom", [1].

We believe that efforts should be made to increase uniformity with respect to:

      - the material covered,

      - the manner in which students' work is evaluated, and

      - the manner in which grades are assigned.

To this end we plan to develop very specific recommendations regarding how a student should be evaluated, and provide samples of departmental midterms and finals that interested instructors may use.

All of these recommendations, policies, examples of midterms and finals will be included in an "**Information Sheet for Instructors**" (see page 34) and be given to all instructors.

We also strongly believe that in order to improve our program in all its aspects, very specific and precise standards as to what it constitutes for a student to be ready for a college level class should be set up and procedures enabling uniform and objective evaluation of each student should be created.

Thus we propose that the idea of the Gateway Test part on the Placement Test and the requirement that students pass the Gateway Test in order to enter a credit level course should be extended as follows.

The **Gateway Test** should be **given** not only to prospective students taking the Placement Test but **to all students wishing to take any credit level course**, including those students who take our non-credit classes.

We believe that requiring even those students who have passed non-credit classes to pass the Gateway Test is very important. To have common standardized exams is a common and well known practice throughout the world. Although not perfect, they do provide a first (and good) approximation to the goal of checking the students' knowledge in an objective and uniform manner. They provide a motivation for students not only to pass the class but also to actually learn the material (and we all know that those are two different things). Such exams force students to integrate material taught in different courses into a single body of knowledge. We also believe that a Gateway Test will also provide extra motivation for us, teachers, to make even more effort on behalf of our students.

2. Flexibility of the program

There are a certain number of students in our program who have quite good mathematical ability and readiness but they still lack sufficient knowledge of arithmetic and basic algebra. They need additional preparation before entering more advanced mathematics classes. We believe that they should be able to learn arithmetic and basic algebra in a relatively short time, while students of 016 and 017 need much more time for that. There are also students who although placed in higher level classes they still lack confidence about their foundations and would like to review the material if provided with a good opportunity. We propose to create this opportunity for those students. We checked the practices of a lot of other colleges, and we found out that a significant number of colleges offer two (or in some cases even three, Borough of Manhattan Community College) parallel streams of classes, i.e. students, depending on their ability, could go through the remedial classes in a faster or slower pace.

For example:

A student of Borough of Manhattan Community College can take one of the following sequences of remedial classes:

Mat 010 (6 hours) $\rightarrow$ Mat 051 (4 hours)

or

Mat 012 (6 hours)

or

Mat 011 (3 hours) $\rightarrow$ Mat 051 (4 hours).

A student of Miami Dade College can take:

Mat 0002 (6 hours) $\rightarrow$ Mat 0024 (6 hours)

or

Mat 0020 (8 hours)

We found out that this idea of different paths of study is quite common, to name just a few more colleges: Valencia Community College, Palomar College, Cuyahoga Community College, University College at Augusta State University, Utah Valley State College.

For the purpose of increasing students' options in finding a right class for them and thus increasing flexibility of the system, we propose designing **a new remedial class: Math 067**.

It will allow students to be placed in a class that closely fits their needs and abilities. At the same time, it will create more homogenous classroom environment – an issue that is often a problem now. The level of preparation of students in the same classroom often differs significantly and thus makes it very difficult to attend to all students' needs.

Math 067 will be a new elementary course aiming at the population of students who are relatively mathematically mature, while lacking basic knowledge. **Math 067**, as proposed herein, **will essentially combine the material of Math 016 and Math 017**, with a clear articulation of the connecting thread, and will cover it at a much faster pace.

Math 067 can be also taken in the following situation. Suppose a student took Math 017 and was still not ready for Math 118 (i.e. although he passed Math 017, he failed the Gateway Test). Instead of retaking Math 017, he will now be able to take Math 067, where the material of Math 016 and Math 017 will be presented in a comprehensive and connected way. We believe that a student who has passed 016 and 017 but is still not able to integrate the material on the Gateway Test will benefit from Math 067.

Below is an abbreviated flowchart representing "math paths" of proposed remedial mathematics program For more details, see also Appendix A, page 46.
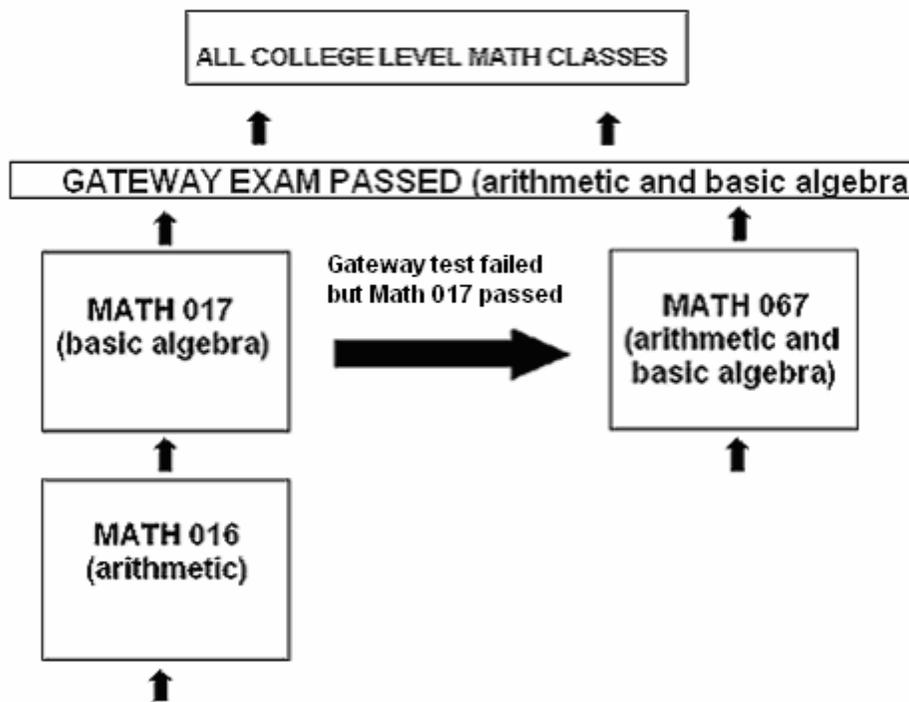
With this structure we are introducing a lot of flexibility to our system. There will be several ways to get to the credit-level classes:
Math 067,
Math 016 + Math 017,
Math 016 + Math 017 + Math 067

Main Ideas of the Proposal:
Identified Problems and Proposed Solutions

Area of attention	Identified problems of elementary math education; Areas with room for improvement	Proposed solutions
Teaching issues	Pedagogical aspects of teaching in remedial classes: not enough emphasis given to understanding of concepts and proper use of mathematical language	Revision of the material covered in Math 016 and 017 with the purpose of creating a curriculum emphasizing the understanding of the concepts. Creation of collection of exercises available to students and instructors
	Pace (too fast) at which the material is covered in remedial math classed. Not enough time to cover topics in greater depth and with more flexibility	Increase of instruction time in remedial classes. (from 3 to 4 hours)
	Lack of utilization of help that is available to students through computers (in and outside the classroom).	Assigning part of instruction time for computer lab instruction. Adopting software and tailoring it to CCP curriculum. Making it available to students outside the classroom to provide them with an opportunity to practice problems related to material (that will be covered in a class), and having instantaneous feedback even outside the classroom.

Faculty: preparation and responsibilities	Lack of forum for faculty to hold discussions concerning the teaching of elementary math	Creation of a web page with links to sites related to the teaching of elementary math and providing means for faculty to exchange their ideas.
	Lack of procedures facilitating familiarization of new faculty (full and part-timers) with departmental expectations and student needs	Creation of written materials for all instructors describing policies, procedures and recommendations adopted for remedial courses by the Department. Introduction of a practice of assigning to each beginning instructor a mentor available to assist in resolving problems and meeting responsibilities.
Students' Issues	Misplacement of students: i.e. students are often placed in classes for which they are not prepared. It should not be a surprise that it becomes extremely difficult for misplaced students to succeed.	Redesigning of the Placement Test: it should include more problems on arithmetic and basic algebra so one can more accurately assess knowledge of students and be able to place them in *an appropriate* class
	Problems with students whose lack of readiness prevents them from succeeding in Math 016 (for example students who do not know multiplication table)	Introduction of some minimum requirements for students entering Math 016: introduction of the Threshold Test
	Insufficient dissemination of information among students about issues vital to their mathematical education	Preparation of detailed information for students about structure of CCP math program, availability of help and practical advice on how to study math (it would be distributed in all elementary classes by the instructors). Making sure that all students are aware of help available to them through the Learning Lab by providing them with the information about it at the very beginning of a semester.

Structure of remedial mathematics program	Not enough flexibility in elementary math offerings: depending on students' abilities students should be allowed to go through remedial classes in a slower or faster pace.	Offering different "paths of study" (alternative choice of classes) to students depending on their math ability and readiness. Creation of additional class, Math 067, integrating the material of Math 016 and Math 017, and allowing some students to get prepared in the course of a single semester.
	Lack of uniform standards for students' assessment in elementary math classes.	Development of uniform recommendations for instructors on how to assess students. Design of departmental midterm and final exams for all remedial classes
	Lack of clearly defined levels of knowledge and proficiency that students "graduating" from remedial classes should have. Nonexistence of tools that would allow one to determine in a uniform manner if those expectations are met by students.	Creation of the uniform test, the Gateway Test, passing of which would be required from every student who would like to register for a credit level class. Creation of a document available not only to instructors, but also to students, describing in a detailed manner the format of the test and the material that is tested.

Details of the Proposal

1. The Courses Math 016, 017, 067

- There will be minimum requirements for Math 016 i.e. students who do not pass the Threshold Test (see page 32) will not be allowed to register for the class.

- The material covered in Math 016 and Math 017 will be revised, see Appendix H and I for a detailed outline of proposed changes (together with sample examples of exercises).

- The importance of using correct mathematical syntax (including proper use of parentheses, the sign of equality and other symbols) will be stressed.

- Calculators will not be allowed in Math 016, Math 017, and Math 067.

- Collection of non-routine problems appropriate for the population of students in CCP remedial classes will be created (see examples of such problems in Appendix G, H, and I)

- Math 017 will assume knowledge of arithmetic. In contrast to the current practice, students who do not know arithmetic will be placed in Math 016; it is expected that many more students will start at Math 016 level.

- A new elementary non-credit course: Math 067 will be created to cover basic algebra and arithmetic. Its content will coincide with the material whose mastery is to be tested by Gateway Test. Math 067 will take comprehensive look at the material of Math 016 and Math 017 and will treat the material in a unified manner (See Appendix I for the suggested outline of the course).

- Math 067 will be offered to:
    - students who passed Math 017 but failed the Gateway Test
    - students who recently took the Placement Test (the Gateway Test included, see page 30) and who (based on their results) are determined to be better prepared for taking Math 067 directly (instead of taking 016-017 sequence first) and learn (or relearn) arithmetic and basic algebra during the course of a single semester

> ➤ students who passed the Gateway Test on the Placement Test but do not feel strong about it and would still prefer to have a one semester review of arithmetic and basic algebra

- The grading system in 016 and 017 will stay the same as what it is now i.e. F, MP, and P, although we would like to make an effort to introduce a certain uniformity in the standards for those grades. We would like to adopt the same system of grading for Math 067.

- Passing Math 067 will consist of two parts. A student must receive a grade of P from the instructor and must also pass the Gateway Test. This rule is designed for avoiding the situation in which a student passes a course but is not admitted to the next class because of failure on the Gateway Test. If a student fails the Gateway Exam but the instructor feels strongly about the student and assigns the student a grade of MP, the student may retake Math 067 (current rules allow students to avail themselves of financial aid to pay for retaking Math 067).

- As mentioned above, to pass Math 067 a student will have to pass the Gateway Test. Except during the summer sessions, a student will be allowed to take the Gateway Test twice. Grades F and MP are to be assigned at the instructor's discretion. Although topics covered in Math 016 and Math 017 together will be the same as in Math 067, in Math 067 students will be held to higher standards than in Math 016 and in Math 017 and the problems treated will be more demanding.

- Passing students of Math 017 will be allowed to take the Gateway Exam (see page 30) and if they pass, to register for any credit level class. It is not expected that all students of Math 017 will pass the Gateway Test; instead, some of them will have to take Math 067.

- Students who are now required to take Math 017 to satisfy some curricular requirements (such as for Nursing) will still have to take only Math 017. In other words, the role of Math 017 in curricula will be maintained.

- A student who gets a P-grade in Math 017, but fails the Gateway Exam (see page 30), and has to (or wants to) take a credit-level math course will be required to take Math 067.

- Each of the classes for Math 016, Math 017, and Math 067 will have four hours of instruction; one of those hours per week will take place in a computer-lab. Problems on the computer (using the same software as for the Gateway Test), relevant to the material being covered at a given week, will be available for use.

- The department will make recommendations (to be adopted by instructors on a voluntary basis) regarding the manner in which students should be evaluated (see page 34); departmental midterms and finals will also be available for use.

2. The Gateway Test as a part of the Placement Test:

- The Gateway Test section will cover comprehensively all arithmetic and very basic algebra.

- A prospective student, upon passing the Gateway Test, will be placed in some credit- level mathematics class, based on the results in the other parts of the Placement Test. Upon failing the Gateway Test on the first attempt, a student will be allowed to retake it exactly one more time. Upon failing the Gateway Test also on the second attempt, a student will be placed in one of 0-credit classes: Math 016, Math 017, or Math 067 or asked to take the Threshold Test (see page 32).

- The Placement Test, as we have determined through experiments, places students in Math 016 relatively rarely, although a large majority of students in Math 017 do not know arithmetic (see Appendix B). This practice will be changed: all students who do not know arithmetic (but pass the Threshold Test; see page 32) will be placed in Math 016.

- Students, whose score on the Gateway Test is relatively high but still not at the passing level, will be placed in Math 067. The exact way of determining whether a student should be placed in Math 067 or not, is still being discussed. It might be that not only the score on the Gateway Test will be taken into account but also the performance on the other parts of the Placement Test. This problem deserves and will be given careful consideration. It will also require some research (like experimental testing).

- Once the Gateway Test is introduced, all new students will be required to pass it before being admitted to any credit-level math class. Students, who at the time of introduction of the obligatory Gateway Test, are already in the CCP system, will not be required to take the Gateway Test, and will continue their study based on the previous rules. Such organization will allow a gradual transition from the old to the new system.

- Tests will be computerized. Maple offers some software designed for computerized tests, and as of now, tentatively, we think this software could

be used for our purposes. (If so, it could perhaps, also be used for the remaining part of the Placement Test.)

- Eventually, we hope to have a computerized non-multiple-choice test. It certainly seems feasible with the part on arithmetic, but even the part on algebra looks promising. If not, some problems would have to be multiple choice. We will have to work on that (different solutions will be considered).

- The Gateway Test section that is not multiple-choice will have the following feature (provided it is technically feasible): an incorrect response will result in an on-screen message asking the student to redo the problem. He will then have exactly one more chance to correct the problem. Needing two attempts to answer a question correctly will not be counted against the student.

- The test will be constructed in the following way: there will be certain number of categories (for example: linear equations, arithmetic of fractions and so on). Each category will contain a certain number of questions (10?, 20?, 30?). Each time a test is taken, questions will be randomly selected from each of the categories and an *individual* test will be created. Thus we will have thousands of versions of the Gateway Test, all structured in the same way.

- Students taking the tests will not be allowed the use of calculators

- There will be no time restrictions, i.e., students can work on exam problems as long as they wish.

- The mathematics department will have full jurisdiction over the content of the Gateway Test (list of categories and questions included). The Gateway Test should be so configured that the department will be able to easily add, delete, or change problems on the exam.

- The Gateway Test will have an expiration date: 1 year from the date of taking; after that time it will need to be retaken.

- Before taking the Placement Test (and thus the Gateway Test on it) all prospective students will be informed about the format and the content of the Exam, and of help of which they can avail themselves, in order to prepare for the test. They will be also informed about the consequences of failing the Gateway Test part. All of this information should be sent together with a routine letter that CCP sends to any prospective student informing

him about the dates of the Placement Test. In this way, before taking the Gateway Exam, a student may come to know about:

> ➢ the existence of a web page (to be constructed, of course), containing detailed information about what is required to pass the Gateway Test, together with an extensive list of examples that students could use to refresh their memory of the material
> ➢ the possibility of getting the same information as one can find on the web "on paper" from relevant CCP-offices
> ➢ the existence of sessions free of charge, organized by Learning Lab, offering quick reviews of topics included on the Gateway Test

3. The Gateway Test for students in remedial classes

- The exam will be identical to the Gateway Exam on the Placement Test, both in format and in content.

- The exam will be offered at several different times and locations during fall and spring final exam session and during last day of classes of Summer Sessions I and II.

- Students taking Math 017 will be allowed to take the exam, and if they pass it (and pass Math 017) they will be allowed to register for credit-level math class. Each student will be allowed to take the exam twice, (except during summer sessions).

- Students taking Math 067 must take the Gateway Exam. Passing will be equivalent with passing Math 067.

4. Threshold Test:

- The purpose of the Threshold Test will be to determine if a student has sufficient preparation in mathematics so that he may have a fair chance to succeed in Math 016. It will be given only to students whose score on the Placement Test is low (how low has to be determined). Students who fail the Threshold Test will not be admitted to any of our math classes and be directed to Adult Basic Education.

- The Threshold Test will be included in the Placement Test but, as stated above, only some students will be asked to take it. Since the Test will be administered on the computer, immediate determination of whether a student is required to take it can be made. We would like to explore two options of administering the Threshold Test. The first option is that a student will be asked to take the Threshold Test immediately after taking the Gateway Test (during the same session). The second option is that he will be asked to come back later and take it at some future time. We believe that there are several advantages of the latter case: student will have an opportunity to use some extra help in preparation. Before taking the Threshold Test a student will be asked to retake the Gateway Test (thus having another chance to do well). Only if the low score is confirmed, he will be given the Threshold Test. Although we realize that College tries to make the procedure of the placement as easy as possible and that means that students are able to finish the whole placement process during one time appointment, it should be noted that only students who score low on the Gateway Tests will be asked to return the second time. A majority of students will be able to complete the Placement Test during one day.

- The test will be multiple-choice and (as of now, we are working on it) based on the problems taken from elementary school texts and standardized tests for elementary school children. In summer 2004 a test based on the numerical questions (operations on whole numbers) was administered in several Math 016 and 017 classes, but, as we found out, it hardly distinguishes between prepared and under prepared students. Thus it does not seem to be a good tool for evaluating students' readiness for our classes. This semester we are experimenting with a test based more on the word problems (see Appendix E). Some of our colleagues expressed the concern that the test is based too much on English. We would like to point out that only students who perform very poorly on the Gateway Test will be required to take it. It seems to us that if a person knows very little math AND does not have a working knowledge of English, they could not possibly benefit from our classes.

- Use of calculators will not be permitted on the Threshold Test.

5. Web page to facilitate communication among instructors teaching remedial courses

We plan to create a web page, and a discussion-board, with links to other sources concerning remedial teaching and where interested instructors could discuss all teaching issues.

The web page will also contain the "Information Sheet for Instructors" (see below) as well as teaching materials prepared for the Department and materials of individual instructors willing to share them with others.

6. Information Sheet for Instructors

In order to facilitate the work of instructors by making sure that they all are familiar with College and Department policies and are aware of support services available to students and teachers, we plan to create an "Information Sheet for Instructors". It will be also used to increase uniformity among sections of all remedial classes i.e. it will contain departmental recommendations regarding the manner in which students of Math 016, 017 and 067 are to be evaluated. (Recommendations are expected to address the following issues: number of tests, quizzes and their "share" in the final grade, the way in which tests are to be graded, partial credit and its proper use, specification of the meaning of F, MP, and P grades, sample midterms and finals.) Recommendations will also be made regarding classroom-policies. "Information Sheet for Instructors" will be distributed to all instructors. It will be also posted on the Department web page for remedial education.

The recommendations are to be based on discussions at departmental forums and the questionnaire that has been sent to all members of the Department (see Appendix F)

The materials will be revised every time new policies or recommendations are made.

7. Information Sheet for Students

This will contain a detailed explanation of the structure of the mathematical program at CCP. In particular it will inform all students about the existence of the Gateway Test. It will give information about various resources available to students (web pages, Learning Lab, workshops, tutors, computer-software) and some advice on how to study mathematics. All instructors will be asked to distribute the "Information Sheet for Students" during the first class-meeting.

8. Computerized Homework

We would like to develop computerized, weekly homework-assignments for each elementary mathematics class: Math 016, Math 017 and Math 067. It will be

constructed in a way similar to the Gateway Exam: homework will consist of certain number of various types of questions; for each type there will be 20-30 examples. A student accessing homework from a computer will be issued a random selection of problems from each type. Thus, in practice, students will have similar homework (that is with the same type of questions) but non-identical (that is different version of questions). A student will have a week to work on the homework. Students typing in a wrong answer will be informed of an error and will be allowed to correct it. After a week, the homework will need to be submitted (electronically) and the score of each student will be reported directly to the instructor's account. Since each class meets in a computer-room once a week, it should not be difficult to show students how to use the system. Ideally the software used for this purpose will be web based, allowing students to use it not only on CCP computers, but on any computer available to them. Organization of such computerized homework is possible. There are already universities implementing very similar ideas. As mentioned above we are currently investigating software that could be used for this and other purposes. One promising candidate is software produced by Maple. Maple represents one of two internationally used major mathematical software on the market - the other one being Mathematica.

9. Mentoring system for beginning faculty

In order to facilitate a smooth immersion of beginning faculty (part time or full time) and to make sure that they are well informed about all policies and practices followed in the Department, the following policy will be implemented: any beginning instructor will be assigned a mentor (a full time faculty member). He will be expected to provide all information and help to a new instructor.
In addition, the beginning faculty will be evaluated according to the department evaluation plan.

The Scheme outlined:

Any prospective student is required to take the Placement Test. Contained in the Placement Test there will be a section on arithmetic and basic algebra: the Gateway Test part of the Placement Test.
Successful completion of the Gateway Test will allow the student to be placed in a math class according to the results of the Placement Test.
Failure on the Gateway Test will lead to the following options:
If a student scores below a certain level on the Gateway Test (how low to be determined), the student will be asked to take a Threshold Test.

Successful completion of the Threshold Test will allow a student to be placed in Math 016.

Students exhibiting sound knowledge of arithmetic but insufficient knowledge of algebra will be placed in Math 017. Students demonstrating strong ability, but still lacking sufficient knowledge of arithmetic and basic algebra will be placed in Math 067.

How to exactly determine whether a student who did not successfully complete the Gateway Test should take Math 016, 017 or 067 is a very difficult and important issue and it will be given very careful consideration.

Students with passing grades from Math 017 will be encouraged to take the Gateway Test and if passed, they will be allowed to register for a credit level math class. Those who did not decide to take the Gateway Test or those who took it but unsuccessfully will be asked to sign for Math 067. Passing Math 067 will be equivalent with passing the Gateway Test. If a student fails (after two attempts) the Gateway Test after taking Math 067, he will be advised to retake it.

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Tentative Time Table of Implementation

This proposal presents a comprehensive and cohesive plan for improvement of CCP mathematical remedial program. Given the complexity of the plan, its implementation will require very careful planning and extremely good organization and coordination. Each detail of the plan will have to be developed well in advance, so eventually all parts of the plan will fit together. Throughout the whole process, the results will have to be continually monitored. Full implementation of the plan will require a significant amount of work and time. Some initial work has already been started: study on the Threshold Test, a search for the software that can be used as a platform for the computerized homework, questionnaire sent among the faculty about uniform standards in all remedial classes. In order for this work to be properly continued, institutional support in the form of extended time is needed.

Semester	Deadline for implementation	Work performed during this time	Support needed
ACADEMIC YEAR 2005-2006			
Fall 2005		Planning details of implementation: disseminating the ideas of the proposal among members of the Mathematics Department, organizing teams of people to work on specific parts of the proposal (for example Math 016, 017 and 067), discussing and deciding on details of the implementation of the proposal. Analyzing information gathered on questionnaires about departmental recommendations for remedial classes, circulated among faculty; possibly supplementing them with some additional feedback from the faculty. Organizing mentor system for beginning instructors Preparation of the NSF grant proposal (collecting required information, writing a proposal)	8 hours of extended time

Fall 2005		Researching available software that could be used as a platform for the computerized homework and Gateway Test. Threshold Test: analysis of data collected during Summer I, 2005, preparation of multiple choice versions of the Threshold Test, soliciting help from the instructors, administering tests in their classes, and keeping track of several statistics	
Spring 2006	Mentor system for beginning instructors introduced	Threshold Test: administering in several classes the test designed in the Fall 2005 , monitoring progress of students who took the test in order to collect detailed statistics to analyze the relationship between the performance of a student in a class and his score on the Threshold Test Creation of two new web pages: for instructors and for remedial class students. Preparation of Information Sheet for Instructors and Information Sheet for Students Course revision for Math 016 Course revision for Math 017 Development of materials for Math 016 and Math 017 begins (problems, solutions, explanations, examples of tests, quizzes, homework assignments) (****)	8 hours for work on materials for Math 016 8 hours for work on materials for Math 017 2 hours for work on Threshold Test 4 hours for web pages design and creation of Information Sheets for Instructors and Information Sheet for Students

Summer 2006	A web page of remedial math discussion among faculty A web page for students in remedial classes	Work on materials for Math 016 and Math 017 continuous. Final selection of the material of Math 067. Work on materials for Math 067 begins: set of problems, examples of tests, quizzes, homework etc.	8 hours for work on materials for Math 016 8 hours for work on materials for Math 017 8 hours for work on materials for Math 067
ACADEMIC YEAR 2006-2007			
Fall 2006	Revised materials for Math 016 and Math 017 Information Sheet for Instructors Information Sheet for Students Special session informing advisers and faculty about a new system organized	Work on materials for Math 067 continues Work on the Gateway Test begins (as a part of the Placement Test and as an exam in our remedial math program). Work on computerized homework and preparation of materials for computer labs in all remedial classes begins. This will be done with close cooperation with people working on the Gateway Test as computerized homework and lab exercises are meant to be a preparation for the Gateway Test	8 hours for work on materials for Math 067 4 hours for work on the Gateway Test (as a part of the Placement Test and as an exam for remedial class students)
Spring 2007	Math 067 designed Threshold Test introduced (*) Approval process of revised Math 016 and Math 017	Work on the Gateway Test continues. Computerized homework and preparation of materials for computer labs in all remedial classes continues	grant support (*****)

Summer 2007		Computerized homework and preparation of materials for computer labs in all remedial classes continues	grant support (*****)
ACADEMIC YEAR 2007-2008			
Fall 2007	New Placement Test introduced (with the Gateway Test Part and the Threshold Test)(**). All students taking Placement Test after the last day of Summer II Session, 2007 are subject of a new system. Revised Math 016 and Math 017 offered for the first time. Students in Math 017 classes are still not required to take the Gateway Test to sign up for credit level classes (***) Approval process of Math 067	Work on the Gateway Test: close cooperation with instructors of Math 017 who volunteered to incorporate the Gateway Test in their evaluations method will take place, collected data will be analyzed and used to made the successive improvements of the Test.. Computerized homework and preparation of materials for computer labs in all remedial classes continues	grant support (*****)
Spring 2008	Math 067 offered for the first time The Gateway Test for students in the redial classes takes place. From now on all students signing for credit level classes are required to pass the Gateway Test Several Math 016, 017 classes offered with a computer lab ingredient are being offered for the first time	Computerized homework and preparation of materials for computer labs in all remedial classes continues	grant support (*****)

Summer 2008		Computerized homework and preparation of materials for computer labs in all remedial classes continues	grant support (*****)
ACADEMIC YEAR 2008-2009			
Fall 2008	Several Math 067 are being offered with a computer lab ingredient are being offered for the first time Computerized homework for Math 016 and 017 is offered	Computerized homework and preparation of materials for computer labs in all remedial classes continues	grant support (*****)
Spring 2009	Computerized homework for Math 067 is offered All remedial classes have computer lab ingredient	Computerized homework and preparation of materials for computer labs in all remedial classes – final stage	grant support (*****)
Summer 2009	COMPLETION OF THE PROJECT		

(*) Since at this time the Gateway Test will not be yet included in the Placement Test, the cut-off for existing Placement Test will be determined and students who score below this level will be asked to take the Threshold Test.

(**) From now on, the Gateway Part on the Placement Test is used to determine if a student has to take the Threshold Test

(***) The Gateway Test will be offered on voluntary basis to instructors wishing to take advantage of it. Its use in determining students' final grades will be up to instructor.

(****) We entertain the idea that creating detailed materials for our students might be also self funded project. The revenue generated in this way might be used to fund further improvement and modification of materials in the future.

(*****) We hope to have some funds available through NSF grant. If not, additional college support will be needed.

Assessment

The assessment of quality of education is, in our opinion, very difficult, not well defined and thus should be done in a very careful manner. Students' "success rate" can be improved, but if it is not followed by students' knowledge, the "success" becomes a failure.

A majority of students entering CCP suffered for years from inadequate education. We believe that it is a CCP obligation to reverse this process. Quite often it might mean more remedial classes taken, more hours in the classroom spent, more work, and even more stress related to exams a student has to pass, but we believe that in the long run, benefits surmount the effort.

We propose to base our assessment not only on students' outcomes but also try to assess knowledge of students. Introduction of the Gateway Test will be a very good, uniformly applied tool for measuring that. We will be collecting various statistics and closely monitoring the process. We also plan to administer the same arithmetic test (or very similar versions) in Math 017, Math 118, and now in Math 067 classes to compare the results with those that were collected in spring 2004 (see Appendices B, C, and D).

Holding students to high standards, we will closely watch the following statistics:
- Student-outcome: passing rates in all remedial classes and success rate in subsequent classes taken by students coming from the elementary math classes.
- Student-persistence: A very significant number of students of remedial classes withdraw before the end of a semester; decreasing the percentage of students in this category would be a big success; by placing students in the right classes and making them more interesting, we hope to improve the present situation.
- Student-perseverance: with the introduction of computerized homework, statistics illustrating the effort on a behalf of students can be easily gathered; this type of data could be used to improve the program by monitoring students' involvement.

Some preliminary statistics could be (and will be) collected at the beginning of the implementation stage, continuous monitoring of the project by gathering and analyzing statistics about students' performance at each phase of the implementation will be carried out.

The overall assessment, of course, will start only when the whole proposal is implemented i.e. in fall 2010.

Grant Applications

We plan to apply for NSF grant: Course, Curriculum, and Laboratory Improvement (CCLI) – Adaptation and Implementation (A&I) track. This program "supports the use of innovative materials and practices that have been demonstrated to be effective and the acquisition of instrumentation to achieve pedagogical change". The project can be supported for up to 3 years with a budget up to $200,000 for comprehensive project involving more than one course ($100,000 for a single course). The deadline for the proposal is in December.

We scrutinized the proposals that were recently sponsored by NSF and we found that many of them are implementing the idea of computerized homework (we did not see any proposal implementing it at elementary level of mathematics). Thus we believe that a well prepared proposal has a good chance to be supported.

We also plan to apply for CCP minigrants to hold information sessions for instructors, advisors, and counselors.

Approximate Cost of the Implementation of the Proposal

We anticipate the following costs:

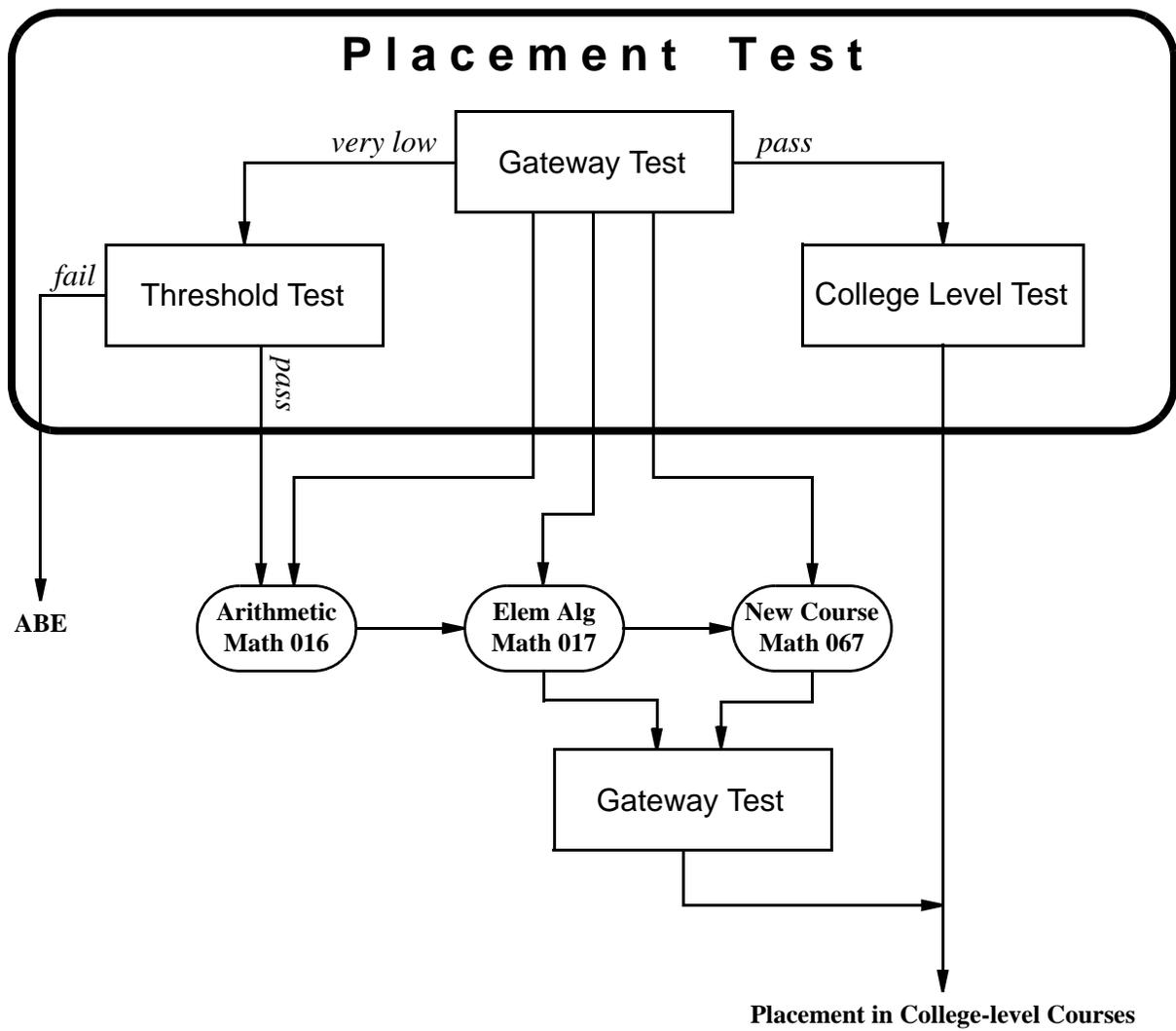- cost of extended hours :
    - Fall 2005   8 hours
    - Spring 2006   22 hours
    - Summer 2006   24 hours
    - Fall 2006   12 hours

- cost of software: potential cost for software $12,000 (for example Maple; other possibilities will be investigated).

APPENDIX A

GRAPHICAL REPRESENTATION OF A PROPOSED STRUCTURE OF REMEDIAL MATH PROGRAM

APPENDIX B

STATISTICS OF ARITHMETIC TEST ADMINISTERESD IN SPRING 2004

At the beginning of the Spring Semester 2004 a test in arithmetic was administered to several sections of Math 118 and 017. Of the students who took the test there were;
- 125 students in Math 017, and
- 175 in Math 118

The tests:
- were not multiple-choice, and
- prohibited the use of calculators
- were administered during the second week of classes, so that the students, very probably, had already had some kind of a review.

Math 017 exam was a subset of Math 118 exam, i.e. the first 25 questions on Math 118 exam were identical with Math 017 exam.

To summarize the results in brief:

If we assume an acceptable level of competence is 60% and above:
  8% of students of Math 017 are competent
  16% of students of Math 118 are competent

If we assume an acceptable level of competence is 70% and above:
  5% of students of Math 017 are competent
  7% of students of Math 118 are competent

RESULTS ON FIRST 13 QUESTIONS (VERY ELEMENTARY QUESTIONS)		
	Math 017	**Math 118**
Percent of students who Scored below 20%	22	5
Percent of students who Scored 20-40%	39	23
Percent of students who Scored 40-60%	18	21
Percent of students who Scored 60-70%	14	21
Percent of students who Scored 70-80%	4	11
Percent of students who Scored 80-100%	3	19

RESULTS ON FIRST 21 QUESTIONS (OVERLAPPING PART OF BOTH TESTS MATH 017 AND MATH 016)		
	Math 017	**Math 118**
Percent of students who Scored below 20%		
Percent of students who Scored 20-40%	44	13
Percent of students who Scored 40-60%	3	31
Percent of students who Scored 60-70%	18	25
Percent of students who Scored 70-80%	2	8
Percent of students who Scored 80-100%	5	13

RESULTS ON THE WHOLE TEST		
	Math 017	**Math 118**
Percent of students who Scored below 20%	26	1
Percent of students who Scored 20-40%	46	35
Percent of students who Scored 40-60%	19	29
Percent of students who Scored 60-70%	3	9
Percent of students who Scored 70-80%	3	1
Percent of students who Scored 80-100%	2	6

RESULTS ON QUESTIONS 26-34 (MORE "SOPHISTICATED QUESTIONS", ONLY ON MATH 118 TEST)	Math 118
Percent of students who Scored below 20%	41
Percent of students who Scored 20-40%	29
Percent of students who Scored 40-60%	16
Percent of students who Scored 60-70%	7
Percent of students who Scored 70-80%	3
Percent of students who Scored 80-100%	4

Math 017				Math 118			
Students placed directly in 017	Students who took 016	Students placed by advisor	All students in 017	Students placed directly in 118	Students who took 017	Students placed by advisor	All students in 118
				AVERAGE SCORES OF 118 STUDENTS (OUT OF 34)			
				15.41	13.87	21.75	15.18
AVERAGE SCORES OF O17 STUDENTS (OUT OF 25)				**AVERAGE SCORES OF 118 STUDENTS ON THE FIRST 25 QUESTIONS (PART OVERLAPPING TEST 017 (OUT OF 25)**			
9.12	7.7	7	8.6	12.61	11.79	17.42	12.59
AVERAGE SCORES ON QUESTIONS 1-13 (OUT OF 13) (ELEMENTARY QUESTIONS)							
5.03	4.81	3	4.86	7.49	7.37	9.83	7.6

THE HIGHEST SCORES:
Math 017 (out of 25):

   20 -one student
   21 - one student
   20 –one student
   19- two students

THE HIGHEST SCORES:
 Math 118 (out of 34):

  34- two students
  31- two students
   29- two students

PERCENT OF STUDENTS WHO ANSWERED CORRECTLY EACH QUESTIONS								
	Math 017				**Math 118**			
	Students placed directly in 017	Students who took 016	Students placed by advisor	All students in 017	Students placed directly in 118	Students who took 017	Students placed by advisor	All students in 118
# of students	86	32	7	125	88	75	12	175
question								
1	20	22	0	19	19	28	42	25
2	33	9	29	25	69	68	92	70
3	48	47	43	45	49	53	75	53
4	74	78	71	72	90	91	100	91
5	42	34	14	38	.63	59	83	62
6	43	53	0	43	76	72	100	76
7	14	.22	0	15	50	39	75	47
8	33	38	14	32	38	47	75	44
9	62	63	86	59	91	81	100	87
10	44	31	14	38	67	69	83	69
11	34	31	14	31	33	33	42	34
12	35	50	14	37	44	49	58	47
13	23	3	0	17	60	48	58	55
14	47	9	71	35	60	48	83	57
15	31	16	29	26	43	31	83	41
16	9	6	0	8	19	8	42	16
17	30	28	43	29	33	29	75	34
18	26	9	29	21	28	36	67	34
19	13	6	14	11	34	31	58	34
20	13	0	0	9	22	20	42	22
21	7	3	0	6	16	11	33	15
22	45	25	43	38	53	43	50	49
23	50	53	57	49	41	41	42	41
24	74	88	86	74	86	84	92	86
25	63	44	29	54	76	60	92	70
26					30	25	58	30
27					28	33	67	33
28					27	12	42	22
29					25	13	25	20
30					33	28	50	32
31					44	45	58	46
32					24	19	33	22
33					28	13	42	23
34					40	19	58	32

APPENDIX C

COPY OF ARITHMETIC EXAM ADMINISTERED TO SEVERAL SECTIONS
OF MATH 017 IN SPRING SEMMESTER 2004

Spring 2004   Name_____

How did you get placed in 017?
Placement test____
Passed 016 ____
Advisor ___
Other: Please state.

Evaluate the following:

1. $1\frac{2}{3} \times 2\frac{1}{7} =$

2. $-4 - 10 =$

3. $\frac{4}{9} \div \frac{2}{9} =$

4. $\frac{-16}{4} =$

5. $7 \times \frac{2}{7} =$

6. $-8 \div (-2) =$

7. $-1 - (-4) + (-2) =$

8. $\dfrac{4}{5} \div 3 =$

9. $-2 + 7 =$

10. $9 \div (-1) =$

11. $6 \div \dfrac{1}{3} =$

12. $\dfrac{4}{5} \times \dfrac{2}{5} =$

13. $-2 + 3 - 10 - 4 + 1 =$

14. What is 10% of 342?

15. $0.1 \times 0.02 =$

16. $\dfrac{0.25}{0.005} =$

17. $1\dfrac{4}{7} + 2\dfrac{5}{7} =$

18. $995\dfrac{2}{5} + \dfrac{4}{9} =$

19. $\dfrac{17}{23} + 444 =$

20. $4321 - \dfrac{2}{5} =$

21. $123456\dfrac{4}{11} - \dfrac{9}{11} =$

22. Are the following numbers equal?

$\dfrac{-4}{5}$ and $\dfrac{4}{-5}$ Yes   No

23. Are the following numbers equal?

$\dfrac{-3}{-4}$ and $-\dfrac{3}{4}$ Yes   No

24. Are the following numbers equal?

$\dfrac{245}{367}$ and $245 \div 367$ Yes   No

25. Are the following numbers equal?

$\dfrac{2}{5}$ and 2.5 Yes   No

APPENDIX D

COPY OF ARITHMETIC EXAM ADMINISTERED TO SEVERAL SECTIONS
OF MATH 118 IN SPRING SEMMESTER 2004

Spring 2004   Name_____

How did you get placed in 018?
Placement test____
Passed 017 ____
Advisor ___
Other: Please state.

Evaluate the following:

1. $1\frac{2}{3} \times 2\frac{1}{7} =$

2. $-4 - 10 =$

3. $\frac{4}{9} \div \frac{2}{9} =$

4. $\frac{-16}{4} =$

5. $7 \times \dfrac{2}{7} =$

6. $-8 \div (-2) =$

7. $-1 - (-4) + (-2) =$

8. $\dfrac{4}{5} \div 3 =$

9. $-2 + 7 =$

10. $9 \div (-1) =$

11. $6 \div \dfrac{1}{3} =$

12. $\dfrac{4}{5} \times \dfrac{2}{5} =$

13. $-2+3-10-4+1 =$

14. What is 10% of 342?

15. $0.1 \times 0.02 =$

16. $\dfrac{0.25}{0.005} =$

17. $1\dfrac{4}{7} + 2\dfrac{5}{7} =$

18. $995\dfrac{2}{5} + \dfrac{4}{9} =$

19. $\dfrac{17}{23} + 444 =$

20. $4321 - \dfrac{2}{5} =$

21. $123456\dfrac{4}{11} - \dfrac{9}{11} =$

22. Are the following numbers equal?

$\dfrac{-4}{5}$ and $\dfrac{4}{-5}$ Yes   No

23. Are the following numbers equal?

$\dfrac{-3}{-4}$ and $-\dfrac{3}{4}$ Yes   No

24. Are the following numbers equal?

$\dfrac{245}{367}$ and $245 \div 367$ Yes   No

25. Are the following numbers equal?

$\dfrac{2}{5}$ and 2.5 Yes   No

26. $\dfrac{90}{11} \times 2 \times \dfrac{33}{9} =$

27. $3 - 2(5 - 7) =$

28. $-1 + .03 =$

29. $\dfrac{1}{2} \times \dfrac{1}{2} + \dfrac{1}{2} \times \dfrac{1}{2} =$

30. $\dfrac{3}{5} \times 5 - 4 =$

31. $(-1)(-1)\left(-\dfrac{1}{2}\right) =$

32. $-1 - 1 - \dfrac{1}{2} =$

33. $\dfrac{2}{3} - 3 =$

34. $2 - (4 - (1 - 7)) =$

APPENDIX E

A COPY OF THE THRESHOLD TEST TOGETHER WITH ITS STATISTICS.

The following test was administered at the very beginning of Spring Semester 2005 to several sections of Math 016 (58 students) and Math 017 (56 students). We will use it as a 0-th approximation of the proposed Threshold Test.
The use of calculators was prohibited.

We recorded the following data:

|	Percent of students who scored above 60% on the test	Percent of students who scored above 70% on the test
Math 016	30.5	19
Math 017	80	62

The Test
Name:
SS#

1. You put 54 marbles into 6 bags, ending up with the same number of marbles in each bag. How many marbles would be in each bag if there were 6 bags?

2. A tree was planted 54 years before 1958. How old was that tree in 1983?

3. There are bags of sand on a truck. Each bag of sand weights 124 pounds. How many pounds do 1000 bags weigh?

4. During the school year 3216 cans were sold in the cafeteria. There were 508 cans left unsold. How many cans of juice did the cafeteria have at the **beginning** of the school year?

5. Which statement describes the pattern of the shapes shown below?
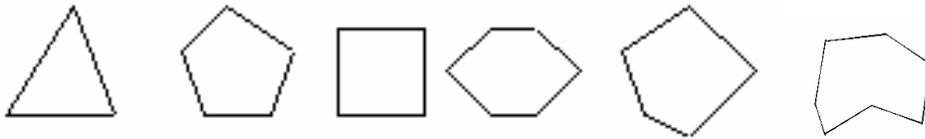
    a) add 1 side, then subtract 2 sides
    b) add 2 side, then subtract 1 side
    c) subtract 1 side, then add 2 sides
    d) subtract 2 side, then add 1 side

6. Mary's class is going to Sturbridge Village for the day. There are thirty-two children and 3 chaperones attending the field trip. If each car holds five passengers, how many cars will be needed to transport everyone?
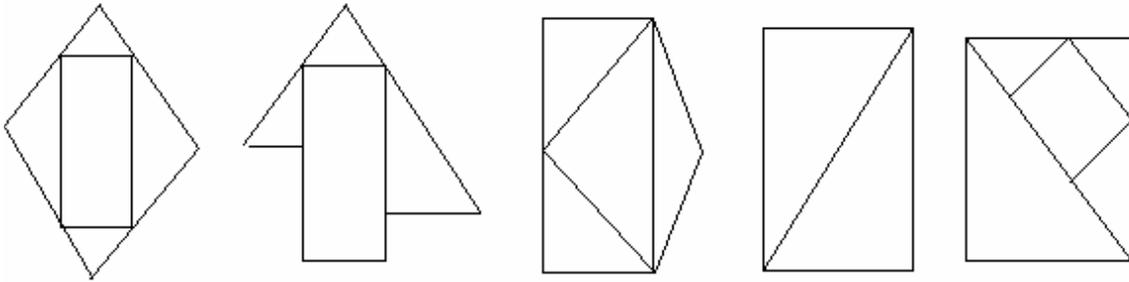
7. In the pattern below, which number belongs in the box?

$$5, \quad 4\frac{1}{2}, \quad 4, \quad 3\frac{1}{2}, \quad 3, \quad \boxed{}$$

8. What fraction of the largest square is shaded?
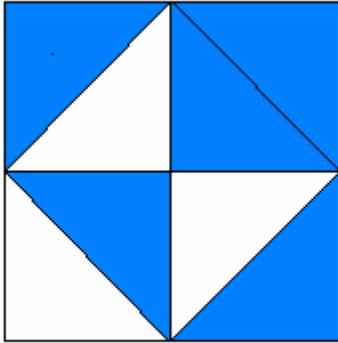
9. Oscar made a drawing that has exactly three triangles and one rectangle. Which drawing below could be the one Oscar made? Please, circle the right answer.

10 Which expression is **not** equal to 3×4. Please, circle the one.

a) $4 + 4 + 4$

b) 4×3

c) $3 + 3 + 3 + 3$

d) $4 \times 4 \times 4$

11. Monica has 7 erasers. She has 5 more pencils than erasers. She has 3 fewer markers than pencils. How many markers does Monica have?

12. Susan has to read a book that is 72 pages long. She wants to read an equal number of pages each day. How many pages per day she has to read, to read the entire book in 2 days?

13. Jason and Jenna were baking cookies for two classes in their school. If each class has 26 students. How many cookies should they bake to give 3 cookies to each student?

14. What number goes in the blank to make the sentence true?

$(8 \times 4) + 7 =$ _____ $+ 7$

15. Complete the pattern by writing the missing numbers in the circles below:
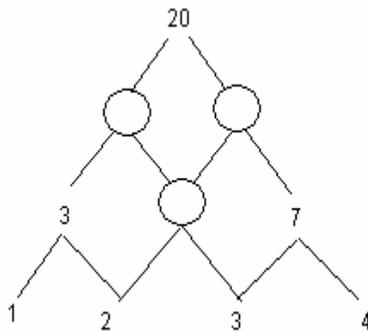
16. What is the missing factor?
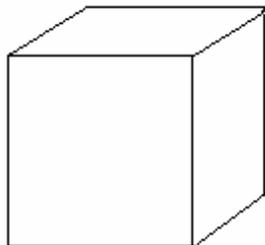
$$12 \times \underline{\quad} = 60$$

17. Bill is 42 inches tall and Tom is twice as tall as Bill. How tall are the two men **together**?

18. What number makes the sentence true?

$$500 + \underline{\qquad} + 7 = 567$$

19. In a group of 50 people three out of every 10 own a CD player. How many people in this group own a CD player?

20. How many numbers do we have to use if we want to put one number on each face of the cube?

21. Mr. Smith is 42 years old. His son is 7 years old. The ages of Mr Smith, his wife, and their son add up to 91. How old is Mr. Smith's wife?

22. 245×13 is more than 244×13. How much more?

23. Give a number that is between 0 and 1.

24. Karen had 238 German, 167 French, and 429 United States stamps. How many did she have in all?

25. In the pattern below, what are the next **two** numbers?
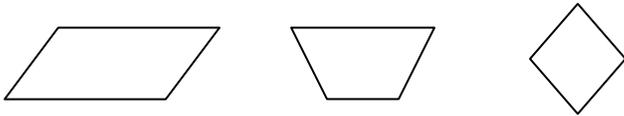
0, 1, 3, 7, 15, 31, …...

26. In his orchard Mr. Butler has 126 rows of trees with 20 trees in each row. How many trees does he have in his orchard?
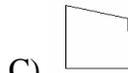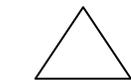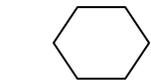
27. Evaluate the following expression:

$7 \times 3 + 2 \times 1000 =$

$22222222 \div 2 =$

28. Look at this group of figures:

Which figure could be included in this group?

A)   B)   C)

APPENDIX F

A COPYOF QUESTIONNAIRE SENT TO ALL MATH INSTRUCTORS

In order to increase the uniformity in the way in which students' work is evaluated and grades are assigned in our remedial classes, we believe that the Department should issue some recommendations about policies in our math classes.

We would appreciate it very much, if you could fill out the questionnaire below. It would allow us to determine where the Department stands on some of the indicated issues and perhaps help find some common ground for future recommendations.

Please send the questionnaire electronically to:

hitczenko@hotmail.com

or place a hard copy in Margaret Hitczenko's mailbox in B2-22.

Name (if you wish to share it with us) _____

1. Fill in the following table with percentage figures in the appropriate columns indicating what, in your opinion, the Department should recommend, as the contribution of the category to the final score of a student, in the indicated course.

	Math 016 (percentage of grade)	Math 017 (percentage of grade)
homework		
quizzes		
tests		
Final Exam		
Other CATEGORY (please specify below)		

Other:

_____
_____
_____

2. Please indicate your answer by selecting appropriate option:
'Passing' grades should be assigned to students whose
- total score is above _____%
- other_____
_____
_____
_____

3. Please indicate your answer by selecting appropriate option:
'Making Progress' grade should be assigned to students whose
- total score is above _____%
- any student who kept coming to class and made an effort
- other_____
_____
_____
_____

4. Please circle one answer:
The Department should recommend that a comprehensive final exam be always given:

Yes

No

The Department should not have any recommendations

5. In your opinion, should the Department recommend each class have a comprehensive final exam written by the Department?
Yes

No

We should not have any recommendations

6. Please indicate your answer by selecting appropriate option:
If there is a final exam then:
- all students should be required to take it
- students who have performed well in class could be exempted (please specify your criteria for exemption:_____

_____
_____
_____).

7. Please indicate your answer by selecting appropriate option:
 The Department should recommend that homework is assigned
- regularly but not necessarily collected, instead, always reviewed i.e. all students'
  questions should be answered (time provided)
- collected and graded at least _____ number of times during a semester
- A small portion of the homework (few problems) should be collected and graded
  at least _____ number of times
- the Department should not give any recommendation
- other (please specify)_____

_____
_____
_____

8. Please indicate your answer by selecting appropriate option.
The Department should recommend that in developmental classes:
- there should be at least _____ number of quizzes during the semester and _____ lowest grades should be dropped
-
- there should be at least _____ number of quizzes during the semester and none of them should be dropped
- the Department should not give any recommendation
- other

__ _____
_____
_____
_____

9. Please circle appropriate choice:
The Department should recommend 'No extra credit policy" ?

Yes

No

The Department should not have any recommendations

10. Please circle appropriate choice:
The Department should recommend 'no partial credit policy' for Math 016 (arithmetic class)?
Yes

No

The Department should not have any recommendations

11. Please circle appropriate choice:
The Department should recommend 'no partial credit policy' for Math 017?

Yes

No

The Department should not have any recommendations

12. Please circle appropriate choice:
Would you be willing to try the policy of not allowing late students into class? Some instructors (John Jernigan for example) do this on a regular basis and claim that if implemented consistently, students will be on time.

Yes   No

13. Please circle appropriate choice:
The Department should recommend that a student can only make-up one exam during the semester ?

Yes

No

The Department should not have any recommendations

14. Please circle appropriate choice:
The Department should recommend that no student be added after the first day of classes.

Yes

No

The Department should not have any recommendations

15. Please indicate what other recommendations, if any, you would you like to have the Department make?

APPENDIX G

OUTLINE OF MATH 016

Math 016

Throughout the whole course, the importance of the use of mathematically correct language should be stressed.

The course does not allow the use of calculators:

Suggested list of topics:

A. Natural numbers.

- The meaning of equal sign. Symmetric and transitive property of equality.
- Addition and multiplication; properties: commutative, associative.
- Multiplication by powers of 10
- Use of properties in computation: e.g. $2 \times 17 \times 5$, or $(149+93)+7$
- Distributive property with examples of its use:
    Mental multiplication: e.g. $14 \times 6 = 10 \times 6 + 4 \times 6$ or $7 \times 17 + 3 \times 17$
- Exponential notation
- Decimal form of a number
- Multiples of a natural number
- Factorization: understanding the meaning 'a is a factor of b' or 'b is divisible by a'
- Prime numbers
- Prime factorization of a composite number
- Understanding that the same number can be written in a lot of different ways: decimal representation, prime representation
- Least common multiple

Examples of problems:

_____Using distributive property, compute the following:

$3 \times 56 + 7 \times 56 =$
$12 \times 7 + 12 \times 3 =$
$70 \times 12 + 12 \times 30 =$

_____

Compute, using commutative and associative law:

$23 + (5 + 73) =$

$(5 \times 8769) \times 2 =$

_____

Perform the indicated multiplication:

$346 \times 1000 =$

$100 \times 56 =$

$10^5 \times 73 =$

_____

Evaluate (please use equal sign symbol):

$3^4 =$

$5^2 \times 2^4 =$

$456^0 =$

_____

Which of the following numbers is equal to 32:

2^5 2×5 $3 \times 10^1 + 2 \times 10^0$ 5^2 16×2

_____

Write the following number in a decimal form: 3429

_____

Write the following number as a product of prime numbers: 180. Find its prime factors.

_____

Count by 6 up to 60 starting with 0. Find ten multiples of the number 6. How many divisors does 6 have?

_____

Write 75 in six different ways. Indicate that all of them are equal to 75 by using the equal sign.

_____

Find least common multiple of 9 and 11.

_____

B. Integers.

- The set of natural numbers is not closed with respect to subtraction: integers
- Natural numbers as a subset of integers
- Additive inverse; understanding why $-(-a) = a, -(-(-a)) = -a$ ...
- Addition and subtraction of integers; understanding
  $a - (-b) = a + b, a + (-b) = a - b$ ...
- Stressing syntax rules: expressions like $a + -b$ are not legal
- Multiplication of integers; knowing that $-a = (-1)a$

- Exponential notation: use of parenthesis, difference between $(-1)^2$ and -1^2.
- Division of integers (staying in the integers); introduction of fraction notation for division; operation of division by zero as undefined
- Order of operations
- Integers on a number line; relation among the integers ('=', '<', '>')

Examples of problems:

Express the following numbers as a product of -1 and another number:

$$45 = (-1) \times \underline{\qquad} = \underline{\qquad} \times (-1)$$
$$-65 = (-1) \times \underline{\qquad} = \underline{\qquad} \times (-1)$$

Please, make sure that you inserted parentheses, if needed.

Evaluate

$-(-3) =$

$-(-(-7)) =$

Evaluate:

$-5 + 6 =$

$34 - 78 =$

$-2 - 5 =$

$-3 - 8 - 7 =$

$-9 + 100 - 3 =$

$-1543876 - 2765 + 9 + 1543876 + 2765 =$

Rewrite the following expressions without 'multiple signs' and then evaluate:

$-4 - (-8) + (-1) =$

$3 - (+4) - (-(-6)) =$

Evaluate:

$(-2)^3 =$

$-2^3 =$

$4^2 =$

$(-1)^{322} =$

Perform the indicated operations:

$(-9) \cdot 8 =$

$99 \cdot (-10) =$

$(-1) \cdot (-2) \cdot (-5) =$

75

Write the following division problem using fraction notation:

$456 \div 33 =$

Perform the indicated operations:

$\dfrac{-36}{6} =$

$250 \div (-10) =$

$-70 \div (-7) =$

List the following numbers in increasing order and then plot them on a number line:

$-7 \qquad \dfrac{-12}{-3} \qquad -5 \qquad -2^4 \qquad (-2)^4 \qquad (-2) \cdot 4$

Perform the following operations:

$-3(-3) =$

$-3 - 3 =$

$-3 - (-3 + 4) =$

$4 \cdot (-50) - 3 \cdot (-11) =$

$\dfrac{3 - 4(-5)}{-2} =$

C. Rational Numbers

- Noticing that neither the set of natural numbers nor the set of integers is closed with respect to division: rational numbers
- Integers as a subset of rational numbers (any integer can be written as a fraction)
- Fractions; fractions as a division; meaning of a denominator and numerator
- Equivalent fractions: lowest term form, mixed numbers, improper fractions; ability to recognize equivalent fractions (use of equal sign)
- Other representations of rational numbers: decimals (all terminating or repeating) and percentages
- Converting from one form to another (decimals, fractions, percentages)
- Arithmetic of fractions (first positive and then positive or negative): addition, subtraction, multiplication, powers, multiplicative inverse (reciprocal), division, all operations combined (order of operations)
- Comparison of fractions (use of '=', '<', '>')

- Arithmetic of decimals: positive or negative (noticing how easy it is to multiply or divide by a power of 10)
- Comparing decimals (use of '=','<', '>')
- All arithmetic operations on rational numbers (order of operations)
- Rational numbers on a number line, relation among the rational numbers ('=', '<','>')

Examples of problems:

_____

Count up to 5 by $\frac{1}{2}$ starting from zero. Count up to 6 by $\frac{2}{3}$ starting from zero.

_____

Write 2 as a fraction. Can you write 2 as a fraction with the denominator equal to 5? How about 7?

_____

Write the division problem '347 divided by 43' using fraction-notation.

_____

Find the missing numerator: $\frac{2}{5} = \frac{}{25} = \frac{}{30}$

_____

Find the missing numerator: $\frac{4}{-7} = \frac{}{-14} = \frac{}{14} = -\frac{}{-14}$

_____

Write as a mixed number: $\frac{37}{7}$, $-\frac{26}{5}$, $\frac{-1002}{100}$

_____

Reduce to lowest terms: $\frac{5}{50}$, $\frac{-26}{39}$, $\frac{100}{-50}$

_____

Write the following as an improper fractions: $3\frac{14}{25}$, 2

Circle all fractions that are equivalent to $5\frac{3}{7}$:

$$-5\frac{3}{7}, \quad \frac{38}{7}, \quad 5\frac{6}{14}, \quad 4\frac{10}{7}, \quad \frac{15}{7}$$

Circle all numbers that are equal to $\quad -\dfrac{4}{9}$

$$\frac{-4}{-9}, \quad \frac{40}{-90}, \quad -\frac{-4}{-9}, \quad -4.9, \quad -.44444444, \quad -0.\overline{4}$$

Circle all numbers that are equal to 20.04

$$2004\%, \quad 0.2004\%, \quad 20\frac{4}{10}, \quad \frac{2004}{100}, \quad 20\frac{4}{100}, \quad 20\frac{1}{25}, \quad 20.04000$$

Evaluate:

$$4 + \frac{7}{8} =$$

$$2\frac{3}{5} + 11\frac{2}{3} + \frac{11}{30} =$$

$$-2 + \frac{2}{3} =$$

$$23456\frac{2}{7} - \frac{8}{9} =$$

$$(-1\frac{2}{11}) \cdot 11 =$$

$$\frac{-36}{6} =$$

$$-\left(-\frac{4}{5}\right)^2 =$$

$$-4\frac{2}{3} \div 2\frac{1}{7} =$$

$$\frac{\frac{-1}{5}}{6} =$$

Perform the indicated operations:

$3.67 + 2.001 =$

$-4.1 - 3.2 + 11 =$

$(-2.3) \times (-0.001) =$

$\dfrac{-3.2}{0.008} =$

$(0.02)^3 =$

___

Find a numerator such that the following fraction is greater than 2:

$$\dfrac{?}{7} > 2$$

___

Convert to decimal notation:

$\dfrac{3}{5}$ $\dfrac{2}{1000}$ $3\dfrac{7}{8}$ $-2\dfrac{2}{3}$ 3% $\dfrac{1}{2}\%$ 0.02%

___

Write as a fraction:

3.56 -4.0002 0.7

___

List the following numbers in increasing order:

0.2 0.23 $.1999$ $\dfrac{1}{5}$ $\dfrac{-3}{4}$ -0.8 $\dfrac{3}{11}$

___

Plot on a real number line:

$\dfrac{5}{7}$ $-3\dfrac{7}{8}$ 0.2

___

**D. Percentage Problems. Perimeter. Area.**
- Review of a definition of percent and operation of converting it to a real number (and back)
- Percentage problems requiring proportions
- Perimeter of polygons
- Area of a triangular, rectangular, trapezoid

Examples of problems:

___

Find the area and perimeter of a rectangle with sides equal 3 and $4\dfrac{3}{5}$ inches.

___

What number is 6% of 666?

_____

2 is what 0.01% of what number?
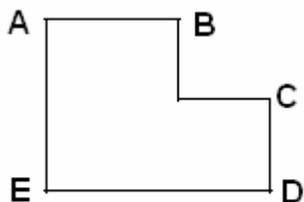
Find the area and the perimeter of the above figure, if the length of the interval AB is 4 inches, ED is $9\frac{2}{3}$ inches, AB is 3 inches, and CD is 1.5 inches.

_____

45 is what percent of 20?

_____

John's lunch cost $25 (without a tip). He gave a 15% tip. What was the total amount John paid?

_____

E. Real Numbers (Irrational Numbers)

- Definition of a square root of a non-negative number
- Definition of an irrational number
- Square roots that are irrational
- Area and circumference of a circle (π as an example of an irrational number)
- Rational numbers as a subset of real numbers; real numbers as a union of rational and irrational numbers
- Representation of an irrational number as non-repeating, non-terminating decimals and understanding that no irrational number is equal to any terminating decimal (so, in particular, $\sqrt{2}$ is not equal to 1.41 or even to 1.414213)
- Real number line, relation among the rational numbers ('=', '<', '>')
- Ability to locate any given root between two integers
- Distance from zero on a real number line: absolute value of a number
- Combination of operations, including absolute value

Examples of problems:

_____

Evaluate:
$\sqrt{36} =$ $\sqrt{100} =$

Find the area and circumference of a circle with the radius $\dfrac{3}{4}$ cm.

List in increasing order and plot on a number line:

$$-\dfrac{2}{3} \qquad \sqrt{19} \qquad -2 \qquad 5 \qquad 5.3 \qquad 5\dfrac{1}{3} \qquad -\sqrt{10}$$

Simplify:

$3\sqrt{7} + 4\sqrt{7} =$

$-\sqrt{17} - 2\sqrt{5} - \sqrt{17} =$

$\dfrac{4\sqrt{11}}{5} - \dfrac{3\sqrt{11}}{2} =$

Evaluate:

$\left(\sqrt{75}\right)^2 =$

$\sqrt{3} \cdot \sqrt{3} =$

Evaluate:

$\left|-3\right| =$

$\left|-3+5\right| =$

$-\left|\dfrac{-2}{3} + 1\dfrac{4}{5}\right| =$

Perform the indicated operations:

$$\dfrac{-2\dfrac{3}{50} - \dfrac{47}{50}}{6} =$$

$-0.2 \cdot (0.1)^2 - 4 =$

$(-2 - \dfrac{4}{7})(4 - \dfrac{2}{5}) =$

$(-1)(-1)(-1) - 1 - 1 =$

$(4 - 22 \cdot \dfrac{3}{11})^2 =$

APPENDIX H

OUTLINE OF MATH 017

Math 017

Math 017 will assume the knowledge of arithmetic.

Throughout the whole course the importance of the use of mathematically correct language should be stressed.

The course does not allow the use of calculators.

Suggested list of topics:

A. Variables. Evaluation of simple algebraic expressions.

Students should:
a) know that variables are treated as numbers because they represent numbers and thus

understand the meaning of: $3 + x,\ x - 3,\ 3x,\ -3x,\ \dfrac{x}{3},\ x^2,\ x^3$,.and so on

b) immediately recognize that
$$1x = 1 \cdot x = x \cdot 1 = x,$$

$$\frac{x}{2} = \frac{1}{2}x = x \div 2,$$

$$\frac{4x}{5} = \frac{4}{5}x = x \cdot \frac{4}{5},$$

$$-x = (-1)x$$

c) understand the meaning of expressions with two or more variables.
d) immediately recognize that (and understand why)
$$xy = x \cdot y = y \cdot x = yx,\ x + y = y + x$$

$$-\frac{x}{y} = \frac{-x}{y} = \frac{x}{-y}$$

but $x - y \neq y - x.$

e) be able to evaluate simple algebraic expressions when the value of variable(s) is given. (in particular, students should know how to evaluate expressions like

$-x,\ x^3,-x^3,\ (-x)^3$, even if x is a negative number and exponential expressions where an exponent is unknown, as, for example, $2^n,\ (-1)^{n+m}$, or when both base and exponent are unknown $: a^n,\ a^{n-m}$ or a^{nm}).

f) understand the meaning of statement $x < 2$ (knowing that it is equivalent to $2 > x$). Students should understand the difference between $x < 2$ and $x \le 2$ and be able to graph the sets on a number line

g) understand that the statement "x is positive" is synonymous with the statement $x > 0$

Examples of problems:

John has eight more nickels than Larry. Express number of nickels John has in terms of number of nickels Larry has.

_____

Evaluate $x^2,\quad -x,\quad \dfrac{x}{2},\quad x-3,\quad x^3,\quad -\dfrac{1}{3}x$, if $x = -2$

_____

Determine which of the following numbers satisfies the inequality: $x \le -.0.6$:

$\dfrac{-1}{2},\quad -.666,\quad -6,\quad -0.6^2,\quad -.5999,\quad 4^0,\quad (-2)^3$

_____

List five numbers satisfying $x \ge 3$. Would your answer be also correct if the questions were: List five numbers satisfying $y \ge 3,\ z \ge 3$ or $3 \le z$?

_____

Assuming that x represents an unknown number, write the following statements, using algebraic symbols:

x is equal to 6,
x is positive,
x is not negative,
x is greater than y,
Additive inverse of x is equal to 3,
Additive inverse of x is positive.

_____

Is $x + 3 = 3 + x$? Why? Is $3x = x \cdot 3$? Why? How about $x - 3$ and $3 - x$?

_____

Determine which of the following expressions are equal to $\dfrac{5x}{6}$:

$5 \cdot \dfrac{x}{6},\qquad \dfrac{5}{6}x,\qquad x \cdot \dfrac{5}{6},\qquad \dfrac{5}{6x},\qquad \dfrac{10x}{12},\qquad 6 \cdot \dfrac{x}{36} \cdot 5$

_____

Find any value of x such that $3x = 9$, $\dfrac{x}{3} = 9$

Suppose that you know that a number x is positive. Of the following expressions, which must be always positive?

$$x, \quad -x, \quad -(-x), \quad -x^2, \quad |x|, \quad -|x|, \quad 3x, \quad \dfrac{x}{3}$$

Repeat the same exercise under the assumption that x is negative.

Graph the following sets on a real number line:

a) $x \le -\dfrac{2}{5}$,   b) $x \ge 1\dfrac{2}{3}$,   c) $-4 < x$

If $x = \dfrac{3}{5}$, $y = -0.1$, $z = -1\dfrac{2}{3}$, evaluate the following expressions:

$$x(z+y), \quad xz+y, \quad xz+xy$$

Evaluate the following expressions:

$$a^3, \quad 4^n, \quad ab^2, \quad (ab)^2, \quad -a^n, \quad a^{n+m}$$

if $a = -1, b = \dfrac{1}{3}, n = 3, \quad m = 2$.

B. Simple logical statements. The meaning of 'and', 'or' and 'if…then'. 'Double inequalities'.

Students should:

   a) know the difference between 'and' and 'or' and be able to determine if a given statement is true or false

   b) know when 'if … then ' statement is true and that it is enough to find just one counterexample to show that 'if … then' statement is false.

   c) know that 'if A then B' does not mean that 'if B then A'

   d) know that 'if A then B' is equivalent to 'if not B then not A'.

   e) be familiar with double inequalities and know how to graph the set of numbers they represent on a real number line.

Examples of problems:

Are the following statements true or false? Why?
a) An apple is a fruit and it grows on trees
b) an apple is a fruit and it is made of wood.

c) an apple is a fruit or it is made of wood
d) an apple is a fruit or it grows on trees.
e) x is an even number and x is an odd number
f) x is an even number or x is an odd number

Can a number x be greater than 4 and less than 2 ($x > 4$ and $x < 2$)?

Are the following statements true or false? Why? If a statement is false, give a specific example showing why it is not true:
a) If one lives in the USA then one lives in North America.
b) If x=3 then 3=x
c) If x=y then y=x
d) If x<y then y>x.
e) If x is a positive number then –x is a negative number.
f) If $x \leq 3$ then $x < 3$.

g) If $x + y = \dfrac{1}{2}$ then $y + x = \dfrac{1}{2}$.

h) If x is divisible by 4, then x is divisible by 2.

i) If $x = \sqrt{2}$ then $x = 1.41$.

j) If $x = \dfrac{-3}{245}$ then $x = \dfrac{3}{-245}$

k) If $a = b$ then $a^2 = b^2$

l) If $x > 0$ and $y > 0$ then $xy > 0$

Find the converse of each of the above statements (so, for example: 'If $x \leq 3$ then $x < 3$.' becomes 'If $x < 3$ then $x \leq 3$.') and then determine if the converse is true or not.

Are the following statements true or false? If a statement is false, give a specific example showing why it is not true:

a) If x=1 and y= $\dfrac{1}{2}$ then xy = 2.

b) If xy=0 then x=0 or y=0
c) If xy=0 then x=0 and y=0
d) If $a = b$ and $b = 3$ then $a = 3$

e) If $a = b$ and $b = 3\dfrac{1}{2}$ then $a = \dfrac{7}{2}$

d) If $x \geq 0$ and $y \geq x$ then $y \geq 0$

Determine which of the following numbers $- 3.45, - 3\dfrac{1}{2}, 3, 0, 2, 1.999, - 3$

satisfies the following condition:

$$x < -3\dfrac{1}{2} \quad \text{or} \quad x \geq 2 .$$

Repeat the same exercise for the following condition:

$$x < -3\frac{1}{2} \quad \text{and} \quad x \geq 2$$

List five numbers that satisfy the following condition:
$$x > 0 \quad \text{and} \quad x < 1$$
Would you be able to list more numbers like that? How many are there?

Determine which of the following statements are true. Why? If a statement is not true, give a specific number showing why it is not true:

a) If $x = 0.09$ then $x \leq 0.009$ and $x > 0$

b) If $x = -2$ then $x \leq -2$ or $x = 5$

c) If $x > 2$ then $x > 1$ and $x < 3$.

d) If $x > 2$ then $x > 1$ or $x < 3$.

Graph the following sets on a real number line:

a) $\quad x \leq -\dfrac{2}{3}$ or $x \geq 4$

b) $\quad -0.1 < x$ and $x < 1\dfrac{2}{7}$ (or equivalently denoted by $-0.1 < x < 1\dfrac{2}{7}$)

C. Operations on power expressions with positive integer exponents

Students should:

a) know that $a^0 = 1$ and recognize that $a^1 = a$

b) understand why

If n and m are positive integers then
$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

$$a^n a^m = a^{n+m}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$(a^n)^m = a^{nm}.$$

(we can show students how to derive the above properties, tell them that this is an example of a mathematical theorem and that mathematics is about proving theorems).

c) have the practical ability to use the above properties to simplify exponential expressions

<u>Examples of problems:</u>

Simplify.

$$n^3 \cdot n^{20} =$$

$$a^{11} \cdot a^{-3} \cdot a^{-18} =$$

$$\left(\frac{x^2 y}{x}\right)^3 =$$

$$(ab)^0 =$$

$$ab^0 =$$

$$\frac{s^{12}}{2s^3} =$$

$$(x^5 y^3)^2 =$$

$$\frac{a^{13} b^2 c^4}{a^5 c^2 b} =$$

D. Manipulation on symbols: addition, subtraction, multiplication. The Distributive Property. Factorization. Simplification of rational expressions.

Students should:
a) be able to recognize like terms (in particular students should remember that xy=yx and be able to apply the previously introduced rules for exponential expressions to recognize like terms, like for example, $x^3 yx$ and yx^4).
b) be able to simplify expressions by combining like terms
c) be able to state the distributive property and know why the distributive property
 implies that $(a + b)(c + d) = ac + ad + bc + bd$, $a(b + c + d) = ab + ac + ad$ and so on.
d) be able to apply the distributive property to multiply algebraic expressions
e) be able to apply the distributive property 'in reverse' to factor expressions
f) be able to factor out -1 (or any other number) from any expression

g) understand why, for example, $\dfrac{x + yx}{x} \neq yx$ and why factorization can be of help in the

process of simplification of rational expressions
h) be able to write their solution using correct mathematical language; be aware of the use 'equals can be substituted for equals'

Examples of problems:

_____

The expression $a \cdot b + c + 25$ is a sum with $a \cdot b$, c and 25 as its summands. Using the commutative property, create three different expressions equal to $a \cdot b + c + 25$.

_____

Simplify:

$3a - \dfrac{2}{3}a =$

$4a + 7a - (8a + 9) =$

$[8(q + 6) - 21] - [3(q - 5) + 4] =$

$(1 + 4x + 6x^2 + 7x^3) + (5 - 4x + 6x^2 - 7x^3) =$

$(6x^4 + 3x^3 - 1) - (4x^2 - 3x + 3) =$

$3a - 9b - (4a - \dfrac{4}{5}b) =$

$-3y + \dfrac{1}{2}x - x + 4y =$

$5(6a - 2) + 12a =$

$-\dfrac{0.1}{0.001}x + x =$

$-3xy + 7yx - (xy + 3) =$

$-\dfrac{3x}{2} + \dfrac{1}{2}x =$

$2x + y + x - 2y =$

$3\sqrt{x} - 4\sqrt{x} =$

$$\frac{3}{38}ab - \frac{7}{19}ba + 2\frac{3}{4} + 3\frac{1}{8} =$$

Perform the indicated operations:

$$(x^3 + x^2 + x + 1)(x - 1) =$$

$$3y^2(6y^4 + 8y^3) =$$

$$(6x^4 + 4)^2 =$$

$$(2x^3 y + \frac{3}{7} - xy^4)xy =$$

$$-\frac{2}{3}(3c - 33d) =$$

$$(3ab - \frac{2}{5}b)(10 - a^3 b) =$$

$$(a + b - g)(c - d - f) =$$

$$(2a - 1)(1 - 3a)(a - 2) =$$

Remove parentheses:
$$-(a + bc - \sqrt{7})$$
$$-[-(x - 1) - (-y)]$$

Factor:
$$10a - 15a^2 =$$

$$5x^5 + 10x^3 =$$

$$17x^5 y^3 + 34x^3 y^2 + 51xy =$$

$$3x^2 y - 6xy^{14} + 9zxy =$$

$$a^3 b^4 + 5b^3 a^7 - a^3 b^3 =$$

Factor out -1 from:
a) $3x - \sqrt{2}y =$

b) $-a+b+1=$

___

Simplify:

$$\frac{5xy^4}{20y^3}=$$

$$\frac{xy+xz}{3x}=$$

$$\frac{a^3b-4b^4a^4}{ab}=$$

$$\frac{a-3b}{-3b+a}=$$

$$\frac{x}{2x-x^2}=$$

$$\frac{12x-4}{3x-1}=$$

___

Simplify the following expressions:

$$\frac{4x-5x}{x}=$$

$$\frac{4x(-5x)}{x}=$$

$$(x-\frac{2}{3})(x+\frac{2}{3})-x^2=$$

$$x-\frac{1-2x}{3}=$$

$$3x-(2x-1)(\frac{1}{2}-x)=$$

$$(2x-1)^2-(x-2)^2=$$

E. Evaluations of algebraic expressions. Substitution of not only numbers but algebraic expressions.

Students should:
   a) Recall the symmetric and transitive properties of equality
   b) be able to evaluate any algebraic expression
   c) be able to substitute an algebraic expression for any given variable and then simplify the resulting expression

Examples of problems:

For the following problems, evaluate if $x = 2$, $y = -5$, $a = 3$ Please write as clearly as you can and make sure that each equal sign is true.

$$(x+y)^2 \qquad \frac{ax^2 + y}{7} \qquad x^2 + 2xy + y^2$$

If $x = -1$ and $y = -2$ evaluate

$2(x - y)$ \qquad $2x - y$

Evaluate

$\frac{1}{3}y - \frac{x}{2}$ if $x = 2\frac{2}{5}$ and $y = -6$

Rewrite the expression $a + b + c$ in terms of x, if it is given that $a = 2x, b = -x, c = 4x$. Simplify.

Repeat the exercise when $a = x^2, b = \frac{x}{2}, c = -2x^2$.

Rewrite the expression $\frac{3a}{2}$ in terms of x, if it is given that $a = 2x$. Simplify

---

Simplify the expression $\frac{-xy}{2}$ if $y = x^2$

Rewrite the expression $a^2 - 1$, if it is given that $a = 5x$.

Rewrite the expression $a^2 - b^2$, if it is given that $a = 5x$ and $b = 1$

Rewrite the expression $a^2 - b^2$, if it is given that $a = 5x - 2$ and $b = -x$. Simplify.

Rewrite the expression $(a - b)(a + b)$, if it is given that $a = 5x$ and $b = -1$

Eva luate $(x + y)^2$ if $x + y = -1$. Evaluate it, if $x + y = 2\frac{2}{3}$.

F. Recognizing and matching patterns (primarily to recognize that a certain formula could be applied in a given situation).

Students should:

be able to rewrite expressions according to prescribed rules (usually with the goal of making it match some given pattern or expressing information in some canonical form) and recognize the value of appropriate coefficients in their representation (for example, if we represent $5x + 3$ in the form of $ax + b$, they should know that a=5).

Examples of problems:

The expression $3x + 4y$ is written in the form:

What are the values of a and b.
$$ax + by$$

The expression $3x^2 + 4y^2$ is written in the form:
$$ax^2 + by^2$$
What are the values of a and b.

The expression $\frac{4}{5}x^2 + \frac{7}{8}y^2$ is written in the form:
$$ax^2 + by^2$$
What are the values of a and b.

Write the following expressions in the form $ax + by$ and determine the value of a and b.

a) $3x + y + x$

b) $\dfrac{x}{2}+2(3-y)$

c) $0.6x-y+y$

d) x

Write the following expressions in the form $mx+b$, where m and b are any real numbers. Find the value of m and b in your representation:

a) $-3x+7-x$

b) $\dfrac{4-x}{5}$

c) $-(x+2)$

d) $-3(x-\dfrac{2}{3})$

Write the following expressions in the form a^2, where a is any algebraic expression or a number. In each case state what a is equal to.

$36, \quad x^2, \quad 3, \quad 25y^2, \quad \dfrac{9}{49}, \quad \dfrac{b^2}{100} \quad (3x+7)^2, \quad a^2$

Write the following expressions in the form $-a$, where a is any algebraic expression or a number. In each case state what a is equal to.

$-36, \quad 36, \quad -x^2, \quad 5, \quad 4y, \quad -(3x+5), \quad 3x+5$

Write the following expressions in the form a^2-b^2, where a is any algebraic expression or a number. In each case state what a and b are equal to.

$x^2-36, \quad 1-\dfrac{y^2}{4}, \quad 10-4c^2, \quad 16x^2-(5x-1)^2, \quad (16x)^2-(5x-1)^2$

Write the following expressions in the form $y=ax^2$. In each case determine the value of a.

a) $y=3x^2$, \qquad b) $y=\dfrac{-2x^2}{5}$, \qquad c) $y=-x^2$ \qquad d) $y=(2b-1)x^2$

G . Generalities on equations and Linear equation in one variable as a particular type of equation.

Students should:
a) be able to recognize the difference between algebraic expression and equation
b) know what it means to solve an equation (any equation)
c) be able to solve any linear equation and know that any linear equation in one variable can have no solution, exactly one solution or can be in the form of identity (understand the concept of identity)
d) be able to solve any linear equation with non-numerical coefficients.

Examples of problems:

Determine whether the following mathematical sentences represent an equation or an algebraic expression:
a) $5x$,
b) $5x = 2$,
c) $x^2 = 36$,
d) $|x| = -6$,
e) $\dfrac{x-1}{2} + x$
f) $x = -4 + 2x$

___

Determine which of the following numbers is a solution of the equation $-x^4 = 16$:

$$-2, 16, \frac{1}{2}, 2$$

___

Does $x = 7$ make the statement $2(x+1) - x = 7$ true or false?

___

Solve the following linear equations for x:

$2x - 3 = -1$	$x - 5 = 3x + 7$	$\dfrac{3}{5}x = 6$
$2(x-3) + 4 = x - 2$	$.03x + 7.75 = .07x + 3.75$	$-x = 4$
$-x = -7$	$-x = a$	$-x = -a$

___

Solve the following linear equations for x:

$x - 8 = x - 9$	$3(2x + 5) + 1 = 6x + 16$	$x = -x$

$$-5 = \frac{x}{4} \qquad\qquad \frac{3}{4}x = -15 \qquad\qquad\qquad \frac{x}{2} + \frac{2}{3} = \frac{3}{4}$$

Solve: $\qquad 2(3p+1) + 11 = 2(3p+1) + p + 6$

Solve for x: $2(x-4) + 2x = 3x - 2$

Solve for t: $2(t-4) + 2t = 3t - 2$

Solve for d: $2d - 1 = 5$

Solve for d: $2d - a = b$

H . Equations in two variables. Cartesian coordinate system. Graphs of linear equations, as a particular example of equations in two variables.

Students should:
- a. understand the meaning of the solution of any equation in two variables.
- b. be able determine if a given pair of numbers is a solution of an equation
- c. be familiar with the Cartesian coordinate system (x-axis, y-axis, quadrants) and be able to plot any point on it.
- d. understand how the graph of any equation in two variables can be obtained
- e. be able to define (and recognize) a linear equation in two variables
- f. be able to graph any linear equation and know that only linear equations have graphs that are lines.

g. <u>Examples of problems:</u>

Determine if the following are solutions of the equation: $y = \dfrac{x^2}{3}$

$(0, 0), \quad (-2, -\frac{8}{3}), \quad (3, 3), \quad (\frac{-1}{2}, \frac{1}{12})$

Determine if the following represents an equation or algebraic expression. If it is an equation determine if it is a linear equation:

a) $3x - 7y = 5$

b) $x^2 - 2y^2 = 0$

c) $-\sqrt{4x} + 5 - y$

d) $x = y$

Determine if the following equations are linear. If so, express it in the form $ax + by + c = 0$.

Determine the values of a, b, and c in your representation:

a) $3 - 4(y - x) = 1$

b) $3x^2 - y = 0$

c) $y - 5 = 0$

d) $\sqrt{2} - x + y\sqrt{3} = 0$

e) $2yx = 1$

f) $\dfrac{x + 8y}{8} = 3$

Find six solutions of the following linear equation: $3x - 5y = 7$

Find six solutions of the following linear equation: $y - 9 = 0$

Plot the following points:

$(4,-2),\quad \left(-1, \dfrac{3}{4}\right),\quad \left(\dfrac{3}{4}, -1\right),\quad \left(\sqrt{12}, 2\right),\quad \left(-2\dfrac{2}{3}, 10\right)$

Find 3 points on the line given by the equation $y = -x + 2$

Find 3 points on the line given by the equation $x = 7$

Find 3 points on the line given by the equation $y = -5$

Which of the following points, when plotted, will be off the line given by the equation:

$$4x - \dfrac{y}{2} - 2 = 0\,?$$

$(1, 4),\qquad (3, -20),\qquad (-3, 28),\qquad (0, -2)$

Graph the line $x - 2y = 6$ Be sure to label your points. If we multiply the above equation by –2 we get the new equation $-2x + 4y = -12$. Graph this line. What do you notice about the lines?

Determine which of the following statements are true?

a) If (s, t) is a solution of $x + 3 = y$ then (s, t) is a solution of $4(x + 3) = 4y$

b) If (s, t) is a solution of $mx + b = y$ then (s, t) is a solution of $mx + b + 5 = y + 5$

An equation $y = 0$ is an equation of the x-axis (convince yourself about it). Find equation of the y-axis

---

Match the following graphs with equations:

(several graphs, not only of linear equations, would be given. Matching has to be done by reading coordinates of some points and checking if those are solutions of equations)

Graph the following equations:

$$y = 4x - 2 \qquad \frac{3}{7}(x-1) = y \qquad y = -2\frac{1}{4} \qquad x - 8 = 0$$

APPENDIX I

OUTLINE OF MATH 067

Math 067

Math 067 will be offered to:
- students who passed Math 017 but failed the Gateway Test
- students who recently took the Placement Test (the Gateway Test included) and who (based on their results) are determined to be better prepared for taking Math 067 directly (instead of taking 016-017 sequence first)
- students who passed the Gateway Test on the Placement Test but do not feel strong about it and would still prefer to have a one semester review of arithmetic and basic algebra

Throughout the whole course the importance of the use of mathematically correct language should be stressed.

The course does not allow the use of calculators.

Suggested list of topics:

E. Numbers (integers, rational and irrational numbers). Positive integer powers and radicals. Order of numbers. Real number line. Plotting points. Absolute value.

Students should:
a) Understand the meaning of a denominator and a numerator and that a fraction represents an operation of division
b) Know that there are other numbers than integers as well
c) The symmetric and transitive properties of equality
d) Understand that a given number can be written in a lot of different ways and be able to recognize equivalent numbers:
- simplification of fractions (understand why it can be done)
- converting: mixed numerals and improper fractions
- converting: fractional and decimal notation
e) Understand power notation (including the role of parenthesis: for example, the difference between -1^2 and $(-1)^2$).
f) Be able to compare any two numbers (and use '<' and '>' symbols)

g) Be able to plot any number on the real number line (including fractions and square roots: ability to locate any given root between two integers)

h) Be able to evaluate any numerical expression involving absolute value and be familiar with interpretation of an absolute value as a distance between a number and zero.

Examples of problems:

_____

Find missing numerators so the following are equal: $12\dfrac{1}{3} = 10\dfrac{}{3} = 8\dfrac{}{3} = \dfrac{}{3} = \dfrac{}{300}$

_____

Express the following as a decimal:

$$3 + \dfrac{6}{10} + \dfrac{1}{1000}$$

_____

Express the following division problem as a fraction $345 \div 5498$

_____

Is the number $\dfrac{245.006}{245}$ greater or less than 1? How do you know?

_____

Among the following numbers, find all that are equal to $7\dfrac{2}{5}$:

$\dfrac{37}{5}$ 7.25   7.400 $5\dfrac{12}{5}$ 72.5 $7\dfrac{20}{50}$

_____

If you know that a given fraction represents a number greater than 2, what can be said about its numerator and denominator (relationship between its numerator and denominator).

_____

Write as a decimal $4\dfrac{3}{100}$, $\dfrac{2}{3}$, $5\dfrac{3}{20}$, $\dfrac{-7}{8}$

_____

Write as a fraction   0.56   0.0023   3.012   -1.4

_____

Simplify the following fractions: $\dfrac{3}{30}$, $\dfrac{125}{600}$, $\dfrac{36}{6}$, $\dfrac{5^3}{5}$

_____

Write as a mixed number $\dfrac{31}{5}, \dfrac{67}{12}$

Find the value of the following: $-3^2, \quad (-3)^2, \quad (-1)^{247}, \quad \left(\dfrac{-2}{5}\right)^3$

List the following numbers in an increasing order:

$\dfrac{1}{3} \quad 2.75 \quad 2\dfrac{5}{6} \quad 0.334 \quad -2.1 \quad 2.8 \quad -\sqrt{10} \quad 0.33 \quad -2.111 \quad \dfrac{32}{12} \quad \sqrt{3}$

Find such a denominator that the following statement is true: $\dfrac{7}{15} < \dfrac{7}{}$. Can you find another

one? Now, find a denominator for the following statement: $-\dfrac{7}{15} < -\dfrac{7}{}$

Is -1^2 equal to $(-1)^2$? How about $-(-1)^2, \quad -(-1^2), \quad (-1)^0$?

List five positive numbers that are less than zero.

Plot on a real number line: $\sqrt{15}, \quad \dfrac{4}{5}, \quad -\dfrac{13}{3}, \quad -2.1, \quad 4\dfrac{2}{3}$

Evaluate: $|-3|, \quad -|-2|, \quad -\left|\dfrac{1}{2}\right| + \left|-\dfrac{1}{2}\right|$

F. Arithmetic: fractions, decimals, signed numbers and combinations of all three. Order of operations.

Students should:

a) Be able to perform any type of arithmetic operations on positive or negative numbers, including fractions and decimals (know the order of operations)

b) Understand the meaning of an equal sign and be able to present their solutions as a sequence of equations, for example:

$$(-2+1)(-4+3)+1 = (-1)(-1)+1 = 1+1 = 1.$$

(Students often think about equal sign as an indication of performing 'the next operation' and write things like $'5\cdot 3+4 = 15 = 19'$; they should be taught to use mathematically correct

language). Students should understand that we use the rule 'equals can be substituted for equals' and that this always yields an equivalent expression.

Examples of problems:

Evaluate:

$$\frac{\frac{-2}{3}-\frac{1}{3}}{2}=\qquad -3\left(-4+\frac{2}{5}\right)=\qquad -0.1\times0.02-1=\qquad \frac{3\times2-7}{789-788\frac{2}{3}}=$$

$$\frac{-1-1}{(-1)(-1)}=\qquad \frac{(-1)^3-5^2}{3-2\cdot2}=\qquad \frac{33}{45}\cdot(-2)\cdot\frac{5}{3}\cdot\frac{9}{22}=$$

___

Explain why $\dfrac{-4}{5}=\dfrac{4}{-5}=-\dfrac{4}{5}$ and $\dfrac{-4}{-5}=\dfrac{4}{5}$

G. Variables. Evaluation of simple algebraic expressions.

Students should:
h) know that variables are treated as numbers because they represent numbers and thus understand the meaning of: $3+x$, $x-3$, $3x$, $-3x$, $\dfrac{x}{3}$, x^2, x^3,.and so on

i) immediately recognize that
$$1x = 1\cdot x = x\cdot 1 = x,$$
$$\frac{x}{2}=\frac{1}{2}x = x\div 2,$$
$$\frac{4x}{5}=\frac{4}{5}x = x\cdot\frac{4}{5},$$
$$-x = (-1)x$$

j) understand the meaning of expressions with two or more variables.
k) immediately recognize that (and understand why)
$$xy = x\cdot y = y\cdot x = yx,\ x+y = y+x$$
$$-\frac{x}{y}=\frac{-x}{y}=\frac{x}{-y}$$
but $x-y\neq y-x.$

l) be able to evaluate simple algebraic expressions when the value of variable(s) is given. (in particular, students should know how to evaluate expressions like

$-x, x^3, -x^3, (-x)^3$, even if x is a negative number and exponential expressions where an exponent is unknown, as, for example, $2^n, (-1)^{n+m}$, or when both base and exponent are unknown : a^n, a^{n-m} or a^{nm}).

m) understand the meaning of statement $x < 2$ (knowing that it is equivalent to $2 > x$).
Students should understand the difference between $x < 2$ and $x \leq 2$ and be able to graph the sets on a number line

n) understand that the statement "x is positive" is synonymous with the statement $x > 0$

Examples of problems:

Susan is 3 times older than Ann. John is 5 years older than Susan. Express Susan's and John's age in terms of Ann's age.

_____

Half of Bob's salary is $\dfrac{4}{5}$ of Tom's salary. Express Bob's salary in terms of Tom's salary.

_____

Evaluate x^2, $-x$, $\dfrac{x}{2}$, $x-3$, $-(-x)-1$, $-\dfrac{1}{3}x$ if $x = -\dfrac{1}{2}$

_____

Determine which of the following numbers satisfies the inequality: $x \leq -\dfrac{2}{3}$:

$\dfrac{-4}{5}$, $-.666$, $\dfrac{1}{-2}$, -2^2, $\left(-\dfrac{2}{3}\right)^{45}$, -4^0, $\dfrac{-2}{3}$

_____

List five numbers satisfying $x \geq 3$. Would your answer be also correct if the questions were: List five numbers satisfying $y \geq 3$, $z \geq 3$ or $3 \leq z$?

_____

Explain why $\dfrac{x}{2} + \dfrac{y}{2} = \dfrac{x+y}{2}$. Write $\dfrac{a+b}{3}$ as a sum of two fractions . Can one do the same with $\dfrac{a-b}{3}$ or with $\dfrac{a+b+c}{3}$? Can you express $\dfrac{a+b+c}{3}$ as a sum of three fractions?

_____

Explain why $\dfrac{x}{3} = \dfrac{1}{3}x$. Write the following expressions as a product of a number and the variable: $\dfrac{y}{4}$, $\dfrac{3z}{5}$, $\dfrac{2x}{\frac{1}{2}}$

Is $x + 3 = 3 + x$? Why? Is $3x = x \cdot 3$? Why? How about $x - 3$ and $3 - x$?

Determine which of the following expressions are equal to $\dfrac{x+y}{6}$:

$\dfrac{x}{6} + \dfrac{y}{6}$, $\dfrac{1}{6}(x+y)$, $\dfrac{1}{6}x + y$, $\dfrac{1}{6}(y+x)$, $6(x+y)$, $\dfrac{y}{6} + \dfrac{x}{6}$

Find any value of x and y such that x+y=7. How many different correct answers can you give? Can you find such value for x and y that x+y=7 and at the same time x=y? What if we consider x+x=7 (why is it different from x+y=7), can you find a value of x satisfying this equation? How many correct answers can we have now?

Suppose that you know that a number x is positive. Of the following expressions, which must be positive? Which must be negative? Which are of undetermined sign?

x, $-x$, $-(-x)$, $-x^2$, $|x|$, $|x-3|$, $|x| - 3$

Repeat the same exercise under the assumption that x is negative.

List all numbers that satisfy $x \geq 2\dfrac{1}{3}$ but do not satisfy $x > 2\dfrac{1}{3}$.

Describe the following set of numbers using $<, >, \leq, \geq$ notation:
        a) all numbers x that are greater or equal to 5
        b) all numbers x that are at least equal to 3
        c) all numbers x that are at most 4
        d) all numbers x that are not more than 8

Graph the following sets on a real number line:

a) $x \leq -\dfrac{3}{4}$,   b) $x \geq 1\dfrac{2}{3}$,   c) $-3 < x$

If $x = \dfrac{3}{5}$, $y = -0.1$, $z = -1\dfrac{2}{3}$, evaluate the following expressions:

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$$xz + y, \quad \frac{2x}{y}, \quad \frac{-y^3}{0.0001}, \quad -x + y - z,$$

Evaluate the following expressions:

$$a^3, \quad 4^n, \quad ab^2, \quad (3ab)^2, \quad -a^n, \quad a^{n+m}, \quad 2^{nm}$$

if $a = -2, b = \dfrac{1}{3}, n = 3, \quad m = 2$.

H. Simple logical statements. The meaning of 'and', 'or' and 'if…then'. 'Double inequalities'.

Students should:

a) know the difference between 'and' and 'or' and be able to determine if a given statement is true or false

b) know when 'if … then ' statement is true and that it is enough to find just one counterexample to show that 'if … then' statement is false.

c) know that 'if A then B' does not mean that 'if B then A'

d) know that 'if A then B' is equivalent to 'if B then not A'.

e) be familiar with double inequalities and know how to graph the set of numbers they represent on a real number line.

Examples of problems:

IRS gives (among others) the following circumstances as ones that qualify a person for an automatic extension of time to file his taxes:
'You live outside the United States and Puerto Rico and your main place of business or post of duty is outside the United States and Puerto Rico.'
John lives in Canada and works in Buffalo, NY. Is he eligible for automatic extension?

Tom received the following coupon in his mail:
'If your friend uses our Learn to Ski Clinic and our equipment rental, you get a free all-day lift ticket'.
Tom invited his friend Susan for a ski trip. He lent her his sister's skis and signed her for Learn to Ski Clinic. Is Tom eligible for a free all-day lift ticket?

Are the following statements true or false? Why?
a) Washington is the capital of the USA and Washington is a name of the first American president.
b) Any dog is a mammal or any dog is a bird.
c) The division by zero is not defined and zero divided by any number different from zero is always equal to zero.
d) The division by zero is not defined and zero divided by any number is always equal to zero.
e) Any number is greater than 2 or it is less than 3.

f) $xy > 0$ or $xy < 0$

___

Are the following statements true or false? Why? If a statement is false, give a specific example showing why it is not true:
a) If one is a student of this math class then one is a student of CCP.
b) If x<y then y>x.
c) If x is a positive number then –x is a negative number.
d) If $x \le 3$ then $x < 3$.
e) If $xyz = 5$ then $zyx = 5$.
f) If x is divisible by 4, it is divisible by 2.
g) If $x \ge 5\frac{3}{7}$ then $\frac{38}{7} \le x$.
h) If x+y=4 then x+y+1=5.
i) If $x \le 2$ then $|x - 1| \ge 0$.
j) If $a = b$ then $a^2 = b^2$
k) If $x > 1$ and $y > 1$ then $x + y > 2$

Find the converse of each of the above statements (so, for example: 'If $x \le 3$ then $x < 3$.' becomes 'If $x < 3$ then $x \le 3$.') and then determine if the converses are true or not.

___

Are the following statements true or false? If a statement is false, give a specific example showing why it is not true:
a) If xy=1 then x = 2 and y=$\frac{1}{2}$.
b) If xy=0 then x=0 or y=0
c) If xy=0 then x=0 and y=0
d) If $xy \ge 0$ then $x \ge 0$ and $y \ge 0$
e) If $xy \ge 0$ and $x \ge 0$ then $y \ge 0$
f) If $xy > 0$ then both x and y are positive or both x and y are negative.

___

Determine which of the following numbers $-6, -4\frac{10}{11}, 7, 0, 6, 5.999, -3$

satisfies the following condition:
    $x < -3$ or $x \ge 6$.
Repeat the same exercise for the following condition:
    $x < -3$ and $x \ge 6$

___

List five numbers that satisfy the following condition:
    $x > 0$ and $x < 2$
Would you be able to list more numbers like that? How many are there?

___

Determine which of the following statements are true. Why? If a statement is not true, give a specific number showing why it is not true:
a) If $x = 5.256$ then $x \le 5.257$ and $x > 5.25$

b) If $x = -2$ then $x \le -2$ or $x = 5$

c) If $x > 2$ then $x > 1$ and $x < 3$.

d) If $x > 2$ then $x > 1$ or $x < 3$.

Graph the following sets on a real number line:

a) $x \le \dfrac{2}{5}$ or $x \ge 3$

b) $-1 < x$ and $x < 1\dfrac{2}{3}$ (or equivalently denoted by $-1 < x < 1\dfrac{2}{3}$)

I. Operations on power expressions with positive integer exponents

Students should:

a) know that $a^0 = 1$ and recognize that $a^1 = a$

b) understand why

     If n and m are positive integers then

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

$$a^n a^m = a^{n+m}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$(a^n)^m = a^{nm}.$$

(we can show students how to derive the above properties, tell them that this is an example of a mathematical theorem and that mathematics is about proving theorems).

c) have the practical ability to use the above properties to simplify exponential expressions

Examples of problems:

Below is an example of mathematical theorem:

Pythagorean Theorem:

If c is equal to the length of the hypotenuse of a right-angled triangle and a and b are the remaining legs of the triangle, then

$$a^2 + b^2 = c^2.$$

Suppose that you know the length of two sides of a triangle, can you always use the Pythagorean Theorem to find the length of the third one?

Find the length of the hypotenuse of a right-angled triangle, if you know that two other sides are 4 in. and 3 in long.

Simplify:

$$(ab)^0 = \qquad ab^0 = \qquad (0.1x)^3 \cdot \frac{x^5}{x^2} = \qquad \frac{(a+3b)^6}{2(a+3b)^5} =$$

$$x^7 x^3 = \qquad \frac{s^{12}}{2s^3} = \qquad \frac{(a^5)^3 x^2 a}{xa^4} = \qquad 2y\left(\frac{2}{3}x\right)^2 x^7 (-y)^5 =$$

J. Manipulation on symbols: addition, subtraction, multiplication. The Distributive Property. Factorization. Simplification of rational expressions.

Students should:
a) be able to recognize like terms (in particular students should remember that xy=yx and be able to apply the previously introduced rules for exponential expressions to recognize like terms, like for example, $x^3 yx$ and yx^4).
b) be able to simplify expressions by combining like terms
c) be able to state the distributive property and know why the distributive property implies that $(a+b)(c+d) = ac + ad + bc + bd$, $a(b+c+d) = ab + ac + ad$ and so on.
d) be able to apply the distributive property to multiply algebraic expressions
e) be able to apply the distributive property 'in reverse' to factor expressions
f) be able to factor -1 (or any other number) from any expression

g) understand why, for example, $\frac{x + yx}{x} \neq yx$ and why factorization can be of help in the

process of simplification of rational expressions
h) be able to write their solution using correct mathematical language; be aware of the use 'equals can be substituted for equals'

Examples of problems:

Simplify:
$$3a - \frac{2}{3}a =$$

$$-\frac{0.25}{0.005}x + x =$$

$$-3y + \frac{1}{2}x - x + 4y =$$

$$-\frac{3x}{2} + \frac{1}{2}x =$$

$$x + 4y^3 - \frac{1}{3} - y^3 - 7x =$$

$$\frac{5x^3 x}{-2} - \frac{5}{2}x^4 =$$

$$\frac{3}{20}ab - \frac{7}{10}ba + 2\frac{3}{5} + 3\frac{1}{10} =$$

$$2x^3 yx^7 + xy - yx^{10} - 0.1yx =$$

$$xyz + zyx + \frac{-7}{8}xy - 2yxz + \frac{7yx}{8} =$$

$$3\sqrt{x} - 4\sqrt{x} =$$

Perform the indicated operations:

$$3x^3(\frac{2}{3}x - x^4 - 1) =$$

$$(2x^3 y + \frac{3}{7} - xy^4 - 4x^5)xy =$$

$$-\frac{2}{3}(3c - 33d) =$$

$$(2x - 1)^2 =$$

$$(3ab - \frac{2}{5}b)(10 - a^3 b) =$$

$$(a + b - g)(c - d - f) =$$

$$(2a - 1)(1 - 3a)(a - 2) =$$

Remove parentheses:

$$-(a + bc - 7.45)$$

$$-[-(3x - 1) - (-y)]$$

Factor:

a) $10x - 5x^2 =$

b) $3x^2 y - 6xy^{14} + 9zxy =$

c) $a^3 b^4 + 5b^3 a^7 - a^3 b^3 =$

d) $3(x-1)^4 + 27(x-1)^2 =$

e) $4\sqrt{x} - \dfrac{y\sqrt{x}}{3} =$

Factor out -1 from:

a) $3x^2 - \sqrt{2}x =$

b) $-y + \dfrac{4z}{5} + 1 =$

Factor out the number 2 from:

$4a + \dfrac{b}{2} - \dfrac{4c^2}{5} =$

Simplify:

$\dfrac{5xy^4}{20y^3} =$

$\dfrac{xy + xz}{3x} =$

$\dfrac{a^3 bc^2 - 4b^4 a^4 c}{abc} =$

$\dfrac{x - 2z}{2z - x} =$

$\dfrac{15(2a-3)^6 a^7}{(2a-3)^4} =$

$\dfrac{-c}{ac^2 - cb + c\sqrt{d}} =$

$$\frac{\frac{3}{4}x^2 - \frac{1}{4}x}{3x - 1} =$$

A student was supposed to simplify the following expression: $\frac{x-y}{y-x}$. He attempted to multiply the numerator by -1 and thus he got $\frac{(-1)(x-y)}{y-x} = \frac{y-x}{y-x} = 1$. Find a mistake in this reasoning. What is the correct answer?

Simplify the following expressions:

$$\frac{4x - 5x}{x} =$$

$$\frac{4x(-5x)}{x} =$$

$$(x - \frac{2}{3})(x + \frac{2}{3}) - x^2 =$$

$$4[3(x-2) - 1] - [2(1-x) - x] =$$

$$x - \frac{1 - 2x}{3} =$$

$$3x - (2x - 1)(\frac{1}{2} - x) =$$

$$-(3\frac{2}{7} - a^2b^3) - 2(ba^2b^2 - \frac{1}{4}) =$$

$$\frac{-2c^2 + 4f^3}{c^2 - 2f^3} =$$

$$\left(\frac{2}{5}x - 1\frac{3}{5}\right)^2 - (x - 2)^2 =$$

If you take any number, multiply it by 4, add 2 to it, divide by 4 and from the obtained result subtract the number you used at the very beginning the result will be always $\frac{1}{2}$. Show why

it is so (hint: call your number x, write an algebraic expression corresponding to operations you perform on your number, simplify this algebraic expression, what do you get?)
Try to come up with a similar 'number trick'.

_____**G**.

Evaluations of algebraic expressions. Substitution of not only numbers but algebraic expressions.

Students should:
  d) Recall the symmetric and transitive properties of equality
  e) be able to evaluate any algebraic expression
  f) be able to substitute an algebraic expression for any given variable and then simplify the resulting expression

<u>Examples of problems</u>:
_____

Evaluate

$-x^2 + \dfrac{1}{2}(y - x)$ if $x = -1$ and $y = \dfrac{1}{2}$

_____

Rewrite the $(a - b)(a^2 + ab + b^2)$ in terms of x, if you know that $a = 2x, b = -1$.

Repeat the exercise when $a = x^2, b = \dfrac{x}{2}$

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Simplify the expression $ax + a^2 x^2$ if $a = \dfrac{1}{x}$.

_____

Simplify the expression $\dfrac{-xy + x^3}{2y}$ if $y = x^2$

_____

If $a - c = 1\dfrac{2}{3}$ and $b + d = -2$, what is the value of $(a - c)(b + d)$? What is the value of $a - c - b - d$?

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aluate (a-b)c if $ca = 2$ and $cb = -1$.

_____

H. Recognizing and matching patterns (primarily to recognize that a certain formula could be applied at a given situation).

Students should:
be able to rewrite expressions according to prescribed rules (usually with the goal of making it match some given pattern or expressing information in some canonical form) and

recognize the value of appropriate coefficients in their representation (for example, if we represent $5x+3$ in the form of $ax+b$, they should know that a=5).

Examples of problems:

The following expression $\dfrac{x^2}{4} - \dfrac{y^2}{5} = 1$ is written in the form:

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, where $a > 0$ and $b > 0$.

Find the value of a and b.

Write the following expressions in the above form and determine the value of a and b.

a) $\dfrac{x^2}{25} - \dfrac{2y^2}{6} = 1$

b) $x^2 - y^2 = 1$

c) $3x^2 - \dfrac{1}{4}y^2 = 1$

Write the following expressions in the form $mx+b$, where m and b are any real numbers. Find the value of m and b in your representation:

a) $-3x + 7 - x$

b) $\dfrac{4}{7}(-1 - 14x) - \dfrac{x-1}{2}$

c) $-(x+2) + x$

Write the following expressions in the form a^2, where a is any expression. In each case state what a is equal to.

$4x^2$, $\quad 3$, $\quad 5y^2$, $\quad \dfrac{9}{31}x^{18}$, $\quad (2b-1)^4$, $\quad 7x^4y^6$

Write the following expressions in the form

$\dfrac{numerator}{2}$, where *numerator* is any algebraic expression:

a) 5, b) $3x$,   c) $\dfrac{5xy}{2} - \dfrac{3yx}{2}$,   d) $-10a - 8a - a$

In each case, determine what *numerator* is equal to.

Write the following expression in the form $-\dfrac{a}{b}$:

$$\frac{3x+4}{2x+1}$$

Write the following expressions in the form $y = ax^2 + bx + c$. In each case determine the value of a, b, and c.

a) $y = \left(x - \dfrac{4}{5}\right)^2$,   b) $y = \dfrac{-2x^2 - 3}{4} - \left(x + \dfrac{x^2}{2} + 2\right)$,   c) y=4

It can be shown that
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2).$$
Use this formula to write $8x^3 - 1$ in the form of the product of two expressions (hint: write $8x^3 - 1$ in the form $a^3 - b^3$; determine what a and b are equal to in your representation and then substitute these expressions in $(a - b)(a^2 + ab + b^2)$ for a and b respectively).

I. Generalities on equations, and a Linear equation in one variable as a particular type of equation.

Students should:
a) be able to recognize the difference between algebraic expression and equation
b) know what it means to solve an equation (any equation)
c) be able to solve any linear equation and know that any linear equation in one variable can have no solution, exactly one solution or can be in the form of identity (understand the concept of identity)
d) be able to solve any linear equation with non-numerical coefficients.

Examples of problems:

Determine whether the following represent an equation or an algebraic expression. If it is an equation, try to find a solution (any solution). If you know one solution of an equation, can you say that you "solved" the equation? Why or why not?
a) $3x - 4x$,
b) $3x - 4x = -1$,
c) $x^2 = 36$,
d) $|x| = -6$,
e) $\dfrac{x}{2} + \dfrac{x}{3} - 1$
f) $-(-(-x)) = -4$

Determine which of the following numbers is a solution of the equation $3x^4 - 5x^2 = -2$:

$$-1, \frac{-2}{3}, \frac{2}{3}, -2, 1$$

Solve for x:

$$\frac{x}{3} = 6 \quad -x = 5 \quad \frac{1}{4}(2x-5) = -\frac{x}{2} \quad -(x+\frac{1}{2}) = 3-x \quad \frac{6x-3}{5} - 1 = -2 + 6(\frac{1}{5}x + \frac{1}{10})$$

Solve for m:

a) $abm + a = F$ b) $\dfrac{m\sqrt{5}}{k} = 2m - s$ c) $(m-t)(t^3 - 2t^5) = 0$

J . Equations in two variables. Cartesian coordinate system. Graphs of linear equations, as a particular example of equations in two variables.

Students should:

    h.   understand the meaning of the solution of any equation in two variables.
    i.   be able determine if a given pair of numbers is a solution of an equation
    j.   be familiar with the Cartesian coordinate system (x-axis, y-axis, quadrants) and be able to plot any points on it.
    k.   understand how the graph of any equation in two variables can be obtained
    l.   be able to define (and recognize) a linear equation in two variables
    m. be able to graph any linear equation and know that only linear equations have graphs that are lines.

n.   Examples of problems:

Determine if the following are solutions of the equation: $y = \left(-\dfrac{1}{2}\right)^x$

$$(0, 1), \quad (2, -\frac{1}{4}), \quad (3, -\frac{1}{8})$$

Find two other solutions of this equation. How many are there?

Determine if the following represents an equation or algebraic expression. If it is an equation find three solutions:

a) $3x - y = 4$

b) $x^2 - y^2 = 0$

c) $-\sqrt{4x} + 5 - y$

d) $x = -y$

How many solutions does the equation $|x|+|y|=-4$ have? Can you come up with another equation that has the same number of solutions?

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Find two solutions of the following equation:
$$x = y + z$$
Find any equation (there are many correct answers) such that one of its solutions is $(1,.1,1,1)$.

_____

The point $(t, \sqrt{2})$ is a solution of the equation $y = -x + 1$. Find t.

_____

Find such s that $(s, -s)$ is a solution of $x = 3y + 2$

_____

Show that for any real number s, the following statements are true:
    a) (s, s) is a solution of the equation $x - y = 0$

    b) $\left(\dfrac{s}{2}, 1-s\right)$ is a solution of the equation $4x - y = -3y - 4$

_____

Determine which of the following statements are true?

a) A solution of the equation $|3x| = y$ is $(\dfrac{1}{3}, 1)$ and a solution of the equation $|3x| = 1$ is

$(-\dfrac{1}{3}, 1)$.

b) If (s, t) is a solution of $x + 3 = y$ then (s, t) is a solution of $x + 5 = y + 2$

c) If (s, t) is a solution of $\dfrac{x+2}{2} = y$ then (s, t) is a solution of $x = y$

_____

Plot the following points:
$$\left(-1, \frac{3}{4}\right), \quad \left(\frac{3}{4}, -1\right), \quad \left(\sqrt{12}, 2\frac{3}{5}\right), \quad \left(-\frac{2}{3}, 100\right)$$

_____

In which quadrants do the following points lie?
    $\left(-3, 5\sqrt{8}\right), \quad (t^2, -1), \quad (|t+1|, -|t|)$, assuming that $t \neq 0$.

_____

Find t, if you know that the following point is located on the x-axis: $(3, 4t - 1)$.

_____

Match the following graphs with equations:
 (several graphs, not only of linear equations, would be given. Matching has to be done by
    reading coordinates of some points and checking if those are solutions of equations)

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Determine if the following statements are true or false:
a) If (x,y) lies in the first quadrant then x>0 or y<0.
b) If (x,y) lies in the third quadrant then x<0 and y<0.
c) If x>0 then (x,y) lies in the first quadrant or the second one.
d) If x>0 and y>x then (x,y) lies in the first quadrant.

_____

Express the following linear equation in its standard form ($ax + by + c = 0$).
$$3x - 8 = y + 5$$
Determine the values of a, b, and c in your representation. Can you rewrite your equation in such way that it is also in the form $ax + by + c = 0$, but the values of a, b, and c are different? Can you rewrite your equation again, so $a = 1$, $a = 2$ (two different representations)

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Determine if the following equations are linear. If so, express it in the form $ax + by + c = 0$.
Determine the values of a, b, and c in your representation:

a) $\dfrac{3x - y}{7} - x = 1$

b) $3x^2 - y = 0$

c) $y = 5$

d) $|3x + y + 1| = 0$

e) $3^x + 2yx = 1$

_____

Graph the following equations:
$$y = 4x + 5 \qquad 3(x - 1) = \frac{5}{6}y \qquad y = 2\frac{1}{7} \qquad x - 8 = 0$$

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