Community College of Philadelphia Mathematics Department

Report on the Pilot Project Spring and Fall 2007

The Committee on Elementary Mathematics and Its Effect on the Curriculum (CEMEC)

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Background

Over the past thirty years, the Mathematics Department and the College Administration have been grappling with the teaching of elementary mathematics. There has often been a dichotomy in the perception of the problem. Members of the Mathematics Department have focused on questions of how we can get students to better understand the mathematics that we are teaching and be better prepared for future courses and for life. Members of the College community outside the Mathematics Department have focused on improving perceived shortcomings in student outcomes such as pass rates. The distinction is subtle but significant. While members of the administration have been concerned with whether pass rates in a course are 55% or 60%, members of the Mathematics Department have been more concerned with what the 55% who passed have actually learned. During these 30 years the Mathematics Department has embarked on a number of different attempts to improve both what students are learning and the outward markers of this, student outcomes. These attempts have run the gamut of proposed solutions that have been around in the mathematics education community, some for more than 40 years, from self-paced mastery learning modules to computer delivered packaged teaching/learning systems. In the early 1980s, under intense pressure from the College administration to increase pass rates in elementary mathematics courses, the Mathematics Department learned that this is not hard to do. By basically giving many short tests, immediately following short instruction units, and allowing multiple retests, the Mathematics Department was able quickly to raise pass rates in elementary mathematics courses to a level deemed acceptable by the Division Director (Dean) and Vice President for Academic Affairs at the time. The fact that, 4 weeks later, the students couldn't pass the same tests that they had passed immediately following instruction seemed to bother only those in the Mathematics Department who were concerned with what students had actually learned. Although we have tried almost every "solution" that has come down the pike, the Mathematics Department has looked with a jaundiced eye at each new package or "solution" which promises to improve student outcomes. The Mathematics Department has focused on what we ask students to learn and why they have such difficulty learning it.

Earlier in this decade, two members of the Mathematics Department, Margaret Wojcicka-Hitczenko and Wimayra Luy, documented a specific problem in students' mathematical education: that students throughout the curriculum had fundamental misunderstandings of basic arithmetic and algebra. They proposed (to the Mathematics Department) an extensive plan to revise the mathematics curriculum at Community College of Philadelphia. While the specifics and merits of this proposal were being considered and debated in the Mathematics Department, the Vice President for Academic Affairs challenged the Mathematics Department to develop a plan to improve student outcomes in developmental mathematics courses. In May 2005, the Mathematics Department submitted a document, Proposal Concerning Elementary Mathematics and its Effects on the Curriculum, to the Vice President for Academic Affairs. This proposal laid out a comprehensive plan, based on local and national research, to revise the mathematics curriculum at Community College of Philadelphia. Some of the major recommendations of this proposal were (for a full presentation see the Proposal, p 9-24):

- Revision of the material covered in Math 016 and Math 017 to
 - adopt the principle that fluency in arithmetic is the gateway to higher mathematics
 - emphasize the understanding of concepts
 - reduce the range of topics, but deepen and broaden the ones that remain
 - emphasize the importance of mathematically correct language
 - expose students to the fundamentals of logic
 - eliminate the use of calculators in arithmetic
- Increasing instruction time in elementary mathematics classes from 3 to 4 hours
- Development of appropriate technology and its incorporation in instruction
- Revision of the Mathematics Placement Test to
 - establish a minimum threshold for students entering Math 016
 - have questions correspond more closely to course placement levels
- Progress towards the attainment of uniformity across sections of the same course by the
 - development of uniform assessment standards and recommendations on their use by instructors
 - design and administration of common exams
- Development of gateway test for entrance into college level mathematics courses
- Development of a new course to help students pass the gateway test

The Vice President for Academic Affairs proposed that the Mathematics Department run a two semester pilot testing a few key ideas in the larger proposal (to be selected by the Mathematics Department). The Mathematics Department committee that was established to design and run the pilot was called the Committee on Elementary Mathematics and its Effects on the Curriculum (CEMEC). CEMEC proposed an ambitious project that included many, but not all, of the key ideas of the larger proposal. Specifically, the pilot project proposed developing new materials for Math 016 and Math 017 that would incorporate the philosophical approach outlined above, increasing instruction time, establishing a threshold for Math 016, using a common syllabus, common exams and common assessment of students.

Implementation

The pilot project was implemented according to the following, very broad time line:

Summer and Fall 2006 – development of Math 016 and Math 017 instructional materials, Arithmetic Test and Threshold Test Spring 2007 - first semester of instruction using new materials Summer 2007 - revision of instructional materials Fall 2007 – second semester of instruction using new materials Spring 2008 - collection and analysis of data and evaluation of pilot project

Perhaps the most ambitious part of the pilot proposal was the development of new instructional materials for Math 016 and Math 017 that would incorporate the principles listed in the previous section. These principles were in accordance with the national research most notably the recommendations of the Fordham Foundation Report released in 2005. These materials were initially developed in Summer and Fall 2006 for use starting in Spring 2007. The Math 016 materials were principally written by Wimayra Luy and the Math 017 materials by Margaret Wojcicka-Hitczenko. The complete materials developed are included in Appendices E and F, but a few examples of how these ideas were implemented might be instructive:

Adopting the principle that fluency in arithmetic is the gateway to higher mathematics:

In the material students are constantly reminded that algebra is an abstract generalization of arithmetic, where numbers are 'replaced' with variables. Any new rules or procedures introduced in Math 017 are presented with their counterparts in arithmetic. To give an example, the explanation of how to simplify any algebraic fraction is preceded by the reminder of how to simplify a numerical fraction:

Recall the operation of simplifying fractions:

$$\frac{15}{25} = \frac{5 \times 3}{5 \times 5} = \frac{\cancel{5} \times 3}{\cancel{5} \times 5} = \frac{3}{5}$$

To simplify this fraction we divide the numerator and denominator by their common factor 5. We 'cancel 5'.

To simplify an algebraic fraction divide the numerator and denominator by all of their common factors. For example:

Consider

$$\frac{3a(b+c)}{ad}, \qquad ad \neq 0$$

Since a is the common factor of the numerator and the denominator, we divide both, the numerator and the denominator, by a. We "cancel a".

Emphasizing the understanding of concepts:

In most of traditional texts, students are usually asked questions that are a repetition of the same idea. After solving two or three of such exercises, students solve them mechanically, without even thinking. Although a certain number of routine questions are needed, there should also be plenty of non-rote exercises. Such exercises do not have to be difficult, but they must lead students to deeper analysis of the studied concept. For example, in the exercise:

Write using exponential notation whenever it is possible:

k)
$$\frac{-z-z-z}{zzzz}$$
 I) $\frac{(-z)(-z)(-z)}{z+z+z}$

students must not only recognize the operations that are performed on the variable z but also recall that exponential notation applies only to multiplication. Each time a student decides whether exponential notation can be used, he reinforces his understanding of this concept.

Reduction of the number of topics, but deepening and broadening the ones that are taught:

All sections covering new topics include exercises that require some deeper understanding of the subject. For example, in the section covering linear equations, one can find the following exercise:

Let P = 3(x-1), Q = 4x+5. Find x such that the following is true:

a)
$$P = Q$$
 b) $P = -Q$

Or in the section on linear inequalities:

If n > 3, what inequality is true for

a) n+7 b) n-5 c) -2n

Teaching mathematically correct language:

It is absolutely essential that students learn correct mathematical language and use it with proper syntax. In the materials, there are many exercises aimed at reinforcing familiarity with proper use of mathematical language and symbols. For example, to be able to work on the following exercise:

Write the following expressions without using exponential notation:

c) $(-m)^3$ d) $-m^3$

students must understand the difference in meaning of the above two expressions caused by the use of the parentheses.

Another example is an explanation that can be found in the materials concerning a common misconception among students that equal sign (=) means 'the next step is'.

When evaluating $2\!\times\!10\!+\!1$, it is INCORRECT to write

 $2 \times 10 + 1 = 20 = 21$ (20 \neq 21)

Instead, one should write:

$$2 \times 10 + 1 = 20 + 1 = 21$$

Numbers or expressions not involved in the operation that is being carried out *must always be rewritten*. Equal sign means that the quantities on either side are equal.

Elimination of the use of calculators:

One of the findings of the Fordham Report was that early and inappropriate over-use of calculators without allowing adequate mastery of simple calculations by hand, probably hinders the development of a sense of numbers. Anecdotally, many math faculty report that students at all levels, when presented with a problem, will frequently reach for a calculator and start pushing buttons, as something to do, instead of thinking about the problem. The exercises included in the CEMEC materials emphasize concepts and use only relatively small numbers not requiring extensive computations, thus the use of calculators was eliminated from both Math 016 and Math 017.

Spring 2007

To prepare for the Spring 2007 semester four sections of Math 016 and four sections of Math 017 were designated as pilot CEMEC sections with an extra hour of class time (4 hours). Each of these 8 sections was paired with a section of the same course at the same time, designated as a pilot non-CEMEC section. Letters were sent to advisors and counselors explaining the pilot and a letter for prospective students was also sent to counselors and advisors. (See Appendix E.) Instructors were assigned to the 16 pilot sections (8 CEMEC and 8 non-CEMEC) who agreed to administer tests on the first and last day of class and for the CEMEC section, to use the CEMEC materials and cooperate throughout the semester with the pilot.

Math 016

Before the Spring 2007 semester started, two of the pilot Math 016 sections were cancelled due to low enrollment so that there remained 4 pilot CEMEC sections and 2 pilot non-CEMEC sections. On the first day of class, students who showed up were administered the Threshold Test. In each of the two remaining pairs of sections 27 students in each pair took the Threshold Test. The larger CEMEC plan includes using the Threshold Test to redirect students who are not ready for Math 016 to other programs. In order to simulate this situation, an attempt was made to only allow students who passed the Threshold Test into CEMEC sections. This attempt was only partly successful. Using a cut off of 10 out of 28, in one pair 24 students scored 10 or higher and of these, 18 were randomly assigned to the CEMEC section. The other 6 who scored 10 or higher, plus the 3 who scored below 10 and all other students (except 1) who did not come on the first day of class were assigned to the non-CEMEC section. Another untested student joined the CEMEC section after the first day. In the other pair, 22 students scored 10 or higher and of these 19 were randomly assigned to the CEMEC section. The other 3 who scored 10 or higher, plus the 5 who scored below 10 and all other students (except 1) who did not come on the first day of class were assigned to the non-CEMEC section. Another untested student joined the CEMEC section after the first day. In the two unpaired CEMEC sections, 23 students took the Threshold Test. Of these, 13 scored 10 or higher. These 13 students, plus the 10 who scored below 10 and all other students who did not come on the first day of class remained in the unpaired CEMEC sections.

Before the semester and periodically during the semester, CEMEC met with the instructors teaching the CEMEC sections to prepare a common syllabus, discuss the progress of the students, prepare common exams and to suggest improvements to the materials. Students in the CEMEC sections were given common exams that were graded by the instructors using a common rubric, and they were given the Arithmetic Test as a common final exam at the end of the semester. Students in the non-CEMEC pilot sections of Math 016 were also given the Arithmetic Test at the end of the semester.

Math 017

In the 4 pairs of pilot sections (4 CEMEC and 4 non-CEMEC), students who showed up on the first day of class were given the Arithmetic Test. Since the CEMEC 017 materials did not begin with an extensive review of arithmetic, unlike the traditional sections of Math 017, it was felt that the students who had a stronger background in arithmetic would have a better chance for success using the more challenging CEMEC 017 materials. Therefore, the students with the highest 20 scores were to be assigned to the pilot sections. In the first pair, 31 students took the Arithmetic Test. Of these, 19 students with scores from 6 to 17 were assigned to the CEMEC section and 12 students with scores from 1 to 8 were assigned to the non-CEMEC section along with 3 students who did not come on the first day. In the second pair, 29 students took the Arithmetic Test. Of these, 17 students with scores from 6 to 26 were assigned to the CEMEC section and 12 students with scores from 1 to 8 were assigned to the non-CEMEC section along with 6 students who did not come on the first day. In the third pair, 27 students took the Arithmetic Test. Of these, 20 students with scores from 6 to 22 were assigned to the CEMEC section and 7 students with scores from 1 to 5 were assigned to the non-CEMEC section along with 9 students who did not come on the first day. In the fourth pair, 31 students took the Arithmetic Test. Of these, 20 students with scores from 7 to 25 were assigned to the CEMEC section and 11 students with scores from 2 to 7 were assigned to the non-CEMEC section along with 9 students who did not come on the first day.

The Math 017 instructors met with CEMEC regularly during the semester to discuss the progress of the students, prepare common exams and reviews, and to suggest improvements to the materials. Students in the CEMEC sections were given common exams that were graded by the instructors using a common rubric, and they were given the Arithmetic Test as part of a common final exam at the end of the semester. Students in the non-CEMEC pilot sections of Math 017 were also given the Arithmetic Test at the end of the semester.

Summer 2007

Based on the feedback of instructors of the CEMEC sections in the Spring 2007, the materials for both Math 016 and Math 017 underwent significant revision. Syllabi, exercises, reviews and tests were all revised. The Arithmetic Test was also revised to be consistent with the revisions to the Math 016 materials.

Fall 2007

Similarly to what was done for the Spring 2007 semester, to prepare for the pilot in the Fall 2007 four sections of Math 016 and four sections of Math 017 were designated as pilot CEMEC sections with an extra hour of class time (4 hours). Each of these 8 sections was paired with a section of the same course at the same time, designated as a pilot non-CEMEC section. Again,

letters were sent to advisors and counselors explaining the pilot with a letter for prospective students. (See Appendix E.) Instructors were assigned to the 16 pilot sections (8 CEMEC and 8 non-CEMEC) who agreed to administer tests on the first and last day of class and for the CEMEC section, to use the CEMEC materials and cooperate throughout the semester with the pilot.

Math 016

In the 4 pairs of pilot sections (4 CEMEC and 4 non-CEMEC), students who showed up on the first day of class were given the Threshold Test. Using a cut off of 8 out of 25, in the first pair, 34 students took the Threshold Test. Of these, 29 scored 8 or higher, and of these 29, 17 were randomly selected for the CEMEC section. Another 3 untested students joined the CEMEC section after the first day. The other 12 students who scored 8 or higher, plus the 5 who scored below 8 were assigned to the non-CEMEC section along with 2 additional students who did not come on the first day. In the second pair, 33 students took the Threshold Test and all 33 scored 8 or higher. Of these 33, 16 were randomly selected for the CEMEC section. Another 3 untested students joined the CEMEC section after the first day. The remaining 17 students who scored 8 or higher were assigned to the non-CEMEC section. In the third pair, 18 students took the Threshold Test. Of these, 14 scored 8 or higher, and of these 14, 10 were randomly selected for the CEMEC section. Another 10 untested students joined the CEMEC section after the first day. The other 4 students who scored 8 or higher, plus the 4 who scored below 8 were assigned to the non-CEMEC section along with 8 additional students who did not come on the first day. In the fourth pair, 25 students took the Threshold Test. Of these, 21 scored 8 or higher, and of these 21, 16 were randomly selected for the CEMEC section. Another 4 untested students joined the CEMEC section after the first day. The other 5 students who scored 8 or higher, plus the 4 who scored below 8 were assigned to the non-CEMEC section along with 9 additional students who did not come on the first day.

As in the Spring 2007 semester, before the semester and periodically during the semester, CEMEC met with the instructors teaching the CEMEC sections to prepare a common syllabus, discuss the progress of the students, prepare common exams and to suggest improvements to the materials. Students in the CEMEC sections were given common exams that were graded by the instructors using a common rubric, and they were given the Arithmetic Test as a common final exam at the end of the semester. Students in the non-CEMEC pilot sections of Math 016 were also given the Arithmetic Test at the end of the semester.

Math 017

In the 4 pairs of pilot sections (4 CEMEC and 4 non-CEMEC), students who showed up on the first day of class were given the Arithmetic Test. As in the Spring 2007, the students with the highest 20 scores in each pair were to be assigned to the pilot sections. In the first pair, 32 students took the Arithmetic Test. Of these, 18 students with scores from 8 to 25 were assigned to the CEMEC section and 14 students with scores from 2 to 10 were assigned to the non-CEMEC section along with 6 students who did not come on the first day. In addition, one student who did not come on the first day ended up in the CEMEC section. In the second pair, 25 students took the Arithmetic Test. Of these, 20 students with scores from 7 to 22 were assigned to the CEMEC section and 5 students with scores from 3 to 7 were assigned to the non-CEMEC section along with 12 students who did not come on the first day. In the third pair, 28 students took the Arithmetic Test. Of these, 20 students with scores from 7 to 16 were assigned to the CEMEC section and 8 students with scores from 2 to 6 were assigned to the non-CEMEC section along with 11 students who did not come on the first day. In the fourth pair, 29 students took the Arithmetic Test. Of these, 19 students with scores from 6 to 17 were assigned to the CEMEC section and 9 students with scores from 1 to 6 were assigned to the non-CEMEC section along with 9 students who did not come on the first day.

The Arithmetic Test was also given on the first day, to students in 6 non-pilot sections to allow for comparison of results in CEMEC Math 017 with a group with comparable scores on the Arithmetic Test.

As in the Spring, the Math 017 instructors met with CEMEC regularly during the semester to discuss the progress of the students, prepare common exams and reviews, and to suggest improvements to the materials. Students in the CEMEC sections were given common exams that were graded by the instructors using a common rubric, and they were given the Arithmetic Test as part of a common final exam at the end of the semester. Students in the non-CEMEC pilot sections of Math 017 were also given the Arithmetic Test at the end of the semester.

<u>Results</u>

Grade distributions and pass rates were compared for CEMEC and Non-CEMEC sections of Math 016 and Math 017 in both semesters.



Spring 2007 Math 016 Grade Distribution



Non-CEMEC



Pass rates for students who finished the course (without Ws).

CEMEC



Non-CEMEC





Spring 2007 Math 017 Grade Distribution

Pass rates for students who finished the course (without Ws).









Fall 2007 Math 016 Grade Distribution



Non-CEMEC

Pass rates for students who finished the course (without Ws).



Non-CEMEC







Pass rates for students who finished the course (without Ws).



Non-CEMEC



Fall 2007 Math 017 Grade Distribution



Combined Spring and Fall 2007 Math 016 Grade Distribution

Pass rates for students who finished the course (without Ws).



CEMEC

Non-CEMEC





Combined Spring and Fall 2007 Math 017 Grade Distribution

Pass rates for students who finished the course (without Ws).







In the Spring 2007 semester, grade distributions and pass rates were compared for CEMEC and all other sections of Math 016 and Math 017 at the main campus.







All Other Main Campus

Pass rates for students who finished the course (without Ws).

CEMEC









Spring 2007 Math 017 Grade Distribution

Pass rates for students who finished the course (without Ws).





All Other Main Campus



In the Fall 2007 semester, grade distributions and pass rates were also compared for CEMEC Math 017 students and other Math 017 students, both Non-CEMEC and Non-Pilot, who scored comparably to CEMEC Math 017students (scores above 7) on the Arithmetic Test at the beginning of the semester.



Fall 2007 Math 017 Grade Distribution

Others with Comparable Arithmetic Test Scores





Pass rates for students who finished the course (without Ws).

CEMEC



Others with Comparable Arithmetic Test Scores



Students in Math 017 were given the Arithmetic Test at the beginning of the semester. The table contain scores for all Math 017 students who took the Arithmetic Test.

Arithmetic	Number
Test	of
Score	Students
1	8
2	12
3	21
4	20
5	23
6	29
7	25
8	39
9	26
10	19
11	20
12	16
13	16
14	13
15	14
16	8
17	4
18	2
19	2
20	3
21	1
22	4
23	1
24	1
25	4
26	1
28	1

333

Total

Math 017 Initial Arithmetic Test Scores (All Sections)

Students in Math 016 were given the Arithmetic Test at the end of the semester. Scores for students in CEMEC, Non-CEMEC and Non-Pilot sections of Math 016 were compared.

Math 016 End of Semester Arithmetic Test Performance

	Average	Median	Pass rate	
CEMEC	20.9	21.5	65.1%	
Non-CEMEC	11.2	8.5	19.0%	
Non-Pilot	9.4	8	11.1%	

Math 016 End of Semester Arithmetic Test Pass Rate



Course pass rates for the same sections were compared.



Math 016 Course Pass Rate

Students in Math 017 were given the Arithmetic Test at the end of the semester. Scores for students in CEMEC, Non-CEMEC and Non-Pilot sections of Math 017 were compared.

Math 017 End of Semester Arithmetic Test Performance

	Average	Median	Pass rate
CEMEC	20.3	21	62.0%
Non-CEMEC	15.6	15.5	20.0%
Non-Pilot	15.4	15	26.7%

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Math 017 End of Semester Arithmetic Test Pass Rate



	Percent of Passing Students Who Attempt Subsequent Course		Percent of Attempters Who Pass Subsequent Course		Percent of Passing Students Who Pass Subsequent Course	
	CEMEC	Non-CEMEC	CEMEC	Non-CEMEC	CEMEC	Non-CEMEC
Math 016	71.4%	73.3%	73.3%	72.7%	52.4%	53.3%
Math 017	47.6%	66.7%	80.0%	41.7%	38.1%	27.8%

Spring 2007 Passing Student Follow-up

course were compared for CEMEC and Non-CEMEC students.

Spring 2007 Pass Rate in Math 017 for Students Who Passed Math 016



Spring 2007 Pass Rate in Math 118 for Students Who Passed Math 017



Math 016 students were given the Threshold Test at the beginning of the semester. Using a score of 8 as a passing cut-off, pass rates and correlation between passing the Threshold Test and passing Math 016 were calculated.

Math 016 Threshold Test Score and Passing Rate

Threshold Test Score Range	Pass Rate
0-7	15.8%
8-25	53.9%

Math 016 Threshold Test Score and Passing Rate

r = .256

Conclusions

It is clear that pass rates (p. 14-20) for students in CEMEC sections were no better and in almost all cases worse than for students in the non-experimental sections. Since the CEMEC materials are more demanding and since the students in CEMEC sections were consistently held to higher standards than in most other sections, it is not surprising that pass rates in CEMEC sections are lower than in other sections. While passing standards vary substantially from instructor to instructor, students in the CEMEC sections were consistently held to the same high standards. Due to the high degree of variability, it is difficult to use these pass rates as a valid measure upon which to base reliable conclusions.

Performance on the Arithmetic Test given to Math 016 students at the end of the semester (p. 22) shows that students in CEMEC sections achieved much higher scores and pass rates. Unlike course pass rates, the Arithmetic Test applies the same standard to all students. As a measure of what students in these sections actually learned, the Arithmetic Test results are a strong endorsement of the CEMEC approach.

The results of the Arithmetic Test given to Math 017 students at the beginning of the semester (p.21) confirm that students are entering Math 017 without knowing arithmetic. (Only 4.8% of entering Math 017 students passed the Arithmetic Test.) Even though the material on the Arithmetic Test is not *explicitly* taught in Math 017, the CEMEC Math 017 materials repeatedly emphasize the connection of algebra to arithmetic. It is not surprising that the Arithmetic Test scores and pass rates for Math 017 students at the end of the course (p.23) are much higher in CEMEC sections than in non-CEMEC and non-Pilot sections.

The success of students in subsequent mathematics courses was always considered by CEMEC to be a prime indicator of the success of this approach. The results are mixed. (p.24) The success of Math 016 students in passing Math 017 is roughly the same for students in CEMEC and non-CEMEC sections. The success of Math 017 students in passing Math 118 is clearly better for students in CEMEC sections compared to students in non-CEMEC sections.

The Threshold Test results (p. 25) indicate that, although there was very low correlation between Threshold Test score and passing Math 016, students with scores 0-7 on the Threshold Test had much lower passing rate than students with scores 8-25.

The positive reports of faculty in the Pilot (Appendix A) provide qualitative evidence for the sustainability and possible expansion of the CEMEC plan. The materials and methods of CEMEC have been embraced by a diverse group of mathematics instructors with different histories, practices and assumptions.

Recommendations

The CEMEC Pilot Project has shown promise in accomplishing its primary goal: to have students who successfully complete Math 016 and Math 017 obtain a better understanding of Arithmetic and Algebra. Where it has fallen short is in getting a larger percentage of students to successfully complete these courses. It should be noted that the Pilot Project is a small part of a comprehensive larger plan and was "piloting" only a few aspects of the full proposal. As such, it should only be viewed as an initial indicator of what may be seen when the entire plan is implemented. For example, the Pilot Plan did not incorporate any use of technology. Also, due to the fact that the overwhelming majority of students entering Math 017 failed the Arithmetic Test, it was not possible to fill in all CEMEC sections with students with adequate knowledge of prerequisites. Having most students not ready for the material undercut the effectiveness of the CEMEC approach. The use of the improved Placement Test in the future should bring better results. CEMEC remains confident that in addressing the problem at its root, namely, students' (lack of) understanding of arithmetic and algebra, higher pass rates can be achieved. The major recommendation of CEMEC is that the Pilot Project be continued and expanded as follows:

- Continue to have at least four sections each of Math 016 and Math 017 taught using the CEMEC materials each Fall and Spring semester.
- Continue to revise and improve the Math 016 and Math 017 materials. A large volume of material was written in a relatively short time. The experience of instructors using the material will be invaluable in ongoing improvements.
- Keep four hours for CEMEC sections. If this is financially infeasible, explore with the Federation and Administration the possibility of increasing class size in these sections.
- Develop and pilot materials for Math 118.
- Encourage students who have completed CEMEC Math 016 and Math 017 sections to register for CEMEC sections in future courses. This will provide a continuity of instruction across courses and allow for a better assessment of the effectiveness of the CEMEC approach.
- Expand the administration of common exit exams to all sections of Math 016, Math 017 and Math 118, and eventually Math 161, Math 162, Math 171 and Math 172.
- Continue the regular meetings for pedagogical discussions by CEMEC instructors and invite non-CEMEC instructors to participate in the discussions.

- Revise the Threshold Test and explore other means of predicting student success such as looking at Threshold Test scores in conjunction with Reading and Writing Placement Test scores. Ultimately revise the Math Placement Test to include the Threshold Test and to better conform to the content of the mathematics curriculum. (It is noteworthy that the Pilot took place before the raising of cut-off levels for Math 017 on the Mathematics Placement Test (Spring 2008).
- Develop computerized homework for the CEMEC materials, and use the same platform for exit exams and placement testing (as outlined in the original CEMEC proposal).
- Continue to collect and analyze data on pass rates, exit exams and success in subsequent courses. More data is needed to fairly assess the effectiveness of the CEMEC approach. Small sample sizes and intangibles such as the two-week strike during Spring 2007 make it difficult to draw firm conclusions from the results of the Pilot Project.

Appendix A - Reports from Faculty

Connie Dauval

I appreciated very much the opportunity to participate in the Math Department pilot project and teach CEMEC Math 016 on Mondays and Wednesday evenings last Fall. It was an interesting experience for me. Once I gave the pre-test to the class, the students and I looked over the handout text and worked out a way to cover the material, because there was grumbling in the ranks about not understanding the wording. One student said, "Why don't they just say what they mean?" Others agreed. I had the distinct feeling that many would not read the material in preparation for homework, guizzes, and tests, in spite of the fact that it was free, and it was the text! So I made overhead transparencies of the whole text and projected the material, so we could go over it together in class. I translated phrases that they had difficulty understanding by demonstrating examples and homework problems on the blackboard. Several good students volunteered to show their classmates how and why they did their problems that way. Most of the times class moved along well except for sometimes when a student, who attended every class and who sat and quietly slept in the very front row, would wake up disoriented. The no calculator rule also had the effect of bringing out the creativity of some students during guizzes and testssome tried to use their watch or cell phone calculators by turning away from me or pretending to drop something. When I think about it, because of the formality of the text, developing an alternative way of teaching the CEMEC class was a good experience for me, but I am very concerned that only 6 of my remaining 16 students passed. In addition I discovered that one of the most gratifying aspects of teaching the CEMEC class was to meet with the three other instructors, Lilla Hudoba, John Majewicz, and Philip Kennerly on a monthly basis to discuss common issues, and to create and schedule exams. It was fascinating and helpful to see how they approached teaching different topics.

Lilla Hudoba

I taught a CEMEC 017 class in Spring 2007, and a CEMEC 016 class in Fall, 2007. Previously I taught traditional sections of both classes, and I have worked with students taking these courses in the Learning Lab. In both CEMEC classes I retained more students than in previous, traditional classes. With the weekly extra hour I had more time to monitor students' success individually. Students learned more about organizing their work and using proper signs and notation. The CEMEC 017 material is designed for students who already mastered arithmetic; since this was not the case in my 017 class, significant amount of time was taken away from algebra to discuss arithmetic issues. I lost some students who could not cope with learning two classes (016 and 017) at the same time. Also, the strike interrupted the Spring '07 semester; some students did not even return to my class after the strike. My CEMEC 016 class was more successful; mostly because we assumed much less previous knowledge, so the class was able to work on 016 material all the time.

Pluses:

- (a) The material for both classes was well-structured; included huge amount of problems for class work as well as homework. Many problems were comprehensive, including previously covered operations. Instead of solving same kind of problems over and over, CEMEC provided a large variety of problems, approaching the material from different angles.
- (b) The extra hour was extremely helpful, and provided opportunities for various class activities; for example having students working in groups, or discuss solutions at the board. It is critical to provide developmental students with more time on task.

Cons:

Large number of unprepared students in the CEMEC 017 class. Since the program proposed dramatic changes in these classes (new material, Threshold and Arithmetic Tests, eventually a new developmental class), it is desired to change Math 016 and 017 gradually over several semesters. It would benefit our students to receive more comprehensive education with the use of CEMEC material, which would definitely provide them a strong basic knowledge necessary for college level courses.

Phil Kenerley

I enjoyed teaching the Cemec 016 math class in Spring 2007 and Fall 2007. I normally teach the math 016 class using the Bittinger book. With the Cemec class having extra time, I was able to give a lot more quizzes to help the students learn the material. I liked the materials and methods that were used in the Cemec class. The students in my Cemec classes had good attitudes and seemed to be happy with the material. With the Cemec class, I spent a lot more time on order of operations, fractions, sign numbers, and using the equal sign correctly with every step of each problem. I believe that students that pass Cemec 016 class will have a advantage in 017 math because they should be very good with working with sign numbers. In the Cemec class, the students worked with sign numbers almost all semester. The only disadvantage with the Cemec 016 class in my opinion is that the class didn't cover certain topics like percents , proportions. Since teaching the Cemec 016 math classes , I have seen two of my former students Stephanie Desamot and Dung Nguyen and both have pass 017 math and are doing well.

Elena Koublanova

1. Which course(s) did you teach?

I taught Math 017 CEMEC section in Spring 08, and pilot Math 017 section in Fall 07

2. Previous experience teaching the course.

One or two regular sections of Math 017 every semester for 15 years

3. Differences with your previous experience. (Please include experience in Pilot non-CEMEC and non-pilot classes in case you taught these.

I did not see any difference between students' background in pilot and CEMEC sections. Also, cannot tell that there was difference in the final results.

4. Pluses.

More time for work in groups. Variety of problems is much wider than in textbooks. Course materials for in-class problems solving, homework and quizzes. Team work with other instructors teaching parallel CEMEC sections.

5. Minuses.

Course materials in the "theoretical" part: I don't think that suggested handouts present material clearly and logically, and could replace a good book (if such a thing exists).

6. Views on gradual adoption of these materials and methods as described in the CEMEC proposal.

I don't think that material is ready for global adoption within department. I think that this project has good potentials, but there is still a lot of work to be done.

7. Anything else you consider relevant.

It was difficult for me to teach this course since I could not feel it's logic. Course material contains interesting problems which one cannot find in a standard textbook. These problems still need to be adjusted for our audience. In my opinion, too much attention is paid to algebraic techniques some of which are difficult and not necessary for our students.

I also think that in this course we are trying to feed people everything they missed before, which is obviously impossible to do. I would suggest a mandatory sequence Math 016 – Math 017 for those students who were paced in developmental mathematics. Then we could teach arithmetic and a majority of "real life" applications in Math 016, and be more consistent in teaching algebra in Math 017.

In conclusion, I believe that CEMEC has done a tremendous work which should be continued by the department.

Wimayra Luy

I taught one of the CEMEC Math 016 classes in Spring 2007. Overall it was a good experience, despite the fact that the strike had a negative effect. Some aspects that I can mention are the following.

- (a) Students showed interest in reading the materials provided.
- (b) The extra hour allowed detailed presentation of the topics as well as enough time to practice and address any misconception that they brought from high school .
- (c) Compared to other Math 016 classes I taught in the past, students in the CEMEC class were more persistent. They seemed more motivated and determined to succeed in the class.
- (d) In general students had a better attitude, they showed understanding of what was expected from them and what things they were supposed to work on. In that sense the materials were very specific and provided the necessary guidance.
- (e) The fact that calculators were not allowed worked in favor of the students since they realized that they needed to understand how the operations worked. In my opinion this is a key factor for them to succeed in later Mathematics classes.

John Majewicz

I taught a CEMEC Math 017 in Spring 2007 and a CEMEC Math 016 in Fall 2008. I have a lot of experience teaching Math 017. It's a course I enjoy teaching very much. I have taught Math 016 numerous times as well. The additional hour each week made a huge difference. I found it much easier to have students working in groups on a regular basis, which I think, helped many students a lot. It seemed to me that the CEMEC Math 017 required students to be much more disciplined and mature. I didn't have much success because not many students were willing at all to make their best effort with the course. In my opinion, it had very little to do with the material. I think I just happened to be unlucky enough to end up with a group of particularly unmotivated and scholastically-immature students. I think I would have had similar results with the same group of students in a non-CEMEC Math 017. By contrast, my experience in Fall 2007 was much better. I ended up working with a group of about 11 very dedicated and very hard-working students who did whatever they were told. The materials were very well adapted to group work and the students for the most part used every little bit of their time in the classroom very wisely. It seemed that the amount of material was just enough to allow students to understand the process in which they were engaging. The materials contained just enough abstraction to be considered mathematically sound while at the same time serving as an important tool for enhancing students' comprehension. I think the 017 materials are also good, but the students seem to require a lot more coaching in order to be successful with them. I think there are FAR too many students who place into Math 017 who should not. Thus, there is a risk of having even more underprepared students in courses that use the CEMEC materials. I would like to see the materials adopted for general use. IN that case, there'd really need to be a huge effort to familiarize faculty-members, both FT and PT, with the materials before teaching the course. I highly commend the authors for the materials. It's obvious that a great deal of time, effort and talent were used in their creation.

Jose Mason

I have taught Math 017 ever since I was hired at CCP in 1976, at least one course per semester and most summers. Teaching 017 in the CEMEC pilot class in the fall of 2007 was a welcome change because the intention to address the most glaring weaknesses of the regular course was evident from the way topics were chosen and the presentation of the materials students received. The regular meetings of the team allowed for discussion and coordination.

I perceive this experience as a welcome effort to solve a problem that has haunted Math Departments the world over for the past eighty years at least. The materials do need discussion and revisions, but I feel that they are a good start to arrive at the goal of proposing a uniform approach to the course.

David Santos

I had the opportunity of teaching Maths 017 in the Fall of 2007 under the CEMEC programme. I have mixed feelings about the experience.

The experience was a positive one because:

- (a) The extra hour afforded more time to be dedicated to answering the questions of the students, students helping each other to clarify queries.
- (b) The streamlining of the material provided a focused and coherent approach to Algebra.
- (c) The common exams gave the students the sense that we all had a common cause, that I was coaching them to do well on the exams, rather than teaching to pass my exams.
- (d) It gave me satisfaction to know that I was doing more or less what everyone else was doing in their classes, and hence, there was no ambiguity about what the goal for the final product should be.

It was not a completely positive experience because:

- (a) Although the material was streamlined, there were some topics I would have liked to include, like long division and factoring of quadratic trinomials.
- (b) At one point there was, in my opinion, too much of a focus on in matching expressions to a given form.

My recommendations for the future:

- (a) Revise the notes, rearrange the topics.
- (b) Include the appendices of sample exams with the notes.
- (c) Change the file format fromWord to TeX, so that those not having compatible versions ofWord can read the files accurately.

Geoff Schulz

I have taught with the CEMEC materials 4 time, twice using the M016 materials developed by W Luy and twice with the M017 materials developed by M Hitczenko.

In the Spring 2007, I taught a M016 section using the CEMEC materials. This was the first iteration of the materials and my experience was a mixed one. While I enjoyed going through the materials with the students and emphasizing the structure and format of a good arithmetic problem and the processes needed to perform the indicated operations, I felt that there were not enough practice problems provided which would lead to a mastery of the topics presented. Teaching arithmetic topics to adults is always a challenge given that most feel they already have a grasp of what to do and a built in fear of those areas they have not been successful with in the past. Getting them to embrace a new approach (structured and mathematically correct) is often an uphill battle. Correct use of symbolism and the correct formatting of the problem in stages is a hallmark of the materials and it took diligence on my part to make sure the students were following those procedures and given adequate feedback to correct mistakes. Even though detailed solutions were provided, the students seemed reluctant to use them to guide themselves. Learning to do that is the keystone to learning to do mathematics well and begin to understand what is happening. I felt the order of the materials was excellent, beginning with the natural numbers, moving to integers (and its inclusion of negative numbers) and then going through rational numbers and decimal representations of numbers, always with an eye to the possibility that the answer could be negative.

Staying withM016 material for the moment, I taught with the 2nd iteration of the materials in the Spring 2008 and found a much improved version including many sets of practice problems. These provided more time on task with the materials helping to provide mastery of those who did the problems. Both semesters were afforded the luxury of a 4th hour, the Spring 2007 being built in with me as the instructor and in the Spring 2008, a built in Lab hour due to the fact that I used the materials with a Gateway to College class which provided a mandatory lab session once a week. That 4th hour in both semesters provided an opportunity to have the students work in groups and in collaborative activities under either my guidance or the lab instructor's guidance.

In the Fall 2007, I taught a M017 section using the CEMEC materials and I was delighted with the breadth and depth of the materials and the challenge to the students. Of course, getting the students to accept the challenge was the key. One student in particular made remarkable improvement working closely with the materials and regular math lab help. That student truly embraced the study of algebra and grew both in confidence and ability. There were plenty of exercises of both routine and non-routine nature which provided not only the students an opportunity to practice but lead to wonderful discussion of algebraic principles within the classroom. The 4th hour was invaluable in that collaborative activities and more time for questions and discussion was provided.

In the Spring 2008, I again used the M017 materials with a Gateway to College (GTC) M017 class which was seeded with regular students. The majority of the GTC students stayed with the class but all the other students dropped away. One of the hallmarks of the M017 CEMEC materials is that they do not begin with a review of Arithmetic but incorporate the Arithmetic of numbers within algebraic problems at the onset (evaluation of expressions) and the GTC students were all prepared for that. Those students came to the class (as all students should!) with a fundamental knowledge of working with integers, rationals (fractions) and decimals. Not that

some review was ignored but they didn't need to be "taught" those prerequisites. The discussions in class were focused on the algebraic principles and they did their homework. The results at the end of the semester were good to excellent averages and confidence in their ability to learn the mathematics. A small sample but success came to all who finished. This class did not have the 4th hour and though I deem the class a success, the extra hour per week would have been wonderful.

What each of these courses using these materials lack in breadth they more than make up in depth. There is plenty of materials to engage the students and more importantly, the emphasis on correct format cannot be undervalued in achieving a goal of increasing the understanding of mathematics within the students and improving their confidence that they can indeed learn mathematics and move forward.

I plan to continue to use these materials in my work with the M016 and M017 classes that I teach. Exams have been developed by a team of instructors working together and the two assessment tools (Threshold test and Arithmetic Test) can play a role in pre and post assessment of student's ability.

By using these materials I have come to realize that our role as instructors of these two courses is not to try and cover every topic under the sun but to give the students an opportunity to work with core topics of arithmetic and algebra using good structure and format to build their confidence for the future study of mathematics. Without that, many will be stymied in their attempt to achieve their educational objectives. I am not saying these materials are easier to work with or that all students embrace them. However, by using them and holding students accountable (never an easy task) we just might go a long way to improving our elementary mathematics offerings here at the Community College of Philadelphia and come together as a mathematics department. That was indeed the premise and the vision of CEMEC.

Kudos to Wimayra Luy and Margaret Hitczenko for the development and revision of the materials, to Gail Chaskes and Lilla Hudoba for their invaluable contributions to the materials and the committee, to Atish Bagchi for keeping the committee on task., and to Dan Jacobson for his unwavering support. And a very big thanks to all those faculty members who aided the committee in teaching sections using the materials, in teaching control sections and in allowing the gathering of data for the assessment of the project.

Yun Yoo

I taught CEMEC math 016 for spring 2007. It was an evening class on Mondays and Wednesdays between 6:00-8:00 pm. It was the first time I taught Math 016 at CCP. I felt that most of students could not follow the given CEMEC text and that they had a lot of difficulty in reading it. I think that more practice problems are needed for homework sets. Students were glad that they studied a small amount of material and that I concentrated on important matters like operating with integers and fractions. The extended hour was very helpful to go over all homework problems in the classroom. My class was two hours per day on Mondays and Wednesdays. I think that it was ineffective for students who studied basic mathematics. I think that we need to focus more on operating with real numbers in CEMEC Math 016 to make sure that students learn how to operate with numbers clearly.
Appendix B - Letters to counselors, advisors and students

Community College of Philadelphia

MATHEMATICS DEPARTMENT

MEMORANDUM

TO:	Academic Advisors
	Counselors
FROM:	Dan Jacobson, Math Dept Head Margaret Wojcicka, Developmental Math Pilot Project Coordinator
DATE: 2 Octo	ber 2006
SUBJECT:	Developmental Math Pilot Project

In the Spring 07 and Fall 07 the Mathematics Department is going to run a Developmental Math Pilot Project to assess the impact of proposed changes in the developmental mathematics courses (Math 016 and Math 017). Proposed changes include, among others, an increase in the hours of instruction per week, a new approach to the material, and the application of uniform criteria for assessment of students' work.

Please, watch for the following "special sections" offered in Spring 2007:

CRN	Course-Section	Time	Days	Room
11828	Math 017-015	9:05-10:20 am	MWF	M3-3B
11851	Math 017-046	9:05-10:00 am & 10:10-10:30 am	MWF	BR-71 & B2-36
11870	Math 017-179	10:30-11:45 am MWF	M3-3B	
11869	Math 017-178	11:15-12:10 pm & 10:45-11:05 am	MWF	M3-1 & B2-36
11821	Math 017-008	11:55-1:10 pm	MWF	M3-3B
11832	Math 017-019	12:20-1:15 pm & 11:50-12:10 pm	MWF	M3-16 & B2-36
11817	Math 017-002	6:45-8:35 pm	TR	M3-16
11831	Math 017-018	6:45-8:05 pm & 8:15-8:35 pm	TR	M3-11
11786	Math 016-190	9:05-10:20 am	MWF	M3-3A
11806	Math 016-164	9:05-10:00 am & 10:10-10:30 am	MWF, WF	MG-12 & B2-36
10104	Math 016-164	10:10-11:05 am	Μ	B2-36
11792	Math 016-009	10:30-11:45 am	MWF	M3-3A
11801	Math 016-018	11:15-12:10 pm & 10:45-11:05 am	MWF	MG-11 & B2-36
11802	Math 016-019	11:55-1:10 pm	MWF	M3-3A
11719	Math 016-016	12:20-1:15 pm & 11:50-12:10 pm	MWF	S2-12A & B2-36
11790	Math 016-007	6:00-7:50 pm	MW	M3-3A
13545	Math 016-003	6:00-7:50 pm	MW	B1-8

Note that the times for these courses are incorrect in the Spring 2007 Course Listing booklet.

All students registering in these sections will have 4 hours of instruction scheduled (instead of the usual 3), but only half of the students will be assigned to a 'pilot' section. Whether a student is assigned to a 'pilot' section or a 'regular' section will be determined after the first day of classes (based on results of a test administered during the first class period). Although a student's classroom may change, scheduled class times will remain the same for all students. Students assigned to a pilot section will have 4-hours of instruction per week with all required materials free of charge. Students assigned to a regular section will, in most cases, revert to a 3-hour schedule. No times will be added, but some time may be removed. Please note that students may not select or request either the pilot or regular section, but must be prepared to take whichever section they are assigned.

We believe that students who are assigned to a pilot section will benefit, not only because of the additional hour of instruction, but also because of the new presentation of the material. **Please encourage students to register for these classes.** There is **no** extra cost to students for the additional hour. At the same time, please be aware that pilot classes are **not** intended to be easier or harder than regular classes. The additional hour will be used to increase the understanding of presented concepts and deeper understanding will be expected from the students (which, we believe, will benefit them greatly in subsequent math classes). Please, do not assume that these sections are especially good for weak students or for stronger students. To assess the impact of the proposed changes we would like to see the **entire spectrum of students** in these classes.

Please provide students who enroll (or are considering enrolling) in any of these sections with a copy of the attached letter to prospective students. Thank you for your cooperation. If you have any questions or would like additional information about the Developmental Math Pilot Project, please feel free to contact us.

Dan Jacobson Math Dept Head Office W2-7F 215-751-8792 djacobson@ccp.edu Margaret Wojcicka Developmental Math Pilot Project Coordinator Office B1-9E 215-751-8943 <u>mwojcicka@ccp.edu</u>

Mathematics Department Pilot Study – Spring 2007

Math 016 and Math 017

To students enrolling in any of the following sections in Spring 2007:

CRN 11828	Math 017-015	CRN 11851	Math 017-046
CRN 11870	Math 017-179	CRN 11869	Math 017-178
CRN 11821	Math 017-008	CRN 11832	Math 017-019
CRN 11817	Math 017-002	CRN 11831	Math 017-018
CRN 11786	Math 016-190	CRN 11806	Math 016-164
CRN 11786 CRN 11792	Math 016-190 Math 016-009	CRN 11806 CRN 11801	Math 016-164 Math 016-018
CRN 11786 CRN 11792 CRN 11802	Math 016-190 Math 016-009 Math 016-019	CRN 11806 CRN 11801 CRN 11719	Math 016-164 Math 016-018 Math 016-016

The Mathematics Department is conducting a pilot study in Spring 2007 in special sections of Math 016 and Math 017. We are testing new materials and a new approach to teaching these courses. The new material covers fewer topics, but requires deeper understanding of the topics covered.

When you register for any of the sections listed above, you will be scheduled for 4 hours per week instead of the usual 3 hours. Not all students will receive 4 hours of instruction. After the first day of class students will be placed in one of two sections, a "pilot" section or a "regular" section, that meet at the same time. Your schedule will not change though your classroom may change. Students assigned to the pilot sections will have 4 hours of instruction per week (at no additional charge) and will receive all required course materials free of charge. Students assigned to a regular section will, in most cases, revert to a 3-hour schedule. No times will be added, but some time may be removed. Since a representative sample of students is needed for both sections, you cannot select or request either the pilot or regular section, but must be prepared to take whichever section you are assigned.

We encourage you to register for the sections listed above. We believe that students who are assigned to a pilot section will benefit, not only because of the additional hour of instruction, but also because of the new presentation of the material. Please be aware that the pilot classes are **not** intended to be easier or harder than the regular classes. The additional hour will be used to increase the understanding of presented concepts and deeper understanding will be expected from the students (which, we believe, will benefit them in subsequent math classes). There is no additional charge for registering for any of these sections.

Community College of Philadelphia

MATHEMATICS DEPARTMENT

<u>MEMORANDUM</u>

TO:	Academic Advisors Counselors
FROM:	Dan Jacobson, Math Dept Head Margaret Wojcicka, Developmental Math Pilot Project Coordinator
DATE:	16 February 2007
SUBJECT:	Developmental Math Pilot Project

In the Fall 2007, the Mathematics Department will be running the second semester of a Developmental Math Pilot Project to assess the impact of proposed changes in the developmental mathematics courses (Math 016 and Math 017). Proposed changes include, among others, an increase in the hours of instruction per week, a new approach to the material, and the application of uniform criteria for assessment of students' work.

Please, watch for the following "special sections" offered in Fall 2007:

CRN	Course-Section	Time	Days	Room
40754	Math 017-006	9:05-10:20 am	MWF	M3-3B
44137	Math 017-032	9:05-10:00 am & 10:10-10:30 am	MWF	BR-46 & B2-36
40680	Math 016-037	10:30-11:45 am	MWF	M3-3B
40650	Math 016-015	11:15-12:10 pm & 10:45-11:05 am	MWF	BR-24 & B2-36
40774	Math 017-012	11:55-1:10 pm	MWF	M3-3B
40891	Math 017-022	12:20-1:15 pm & 11:50-12:10 pm	MWF	B1-10 & B2-36
40592	Math 016-004	8:00-9:50 am	TR	M3-3A
40598	Math 016-005	8:00-9:20 am & 9:30-10:00 am	TR	B1-8 & B2-36
43419	Math 016-038	10:00-11:50 am	TR	M3-3A
40642	Math 016-013	9:30-10:50 am & 11:00-11:30 am	TR	M3-3B & B2-36
40893	Math 017-023	12:00-1:50 pm	TR	M3-3A
40906	Math 017-030	12:30-1:50 pm & 11:50-12:20 pm	TR	B1-7 & B2-36
40765	Math 017-009	5:15-7:05 pm	TR	M3-3B
44261	Math 017-059	12:20-1:15 pm & 11:50-12:10 pm	TR	S2-12A & B2-36
40582	Math 016-003	6:00-7:50 pm	MW	W3-39
40665	Math 016-021	5:15-6:35 pm & 6:45-7:15 pm	MW	M3-3A & B2-36

Note that the times for these courses are incorrect in the Fall 2007 Course Listing booklet, but they are correct in Banner.

All students registering in these sections will have 4 hours of instruction scheduled (instead of the usual 3), but only half of the students will be assigned to a 'pilot' section. Whether a student is assigned to a 'pilot' section or a 'regular' section will be determined after the first day of classes (based on results of a test administered during the first class period). Although a student's classroom may change, scheduled class times will remain the same for all students. Students assigned to a pilot section will have 4-hours of instruction per week with all required materials free of charge. Students assigned to a regular section will revert to a 3-hour schedule. No time will be added, but some time will be removed. Please note that students may not select or request either the pilot or regular section, but must be prepared to take whichever section they are assigned.

We believe that students who are assigned to a pilot section will benefit, not only because of the additional hour of instruction, but also because of the new presentation of the material. **Please encourage students to register for these classes.** There is **no extra cost** to students for the additional hour and **because the materials are free, the students will be saving on textbooks**. At the same time, please be aware that pilot classes are **not** intended to be easier or harder than regular classes. The additional hour will be used to increase the understanding of presented concepts and deeper understanding will be expected from the students (which, we believe, will benefit them greatly in subsequent math classes). Please, do not assume that these sections are especially good for weak students or for stronger students. To assess the impact of the proposed changes we would like to see the **entire spectrum of students** in these classes.

Please provide students who enroll (or are considering enrolling) in any of these sections with a copy of the attached letter to prospective students. Thank you for your cooperation. If you have any questions or would like additional information about the Developmental Math Pilot Project, please feel free to contact us.

Dan Jacobson Math Dept Head Office W2-7F 215-751-8792 djacobson@ccp.edu Margaret Wojcicka Developmental Math Pilot Project Coordinator Office B1-9E 215-751-8943 mwojcicka@ccp.edu

Mathematics Department Pilot Study – Fall 2007 Math 016 and Math 017

To students enrolling in any of the following sections in Fall 2007:

CRN 40754	Math 017-006	CRN 40582	Math 016-003
CRN 40765	Math 017-009	CRN 40592	Math 016-004
CRN 40774	Math 017-012	CRN 40598	Math 016-005
CRN 40891	Math 017-022	CRN 40642	Math 016-013
CRN 40893	Math 017-023	CRN 40650	Math 016-015
CRN 40906	Math 017-030	CRN 40665	Math 016-021
CRN 44137	Math 017-032	CRN 40680	Math 016-037
CRN 44261	Math 017-059	CRN 43419	Math 016-038

The Mathematics Department is conducting a pilot study in Fall 2007 in special sections of Math 016 and Math 017. We are testing new materials and a new approach to teaching these courses. The new material covers fewer topics, but requires deeper understanding of the topics covered.

When you register for any of the sections listed above, you will be scheduled for 4 hours per week instead of the usual 3 hours. Not all students will receive 4 hours of instruction. After the first day of class students will be placed in one of two sections, a "pilot" section or a "regular" section, that meet at the same time. Your schedule will not change though your classroom may change. Students assigned to the pilot sections will have 4 hours of instruction per week (at no additional charge) and will receive all required course materials free of charge. Students assigned to a regular section will revert to a 3-hour schedule. No time will be added, but some time will be removed. Since a representative sample of students is needed for both sections, you cannot select or request either the pilot or regular section, but must be prepared to take whichever section you are assigned.

We encourage you to register for the sections listed above. We believe that students who are assigned to a pilot section will benefit, not only because of the additional hour of instruction, but also because of the new presentation of the material. Also, because the materials are free, you will be saving money on textbooks. Please be aware that the pilot classes are **not** intended to be easier or harder than the regular classes. The additional hour will be used to increase the understanding of presented concepts and deeper understanding will be expected from the students (which, we believe, will benefit them in subsequent math classes). There is no additional charge for registering for any of these sections.

Appendix C - Math 016 Requirements

Examples of questions:	Description of a question	Number of questions
-3+5 = -7-1 = 3-10 =	Addition and subtraction of two integers (without parenthesis)	1
-3-(-2) =	Addition and subtraction of integers (with the use of parenthesis)	1
$(-1)(-4) = -2 \times (-1) =$	Multiplication of two integers	1
$\frac{-10}{2} = -10 \div 2 =$	Division of integers (using both: ÷ and fraction bar as a division sign):	1
$-\frac{2}{3}+\frac{4}{5}=$	Addition and subtraction of fractions (positive and negative)	2
$2 + \frac{3}{4} = 1 - \frac{4}{3} =$	Addition and subtraction of fractions and integers	1
$\frac{3}{5} \times (-10) =$	Multiplication of fractions and integers (positive and negative)	1
$-\frac{345}{5} \times \frac{1}{2} \times \frac{10}{345} =$	Multiplication of fractions (positive and negative)	1
$\frac{\frac{3}{4}}{\frac{1}{2}} = -\frac{3}{5} \div \frac{2}{7} =$	Division (using both ÷ and fraction bar) of fractions (positive and negative)	1
$\frac{\frac{3}{4}}{\frac{2}{2}} = -4 \div \frac{2}{7} =$	Division of (using both ÷ and fraction bar) fractions and integers (positive and negative)	1
$-2667 + 2667 = 0 \times 237 = 0 \div 23 =$	"Zero" category(number/0; 0/number; addition of opposite numbers; multiplication by 0)	1
$0.2 \times 100 = 0.3 \div 10 = 234 \times 1000 =$	"Ten" category: Multiplication and division of decimals and integers by powers of ten.	1
$7 + 2 \times 3 = 4 - (7 - 8) = \frac{5 - 10}{5} = -1 - 1(-1) =$	Order of operations, integers only, two operations (only integers as an answer, no exponents)	4
Which of the following numbers are equal to $-\frac{1}{2}$: a) $\frac{-1}{2}$ b) $1 \div (-2) =$ (*) see below for more explanation of this category.	Number matching:	3
uns calegory		

Arithmetic requirements for students completing Math 016:

Replace the blank with one of the	Comparison of	1
symbols:	decimals	
'<', ">', or '=" to make the following		
statement true: 0.8890.89		
-1.20.8		
Replace the blank with one of the	Comparison of fractions	1
symbols:		
'<', ">', or '=" to make the following		
statement true: $\frac{2}{4}$		
3 <u>5</u>		
7 8		
$-\frac{-3}{3}$		
0.1 - 2.1 = 0.1 + 0.999 =	Addition and subtraction of decimals (and	1
	integers)	
$-3 \times 0.02 = 0.1 \times 0.01 =$	Multiplication of decimals (and integers)	2
$2 = 0.33 \div (0.011) =$	Division of decimals (and integers)	2
$\frac{-0.2}{-0.2}$ - 0.35 ÷ (-0.011) -		
$(2)^2$	Exponential notation (fractions, integers,	2
$\left -1^{6} = (0.1)^{3} = - \left -\frac{2}{3} \right =$	decimals)	
	Addition and ashtas ation using mixed	2
$2\frac{1}{2}-1\frac{5}{2}=2\frac{5}{2}+5\frac{2}{2}=$	representation	2
3 6 5 7		
$-2\frac{1}{2} \times 1\frac{6}{2} = 1\frac{2}{2} \div (-1\frac{1}{2}) =$	Multiplication and division using mixed	1
	representation	
$-2 \times \frac{1}{2} - 2^{0} = (2-3)(-\frac{1}{2} - \frac{2}{2}) =$	Order of operation	3
2^{-2} $2^{}$ $(2^{-3})(-3^{-3})^{}$		
Which of the is equal to $-2-3+4$:	"Rewriting' Category	1
a) $-3+4-2$ b) $-2-(3+4)$		
(**) see below for more explanation		
of this category		
	Total number of questions:	36

Grading of the Exam:

Each question is worth 1 point. No partial credit will be given except in the following categories:

- Multiplication of fractions and integers (positive and negative)

- Multiplication of fractions (positive and negative)

- Division (using both ÷ and fraction bar) of fractions (positive and negative)

- Division of (using both \div and fraction bar) fractions and integers (positive and negative)

In the above categories half credit will be given if a student performs multiplication (division) correctly but does not have the right sign in his answer.

(*) The following will be included in Number Matching Category:

a) Equivalent fractions:
$$\frac{3}{5} = \frac{6}{10}$$

b) Mixed numbers and Improper fractions: $3\frac{1}{2} = \frac{7}{2}$

c)"minus sign" in fractions:
$$-\frac{1}{2} = \frac{-1}{2} = \frac{1}{-2}$$

d) Fraction as a division:
$$\frac{2}{3} = 2 \div 3$$

e) Integers and fractions: $2 = \frac{2}{1} = \frac{4}{2} = \dots$

f) Decimals and fractions:
$$\frac{3}{10} = 0.3$$
, $\frac{2}{5} = 0.4$

g) "zero' in decimals:
$$0.3 = .3 = .30 = ...$$

h) Opposite numbers: -(-2) = 2, -(-(-2)) = 2,...

(**) The following will be included in "Rewriting Category":

a) Rewriting expressions with proper use of parenthesis: Is (3×5)+2 equivalent to 3×5+2?
b) Rewriting expressions using the order of operations: Is -3+4 equivalent to 4-3? Is 34×78 = 78×34?
c) Rewriting expressions involving fractions: Is 2+7/2 = 2/2 + 7/2?

Is
$$-\frac{3}{3} = \frac{3}{3} + \frac{3}{3}$$

Is $2 \times \frac{3}{7} = \frac{2 \times 3}{7}$?

Appendix D - Arithmetic Test

Fall 2007

Name:_____

Section:_____

Instructor name:_____

Did you take Math 016?_____

If you did, when did you take it?_____

Community College of Philadelphia ARITHMETIC TEST

Work carefully. Show all your work on these sheets. Simplify as much as possible. Do not use calculators . Note: " \times " always means multiplication and "." always means decimal point.

 Perform the following operations if possible. If not possible write "undefined". Simplify as much as possible.

ANSWERS

(a) -4+1

(b) 3 - (-2)

(c) 3(-2)

(d)
$$\frac{15}{-3}$$

(f) 0.2×100

(g) $-2+4\times 2$

(h) $12 \div 6 \div 2$

(i) $\frac{0-3}{-3}$

(j)
$$-1 - 1(-1)$$

2. Compute and simplify.

(a)
$$-\frac{2}{3} + \frac{2}{5}$$

(b)
$$\frac{1}{6} - \frac{1}{9}$$

(c)
$$2 - \frac{5}{3}$$

(d)
$$3 \times \frac{5}{2}$$

(e)
$$\left(-\frac{321}{9}\right)\left(-\frac{6}{7}\right)\left(-\frac{14}{321}\right)$$

(f)
$$\frac{\frac{7}{5}}{-\frac{2}{25}}$$

(g)
$$-\frac{1}{5} \div 5$$

- 3. For each of the following decide if the expression is equal to $-\frac{4}{3}$. (Circle YES or NO for each of them.)
 - (a) $\frac{-4}{3}$ YES NO
 - (b) $4 \div (-3)$ YES NO
- 4. For each of the following decide if the expression is equal to $2\frac{3}{4}$. (Circle YES or NO for each of them.)
 - (a) $2 \times \frac{3}{4}$ YES NO
 - (b) $\frac{11}{4}$ YES NO
- 5. For each of the following decide if the expression is equal to $\frac{2}{5}$. (Circle YES or NO for each of them.)
 - (a) 0.4 YES NO
 - (b) 0.400 YES NO
- 6. For each of the following decide if the expression is equal to $-3 + 5 \times 6$. (Circle YES or NO for each of them.)
 - (a) $(-3+5) \times 6$ YES NO
 - (b) $5 \times 6 3$ YES NO

 Fill in the blank between the numbers using either '=', '<' or '>' as appropriate.

8. Fill in the blank between the numbers using either '=', '<' or '>' as appropriate.



 Perform the following operations if possible. If not possible write "undefined". Simplify as much as possible.

(a) $(-1)^4$

(b) $-5 + 10 \div 5 \times 2$

(c) 1.12 - 2.1

(d) -3×0.05

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(e) 0.1×0.01

(f)
$$\frac{2}{-0.2}$$

(g) $1.8 \div 0.01$

(h)
$$-2+3\left(-\frac{1}{3}-\frac{1}{3}\right)$$

(i)
$$2\frac{1}{4} - 1\frac{4}{5}$$

(j)
$$1\frac{1}{3} + 1\frac{5}{6}$$

(k) $2\frac{2}{3} \times \frac{9}{10}$

(l)
$$-\left(-\frac{2}{3}\right)^2$$

(m)
$$-\frac{1}{2} - \frac{2}{3} + \frac{1}{12}$$

<u> Appendix E – Threshold Test</u>

Mathematics Assessment 1 Fall 2007

Name:	
	J#:
	Class:
	Section:
Instructor name:	
Hε	we you taken Math 016 previously?

If you did, when did you take it? _____

Students: Thank you for working these exercises to the best of your ability.

1. Find the sum:	67 + 2008 + 49 + 5	
		1
2. Find the difference:	8003 - 568	
		2
3 Find the product:	67×0	
5. Phild the product.	07 × 9	
		3
4. Find the quotient:	1161 ÷ 27	

5. You put 54 marbles into 6 bags, ending up with the same number of marbles in each bag. How many marbles would be in each?

5_____

4_____

6. There are 1000 bags of sand on a truck. Each bag of sand weights 124 pounds. How many pounds of sand are on the truck?

6			

7. During the school year 3200 cans were sold in the cafeteria. There were 500 cans left unsold. How many cans of juice did the cafeteria have at the **beginning** of the school year?

8. A class is going to Sturbridge Village for the day. There are thirty-two children and 3 teachers attending the field trip. If each car holds five passengers, how many cars will be needed to transport everyone?

8_____

7___

9. What fraction of the large square shown below is shaded?



10. Which expression is **not** equal to 3×4 ? Please circle the one and record your choice.

a) 4 + 4 + 4
b) 4 × 3
c) 3 + 3 + 3 + 3

10_____

d) $4 \times 4 \times 4$

11. Susan has to read a book that is 72 pages long. She wants to read an equal number of pages each day. How many pages per day should she read, to read the entire book in 2 days?

11_____

12. Jamar and Jenna were baking cookies for two classes in their school. If each class has 26 students, how many cookies should they bake to give 3 cookies to each student?

12_____

13. What is the missing factor?

13 × ____ = 91

13_____

14. Bill is 42 inches tall and Tom is twice as tall as Bill. How tall are the two men together?

14_____ inches

15. What number makes the sentence true?

500 + ___ + 7 = 567

15_____

16. In a group of 50 people three out of every 10 own a CD player. How many people in this group own a CD player?

16_____

17. Mr. Smith is 42 years old. His son is 8 years old. The ages of Mr. Smith, his wife, and their son add up to 91. How old is Mr. Smith's wife?

17_____

18. 245×13 is more than 244×13 . How much more?

18_____

19. Crystal had 238 German, 167 French, and 429 United States stamps. How many did she have in all?

20. In his orchard Mr. Butler has 126 rows of trees with 20 trees in each row. How many trees does he have in his orchard?

20_____

21. Find the product: $4 \times 25 \times 9$

21_____

22. Miguel wants to buy 3 bags of potato chips. Each bag of potato chips costs \$2.50. If he uses a coupon for \$1.00 off the price of one bag, how much will Miguel owe for the 3 bags of potato chips?

22_____

23. Harry is taking care of his neighbor's dog for 7 day. Harry needs to let the dog outside 3 times a day. In all, how many times will Harry let the dog out?

23_____

24. What number is half of 10?

24_____

25. What number is double of 50?

Appendix F - Math 017 Materials

Lesson 1

Topics:

Variables and algebraic expressions; Evaluation of algebraic expressions.

Variables and algebraic expressions as symbolical representation of numbers:

Suppose that you thought of a number but you did not tell me what it was. I can think about your number as a number x. Symbol x is an example of a variable.

Variable A variable is a symbol that represents an unknown number.

The choice of the name of a variable is arbitrary: one can as well call it n, m or Ψ . We treat variables as they were numbers we could use. We can, for example, add numbers to variables: m+3, or subtract other variables from them: $m-\Psi$. We can multiply them: $4 \cdot m$, divide: $\frac{\Psi}{m}$, raise to any given power: m^2 and then, if we wish, add all expressions together: $4 \cdot m + \frac{\Psi}{m} + m^2$. The resulting expressions are called algebraic expressions:

Algebraic Expression An algebraic expression is a number, variable or combination of the two connected by some mathematical operations like addition, subtraction, multiplication, division, or exponentiation.

Notice that numbers and variables are also examples of algebraic expressions. We can refer to 3, x, or y as algebraic expressions.

Just like $4 \cdot 5$, 2-5, or $3^2 - 1$ are numbers (written in a 'complicated' way, but numbers), algebraic expressions $4 \cdot m$, x - y or $a^n - b$ are symbolic representation of numbers. Both variables and algebraic expressions could be thought of as unknown numbers.

Correct language and conventions used when forming algebraic expressions:

Algebraic expressions are read using the same terminology as in arithmetic. For example, A + 5 can be read as "A plus 5" or "the sum of A and 5"; y^2 can be read as "y raised to the second power" or "y squared"; -x is read "minus x" or "the opposite of x".

The following convention is commonly adopted to indicate multiplication of a number and a variable, or multiplication of variables:

To denote the operation of multiplication, the sign of multiplication between a number and a variable or between two variables or expressions does not have to be explicitly displayed, so for example:

2A means 2 times A xy means x times y, y(a+b) means y times the quantity a+b

According to the above convention the following is true:

 $x = 1 \cdot x = 1x$

Although x = 1x, it is **customary to write** x **instead of 1**x (just like any time we want to write 4, we just write 4 not 1 · 4).

The following is also true:

 $-x = -1 \cdot x = -1x$

Again, it is **customary to write** – *x* **instead of** –1*x*.

When forming algebraic expressions, we place parentheses according to the notational convention adopted in arithmetic.

For example:

Any time two operation signs are next to each other, parentheses are needed. We write: $2 \times (-1)$, 2 - (-1), or $2 \div (-1)$. It is unacceptable to write any of the following: 2×-1 , 2 - -1, $2 \div -1$. Likewise, when multiplying *a* and -b, we write a(-b). When subtracting -b from *a*, we write a - (-b). Notice that parentheses are needed even if the multiplication sign is not explicitly displayed.

Exponents pertain only to 'the closest' expression. You might, for example, recall that to indicate that the entire fraction is raised to a given power, we use parentheses. We write $\left(\frac{2}{5}\right)^2$. Similarly, whenever you exponentiate any algebraic expression that is not represented by a single symbol, you must use parentheses: $\left(\frac{x}{y}\right)^2$, $(-x)^2 (y-x)^2$. On the

other hand, $5^2 x^2$, y^2 do not require parentheses.

Algebraic expressions allow us to express mathematical ideas in a general form:

Algebraic expressions allow us to write mathematical ideas in symbols, without using specific numbers. For example, the area of a square is equal to the square of the length of its side. Not every square is going to have the same size, so we use a variable to represent the length of a side. If we denote *s* to be a side of a square, then the area of the square can be expressed as $s \cdot s = s^2$

Evaluation of algebraic expressions:

As mentioned, we can use variables and algebraic expressions to describe certain quantitative relationships without information about their specific values:

In a certain store, a cake costs 5 dollars. Let x be a variable that represents the number of cakes we plan to buy in that store. To calculate how much we will pay for such a purchase, we multiply the price of one cake, 5 dollars, by the number of cakes we buy, x. Thus, we can express the cost as $5 \cdot x = 5x$. The algebraic expression 5x represents the cost of x cakes bought at 5 dollars each. From now on, any time we know the number of cakes we wish to buy, i.e. we know the value of x, we can find the price by **evaluating** the expression 5x.

To Evaluate an Expression

To evaluate an algebraic expression means to find its value once we know the values of its variables. Each variable has to be replaced by its value and the resulting numerical expression has to be calculated.

For example:

Evaluate 5x when x = 10 (in the above example it would mean finding the price of 10 cakes at 5 dollars each):

 $5x = 5 \times 10 = 50$

The price of 10 cakes is 50 dollars.

Notice, that what we did was to substitute 10 for x in the expression 5x. We could do that because x is equal to 10. The following fundamental principle underlies the process of evaluation:

If two quantities are equal, you can always substitute one for the other. "Equals can be substituted for equals"

Evaluation of algebraic expressions is not always possible:

Can any algebraic expression be evaluated? Let us consider evaluation of $\frac{1}{a}$ when a = 0. If we replace *a* by its value 0, the operation we would have to perform is division by 0, but division by zero is not defined, so $\frac{1}{a}$ cannot be evaluated if a = 0. Expressions like $\frac{x}{0}$, or $x \div 0$ are undefined.

Examples and Problems with Solutions

Example 1.1 How are the following expressions read?

a)
$$x^{2}$$
 b) xz c) x^{3} d) x^{m} e) $-x$ f) $\frac{x}{y}$

Solutions:

a) 'x raised to the second power' or 'x squared'

b) 'x times z', or 'the product of x and z', or 'x multiplied by z'

c) 'x raised to the third power' or 'x cubed'

d) 'x raised to the m-th power'

e) 'minus x' or 'the opposite of x'

f) 'x divided by y' or 'the quotient of x and y'

Example 1.2 Let x and y denote two different numbers. Express the following statements using algebraic symbols:

b) The difference between x and y

d) The quotient of x and y

a) The sum of x and y
c) The product of x and y
Solutions:

a) x + y
b) x - y
c) xy

d) $x \div y$ or equivalently $\frac{x}{y}$

Example 1.3 Find the algebraic expressions representing the opposite of the following expressions:

a) *x* Solution:

Recall that to find the opposite of a number, we must multiply the number by -1, thus

b) – *x*

a) The opposite of x is $-1 \cdot x = -x$

b) The opposite of -x is $-1 \cdot (-x) = -(-x)$ Please, notice that since the minus sign is directly followed by another minus sign, the parentheses are needed.

<u>Example 1.4</u> Use the letter x to represent a number and write the following statements as algebraic expressions:

a) Double a number b) Two thirds of a number c) A quantity increased by 3 Solutions:

a) 2xb) $\frac{2}{3}x$ c) x+3

Example 1.5 Write the following statements as algebraic expressions:

a) x subtracted from A b) -x added to -A

c) -x multiplied by -y d) $\frac{a}{b}$ raised to sixteenth power

Solution:

a) A - x Notice 'the reversed order' of variables (if you were asked to subtract 3 from 6, you would write 6-3, similarly we write A-x).

b) -x + (-A) Notice the use of parentheses: a plus sign, followed by a minus sign, requires parentheses.

c) (-x)(-y) or -x(-y) Notice the use of parentheses: multiplication sign (even if not explicitly displayed), followed by a minus sign, requires parentheses.

d) $\left(\frac{a}{b}\right)^{16}$ Parentheses are needed to indicate that entire $\frac{a}{b}$ is raised to the sixteenth

power.

<u>Example 1.6</u> Remove all parentheses that are not necessary. If parentheses are needed, rewrite the expression without any changes:

a) a(-c) b) $(-a)^n$ c) $-(a)^n$

Solutions:

a) a(-c). Parentheses are needed. If we remove the parentheses, and write a-c, we would have a subtraction of c from a, instead of the intended operation of multiplication of a(-c).

b) $(-a)^n$. Parentheses are needed. Without the parentheses, only *a* would be raised to the n-th power. With parentheses, we raise -a to the *n*-th power.

c) $-a^n$. We can remove parentheses. The power *n* pertains only to the variable *a* with or without parentheses.

<u>Example 1.7</u> Rewrite each expression replacing variables with their values, and then evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a) $x - 6$, if $x = 5$	b) $-C$, if $C = -2$	c) $2x$, if $x = -3$
d) 2^n , if $n = 3$	e) $y - x$, if $x = -1$ and $y = \frac{1}{3}$	f) $\frac{1}{x+2}$, if $x = -2$

Solutions:

Each variable should be replaced by its assumed value and the obtained numerical sentence has to be evaluated. Please, pay attention to the way parentheses are used:

a)
$$x-6=5-6=-1$$

b) $-C = -(-2) = 2$
c) $2x = 2(-3) = -6$ (remember that $2x$ means multiplication of 2 and x)
d) $2^n = 2^3 = 8$
e) $y-x = \frac{1}{3} - (-1) = \frac{1}{3} + 1 = 1\frac{1}{3}$
f) $\frac{1}{x+2} = \frac{1}{-2+2} = \frac{1}{0}$ The expression cannot be evaluated, since division by zero is not

defined.

Common mistakes and misconceptions

Mistake 1.1

When evaluating $2 \times 10 + 1$, it is INCORRECT to write $2 \times 10 + 1 = 20 = 21$ ($20 \neq 21$) Instead, one should write: $2 \times 10 + 1 = 20 + 1 = 21$

 $2 \times 10 + 1 = 20 + 1 = 21$

Numbers or expressions not involved in the operation that is being carried out *must always be rewritten*. Equal sign means that the quantities on either side are equal.

Mistake 1.2

When x = -1, and you are asked to evaluate -x, you must be careful to write -x = -(-1). Do not forget to recopy the minus sign before -1. It is incorrect to evaluate -x by simply writing -x = -1.

Mistake 1.3

One should NOT think that -x always represents a negative quantity. It depends on the value of x. If, for example, x = -1, then -x = -(-1) = 1. SO we see that if x is a negative value, -x actually represents a positive quantity

Mistake 1.4

When writing a^m , do NOT place *m* at the same level as *a* but slightly higher. Otherwise, a^m becomes *am*. These two expressions have different meanings.

Mistake 1.5

Expression x^2 is NOT read as 'x two' or 'two x'. Instead, read it as 'x raised to the second power' or 'x squared'

Exercises with Answers (For answers see Appendix A)

Exercises 19-33 will help to review basic arithmetic operations using integers, rational numbers (fractions), decimals.

Ex.1 Fill in blanks using the following words: 'variable', 'algebraic expression', 'number(s)' as appropriate:

 $3x+2, y^2, \frac{a+bc}{2}, (-2a+1)^3$ are examples of ______. $\Psi x, y, a, b, c$ are examples of ______ but also examples of ______. Variables represent ______.

If we know the value of x, we can evaluate 3x + 2, and as a result we get a _____.

Ex.2 How are the following expressions read?

a) a^{2}	b) a^{3}	c) a^{12}	d) 2 ^{<i>m</i>}
e) – y	f) <i>cd</i>	g) <i>a</i> – <i>b</i>	h) $\frac{2}{5}x$

Ex.3 Rewrite the following expressions, inserting a multiplication sign whenever multiplication is implied. Whenever there is no operation of multiplication, clearly say so using the phrase "there is no multiplication performed in this expression":

a) 7nb) -5kmc) -x-yd) -x(-y)e) $\frac{3x}{2}$ f) 2x - yz + w(-t)

Ex.4 The operation that is indicated in the algebraic expression a+b is, of course, addition. Name the operation that is to be performed in the following algebraic expressions:

a)
$$3-x$$
 b) $4x$ c) x^5 d) $\frac{q}{s}$ e) ab

Ex.5 Use the letter y to represent a number and write the following phrases as algebraic expressions:

a) Half of a number	b) Two thirds of a number
c) A quantity increased by 5	d) A number subtracted from v
e) A quantity squared	f) Three more than a number
g) A number decreased by x	h) The product of x and a number

Ex.6 Write the following phrases as algebraic expressions. Remember to place parentheses when needed (place them only when needed):

a) The sum of a and $-b$	b) The difference of a and $-b$
c) The product of a and $-b$	d) The opposite of C
f) The opposite of $-C$	g) The opposite of $\frac{-a}{-b}$
h) The product of v , $-t$, and $-p$	i) The quotient of c and $-B$
j) $-x$ raised to m -th power	k) $\frac{x}{y}$ raised to <i>m</i> -th power

*Ex.*7 Give your answer in the form of an algebraic expression:

a) Carlos is x years old at this moment. How old will Carlos be in 10 years?

b) The items in a store cost x dollars. What is the price of each item, if after a discount, its price was reduced to two thirds of the original one?

c) You have x dollars to divide equally among 3 kids. How much will each child get?

d) You have \$100 to divide equally among x kids. How much will each child get?

e) There are 30 books on each shelf. How many books are on x shelves?

f) There are *x* students in a classroom. How many students are still in the classroom, if 3 students leave?

Ex.8 Let d be a variable representing the distance driven by a car, and let t represent the time it took to drive that distance. Write the following phrase as an algebraic expression: The distance divided by time.

Ex.9 Let *m* be a variable representing the mass of a given body, and let *a* represent its acceleration. Use *m* and *a* to write the following phrase as an algebraic expression: The product of the mass of a body and its acceleration.

Ex.10 Let h be a variable representing the height of a triangle, and b represent the base of the triangle. Use h and b to write the following statement as an algebraic expression: One half of the product of the base of a triangle and its height.

Ex.11 Fill in the blanks so the resulting statement is true. Use parentheses if needed:

 $A = \underline{\qquad} \cdot A = 1 \cdot \underline{\qquad}$ $a^{2}b = \underline{\qquad} \cdot a^{2}b = 1 \cdot \underline{\qquad}$ $-M = \underline{\qquad} \cdot M = -1 \cdot \underline{\qquad}$ $-2cde = \underline{\qquad} \cdot 2cde = -1 \cdot \underline{\qquad}$

Ex.12 Write an algebraic expression representing the opposite number of (do not remove parentheses):

a)
$$\frac{x^3}{y}$$
 b) $-\frac{x^3}{y}$ c) $\frac{-x^3}{-y}$

Ex.13 Remove parentheses, if unnecessary. In cases when parentheses cannot be removed, clearly indicate so.

a)
$$y(x)^8$$
 b) $\left(\frac{m}{n}\right)^\circ$ c) $(-a)^4$ d) $-(a)^4$ e) $x + (-b)$
f) $a \div (-b)$ g) $y(-x)$ h) $(-x)y$ i) $3(b)c$ j) $3b(-c)$

Ex.14 Let x = 3. Rewrite the expression replacing the variable with its value and evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a) x+5 b) x-2 c) $\frac{x}{3}$ d) 4x e) x^2 f) $\frac{6}{x}$ *Ex.15* If x=0 the following expression cannot be evaluated: $\frac{1}{x}$. Why not? Can $\frac{1}{x-5}$ be

Ex.15 If x = 0 the following expression cannot be evaluated: $\frac{1}{x}$. Why not? Can $\frac{1}{x-5}$ be evaluated when x = 0? What if x = 5? Find another example of an algebraic expression and a value of a variable(s) for which evaluation is not possible.

Ex.16 Let x = 0. Rewrite the expression replacing the variable with its value and evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a) 3x b) x-2 c) $\frac{4}{x}$ d) $\frac{x}{7}$ e) $\frac{2}{x-3}$

Ex.17 Let x = 2. Rewrite the expression replacing the variable with its value and evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a) 3^x b) x^3 c) x^x

Ex.18 Evaluate -A, if a) A = 2 b) A = -2
Ex.19 Substitute x = 6 in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a) x-8 b) -10-x c) -4+x d) x-6 e) -2+x-6

Ex.20 Substitute x = -2 in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a) 2+x b) 2-x c) -2-x d) -5-x+4 e) 6+x-10-x

Ex.21 Substitute x = 10 in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a)
$$3x$$
 b) $-5x$ c) $\frac{-200}{x}$ d) $-\frac{x}{2}$ e) $-5 \div x$ f) x^4

Ex.22 Substitute x = -12 in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a)
$$-1000x$$
 b) $\frac{x}{6}$ c) $-5x$ d) $\frac{6}{x+12}$ e) $-24 \div x$ f) x^2

Ex.23 Substitute $x = \frac{2}{3}$ in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a)
$$\frac{5}{3} + x$$
 b) $x + \frac{1}{5}$ c) $-x + \frac{2}{7}$ d) $-\frac{5}{12} - x$ e) $2 + x$ f) $-x - 3$

Ex.24 Substitute $x = -\frac{3}{5}$ in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a)
$$\frac{3}{10} - x$$
 b) $-\frac{1}{7} - x$ c) $2\frac{1}{5} + x$ d) $-1\frac{1}{4} - x$ e) $-x - 3\frac{1}{2}$

Ex.25 Substitute $x = \frac{2}{7}$ in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a) 2x b) -7x c) $-\frac{14}{3}x$ d) $\frac{5}{28} \div x$ e) $\frac{5}{x}$ f) $\frac{-x}{2}$

Ex.26 Substitute $x = -\frac{3}{4}$ in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a)
$$x^2$$
 b) $\frac{4}{3}x$ c) $\frac{-x}{1\frac{2}{3}}$ d) $-1\frac{1}{8} \div x$ e) $\frac{x}{-3}$ f) $\frac{0}{x}$

*Ex.*27 Substitute x = 0.2 in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a)
$$x + 3.21$$
 b) $35.01 - x$ c) $\frac{x}{4}$ d) $-40x$ e) $0.3x$ f) $\frac{-x}{0.04}$

Ex.28 Substitute x = -0.6 in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a)
$$-x - 4.5$$
 b) $-2.7 - x$ c) $\frac{1.2}{-x}$ d) $-600x$ e) $0.001x$ f) x^3

Ex.29 If possible, evaluate x + y, if

a)
$$x = \frac{3}{5}, y = \frac{2}{3}$$
 b) $x = \frac{2}{7}, y = -\frac{9}{14}$ c) $x = -.2, y = -1.08$

Ex.30 If possible, evaluate
$$x - y$$
, if
a) $x = \frac{3}{5}$, $y = \frac{2}{3}$ b) $x = \frac{2}{7}$, $y = -\frac{9}{14}$ c) $x = -.2$, $y = -1.08$

Ex.31 If possible, evaluate xy, if
a)
$$x = \frac{2}{11}$$
, $y = \frac{22}{9}$ b) $x = -4$, $y = -\frac{9}{10}$ c) $x = -.2$, $y = 0.01$

Ex.32 If possible, evaluate
$$\frac{x}{y}$$
, if
a) $x = \frac{2}{11}$, $y = \frac{22}{9}$ b) $x = -4$, $y = -\frac{9}{10}$ c) $x = -0.2$, $y = 0.01$

Ex.33 If possible, evaluate
$$(-x)^m$$
 if
a) $x = 10$, $m = 7$
b) $x = -2$, $m = 4$
d) $x = -0.1$, $m = 5$

Ex.34 Is it possible to find a value of K so that -K is positive? If yes, give an example of such a value of K.

Ex.35 Is it possible to find values of a and b so that -(a+b) is positive? If yes, give an example of such values of a and b.

1. Use the letter x to represent a number and write the following phrases as algebraic expressions:

a) one-eighth of a number

b) A number multiplied by -3

2. Rewrite the following expressions, inserting a multiplication sign whenever multiplication is implied. Whenever there is no operation of multiplication, clearly say so using the phrase "there is no multiplication in this expression":

a) 2*x*

b) -2x

c) y - 4

- d) y(-4)
- 3. How are the following expressions read?

a)
$$\frac{a}{4}$$

b) *a*¹⁷

1. Write the following statements using algebraic symbols:

a) x raised to the third power

b) The sum of x, p and y

2. Rewrite the following expressions, inserting a multiplication sign whenever multiplication is implied. Whenever there is no operation of multiplication, clearly say so.

a) A(-B)

b) *A* – *B*

c)
$$\frac{3}{4}m$$

d) (a+bc)6

- 3. How are the following expressions read?
- a) a^{3}
- b) *a* · 3

c) – *a*

- 1. Write the following statements as algebraic expressions:
 - a) The product of x, y, and z
 - b) The opposite of -m
 - c) The quotient of 3 and *B*
- 2. Name the operation that is to be performed in the following algebraic expressions:
- a) *a*(–*b*)
- b) *a b*
- 3. Fill in the blanks with numbers to make the statement true:
- a) _____· x = x
- b) _____ · x = -x
- c) _____ · x = 0

- 1. Write the following statement as an algebraic expression:
- a) The sum of 3a and x

b) The difference of 3a and x

c) The product of 3a and x

d) The quotient of 3a and x

2. Rewrite the following expressions removing all unnecessary parentheses. If removing is not possible, please, clearly indicate so.

a) 4 - (x)

b) 4(-*x*)

- c) (-x) + 4
- d) $4 \div (-x)$

e)
$$\frac{(a)^4}{b}$$

f) $\left(\frac{a}{b}\right)^4$

1. Write the following phrases as algebraic expressions. Remember to place parentheses when needed: a) -8 added to -5

b) -n added to -m

c) -8 multiplied by -5

d) -n multiplied by -m

e) 8 subtracted from 5

f) -n subtracted from m

g) -n divided by -m

2. Give your answer in the form of an algebraic expression:

a) John has x dollars in his bank account. How much money will John have in the bank, after he withdraws 100?

b) John's salary is *x* dollars. He got a new job and his salary doubled. How much does John earn in his new job?

1. Write the following phrases as algebraic expressions. Remember to place parentheses when needed:

a) -2 raised to the third power

b) -x raised to the third power

c) $\frac{2}{3}$ raised to the tenth power

d) $\frac{b}{a}$ raised to the tenth power

e) $-\frac{2x+1}{y}$ cubed

f) $2a^{3}bc^{2}$ squared

2. Give your answer in the form of an algebraic expression:

a) If a family has 2 children, and each of those children has 3 children, how many grandchildren are there?

b) If a family has x children, and each of those children has 3 children of his own, how many grandchildren are there?

c) If a family has x children, and each of those children has x children of his own, how many grand children are there?

1G

a) the sum of 3, -s, and -t

b) -2s squared

c) the product of 3, -s, and -t

d) $a + b^2$ raised to the sixth power

e) The difference of -s and -t

2. Fill in the blanks:

.

a) It is customary to write _____ instead of $1 \cdot x$.

b) It is customary to write _____ instead of $-1 \cdot x$.

c) When one writes ab, it is understood that the operation that is to be performed is

1. Using the letter B to represent a quantity, write each phrase as an algebraic expression. Remember about parentheses:

a) A quantity multiplied by -y

- b) -y subtracted from a number or quantity
- c) A quantity divided by -y

2. Rewrite the following expressions removing all unnecessary parentheses. If removing is not possible, please, clearly indicate so.

a) -(x)yz

b) (-x)yz

c) y(-x)z

3. Let t be a variable representing temperature in Fahrenheit on a given day. Use the variable to write the following statement as an algebraic expression:

The temperature on the given day increased by 5 degrees (assume all temperatures in this example are given in the Fahrenheit system).

1. Let a be a variable representing the length of the side of a square. Use a to write the following statements as algebraic expressions:

a) Four times the side of a square.

b) The side of a square raised to the second power.

- 2. Express as an algebraic expression:
- a) The opposite of 2a
- b) The opposite of -2a
 - c) The opposite of $-\frac{2a}{7}$
 - d) The opposite of $\frac{-2a}{-7}$

1. Rewrite the following expressions removing all unnecessary parentheses. If removing is not possible, please, clearly indicate so.

a) 3 - (x)

b) 3 - (-x)

c) 3(-x)

d) $(-x) \cdot 3$

2. Evaluate, if possible, x + 2, 10 - x, 4x, $\frac{18}{x}$, when x = 9. If not possible, explain why it is not possible to evaluate.

1. Circle all expressions that cannot be evaluated, when x = 10. Explain why they cannot be evaluated:

0	5	x - 10	1	r 10	6	2	3 <i>x</i>
$\overline{3x}$,	$\overline{x-10}$,	,	$\overline{10}^{X}$,	x = 10,	$x + 10^{-1}$,	$\overline{10-x}$,	0

2. Rewrite the following expressions removing all unnecessary parentheses. If removing is not possible, please, clearly indicate so.

a) (-*s*)*t*

b) (-x) + 5



d) $(-x)^{m}$

e) $-x^{(m)}$

- Evaluate the following when x = 1
 a) 2^x
 b) x²
 c) x^x
 c) the following when x = 3
 a) 2^x
 b) x²
 - c) x^x

3. Is it always possible to evaluate an expression? If not, give an example of such an expression and write the value of the variable(s) that prevents the expression from being evaluated.

1. Let a = 10. Rewrite the expression replacing the variable with its value, and then evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a) -12 + a

b) -5-a

c) -3+2-a-7

d) a^{5}

e) – 22*a*

f)
$$\frac{5}{a-10}$$

g)
$$\frac{0}{-a}$$

1. Let v = -1. Rewrite the expression replacing the variable with its value, and then evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a) – v

b) -3v

c) v - 8

d) $v \div (-2)$

e)
$$-3-v$$

f)
$$\frac{-4}{v}$$

g)
$$\frac{-1}{v+2}$$

1. Let a = -2. Rewrite the expression replacing the variable with its value, and then evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a) *a* – 7

b) – *a* + 3

c) *a*+2

d) -2 + x - 7 + 8

e)
$$\frac{4}{a}$$

f) -400a

g) $a \div (-1)$

1. Let $m = \frac{3}{7}$. Rewrite the expression replacing the variable with its value and evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a)
$$\frac{120}{7} - m$$

b)
$$\frac{1}{14} + m$$

c)
$$m + 2$$

d) -2m

e)
$$\frac{4}{m}$$

f)
$$-\frac{9}{21} \div m$$

g)
$$-m \cdot \frac{2}{81} \cdot 9$$

1. Let $x = -\frac{7}{9}$. Rewrite the expression replacing the variable with its value, and then evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a)
$$\frac{1}{3} - x$$

b) -x - 1

c)
$$\frac{2}{3} + x - \frac{8}{9}$$

d)
$$\frac{15}{2}x$$

e) $-14 \div x$

f)
$$-x + \frac{7}{9}$$

g) x^{2}

1. Let y = 0.6. Rewrite the expression replacing the variable with its value, and then evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a) *y* + 72.86

b) 1.3 – y

c) -y - 0.2

d) 0.01*y*

e)
$$\frac{y}{0}$$

f)
$$\frac{-0.12}{y}$$

g)
$$\frac{-y}{0.01}$$

1. Let x = -2.2 Rewrite the expression replacing the variable with its value, and then evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a) -3 - x

b) -0.1 + x + 0.002

c)
$$\frac{x}{1.1}$$

d)
$$\frac{5}{2.2+x}$$

e) $x \div (-0.02)$

f) x^2

g) $a \div (-1)$

1. Let x = -3 and $y = \frac{4}{5}$. Rewrite the expression replacing each variable with its value, and then evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a) x - y

b) y + x

c) y^3

d) *xy*

e) $\frac{y}{3+x}$

f) $x \div (-y)$

g)
$$\frac{y}{x}$$

1. Let $a = -\frac{3}{4}$, $b = \frac{5}{6}$. Rewrite the expression replacing each variable with its value, and then evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a) b-a

b) *a*²

c) a + 2 - b

d) 4*ab*

e) $\frac{b}{a}$

f)
$$\frac{a}{b}$$

g)
$$\frac{-a}{b}$$

1. Evaluate, if possible, when $A = \frac{1}{2}$, $B = -\frac{1}{2}$. Indicate, if not possible. Before evaluating, rewrite the expression substituting the numerical values of the variables.

a) -A

b) -B

c) A + B

d) A-B

e) *B* – *A*

f) 2*A*

g)
$$\frac{A}{B}$$

1. Evaluate, if possible, when $A = \frac{2}{5}$, $B = -\frac{1}{4}$. Indicate, if not possible. Before evaluating, rewrite the expression substituting the numerical values of the variables.

a) *AB*

c)
$$-\frac{3}{4}B$$

d) $A \div B$

e)
$$\frac{1}{A}$$

f) A + B

- 1. Write the following phrases as an algebraic expression:
- a) The sum of -x and -y
- b) The product of -x and -y
- c) The difference of -x and -y

2. Let c = -2. Rewrite the expression replacing the variable with its value, and then evaluate, if possible. If evaluation is not possible, clearly indicate so.

a)
$$c^2$$
 b) $-c^2$

c) $(-c)^2$ d) 10^{-c}

3. Rewrite the following expressions removing all unnecessary parentheses. If removing is not possible, please, clearly indicate so.

a) 2*m*(*n*)

b) 2*m*(−*n*)

c) m - (-n)

1. Use the letter x to represent a number and write the following statements as algebraic expressions. Then evaluate each expression when $x = \frac{9}{11}$

a) A number doubled

b) One third of a number

2. Evaluate -2t, when t = 1, t = 2, t = -7

Based on your results, which of the following are true? a) -2t is always positive

b) -2t is always negative

c) -2t may be positive or negative depending on the value of t

Lesson 2

Topics:

Algebraic expressions and their evaluations: order of operations.

The presumed order of operations:

Recall the presumed order of operations:

When evaluating arithmetic expressions, the presumed order of operations is:

- Perform all operations in parentheses first.
- Next, do any work with exponents.
- Perform all multiplications and divisions, working from left to right.
- Perform all additions and subtractions, working from left to right.

If a numerical expression includes a fraction bar, perform all calculations above and below the fraction bar before dividing the top by the bottom number.

Let x be an unknown number. If we increase the number by 1, the resulting number can be represented by x+1. Now, suppose that after adding 1, we multiply the result by some other number, called y. We may now write the expression y(x+1). Let us analyze why we must place parentheses around x+1. If an algebraic expression involves more than one mathematical operation, then the presumed order of operations is followed. If we simply write yx+1 (without using parentheses), we would only be multiplying y and x, rather than y times the entire quantity x+1. According to the order of operations, using parentheses in y(x+1) indicates that we add 1 to x first, and then multiply the result by y. Expressions yx+1 and y(x+1) have entirely different meaning, and only y(x+1) correctly represents the result of operations performed in this example.

The presumed order of operations is used when algebraic expressions are evaluated. For example:

Evaluate $2x^2 - x$ when	x = 3.
$2x^2 - x =$	Replace each x with 3
$2 \times 3^2 - 3 =$	Raise 3 to the second power
$2 \times 9 - 3 =$	Then, multiply it by 2
18 - 3 =	Finally, subtract 3 from the result
15	

Examples and Problems with Solutions

Example 2.1 Rewrite the following expressions and circle the arithmetic operation together with its operands that has to be performed first. Write the name of the operation next to your expression. a) 3-5y b) $2x^4$

Solutions:

a) We perform multiplication before subtraction

3-5y multiplication

b) We exponentiate before multiplication

 $2x^{4}$ exponentiation

Example 2.2 List, according to the order of operations, all the operations together with operands that are to be performed in the following expressions: a) 4+3xb) $4 \div 8a$ Solutions:

a) There are two operations in 4+3x: multiplication and addition. According to the presumed order of operations, multiplication should be performed first. Thus, the answer is: multiply 3 by x, then add 4.

b) There are two operations in $4 \div 8a$: division and multiplication. According to the presumed order of operations, they should be performed as they appear reading from the left to the right. Thus divide 4 by 8 first, and then multiply by a.

<u>Example 2.3</u> Write the algebraic expressions representing the following:

a) a+b raised to the seventh power

b) a+b subtracted from y

c) 8 times the quantity a + b

d) the opposite of a + b

Solutions:

Parentheses must be used in each of these examples:

a) $(a+b)^7$ (We learned that the exponent pertains only to the closest expression, so if we write $a+b^7$ instead of $(a+b)^7$, only *b* would be raised to the power 7. This rule can be view as a consequence of the presumed order of operations: exponentiation should be performed before addition, hence in $a+b^7$, *b* is raised to the seventh power, and then the result is added to *a*. To ensure that *b* is first added to *a*, we need to use parentheses, and only then the sum is raised to the seventh power.

b) y - (a + b) (Notice the order of expressions)

c) 8(a+b) (Thanks to parentheses, we add first, and then multiply)

d) -(a+b) (Taking the opposite means multiplication by -1; to ensure addition first, we need parentheses)

Example 2.4 Use the letter x to represent a number and write the following as algebraic expressions:

a) Add three to a number, and then divide it by z

b) Seven more than one third of a number

c) A quantity decreased by 9, and then multiplied by A

d) A number cubed, and then decreased by y

Solutions:

Please, notice the use of parentheses in all examples below:

a)
$$(x+3) \div z$$
 or $\frac{x+3}{z}$
b) $\frac{1}{3}x+7$
c) $(x-9)A$
d) $x^{3} - y$

Example 2.5 Remove all parentheses that are not necessary and do not change the order of operations in the following algebraic expressions. If all parentheses are needed, rewrite the expression without any changes: a) (a+b)-cb) (a+b)cSolutions:

a) a+b-c. We can remove parentheses, since addition and subtraction are always performed in the order as they appear from the left to the right.

b) (a+b)c. Parentheses are needed. Without the parentheses, we would first multiply *b* and *c*, and then add *a*. With parentheses we first add *a* and b, and then multiply by *c*.

<u>Example 2.6</u> Rewrite the expression replacing variables with their values. Then, evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a)
$$\frac{x-1}{x+2}$$
 if $x = -2$
b) $3y - x$ if $x = -1$ and $y = \frac{1}{3}$
c) $(n+p)^m$ if $m = 2, n = -5$, and $p = 4$

Solutions:

Each variable has to be replaced by its given value and the resulting numerical expression has to be evaluated. Please, pay attention to the way parentheses are used:

a) $\frac{x-1}{x+2} = \frac{-2-1}{-2+2} = \frac{-3}{0}$, but division by zero is not defined. The expression cannot be

evaluated.

b)
$$3y - x = 3 \cdot \frac{1}{3} - (-1) = 1 + 1 = 2$$

c) $(n + p)^m = (-5 + 4)^2 = (-1)^2 = 1$

Exercises with answers (For answers see Appendix A)

Ex.1 Write the following phrases as algebraic expressions. Remember to place parentheses where needed (please place them only when needed)

a) Multiply 3 by x, and then add y

b) Multiply the sum of *a* and *b* by 4.

c) The opposite of 4x + 2, then divided by 6

d) Subtract 3 from y, and then multiply by z

e) Raise x to the third power, and then multiply by 9

f) Multiply x by 9, and then raise the result to the third power

g) The difference of a and b, then divided by c

h) Divide 3 by y, and then add 3x+1

i) Raise -x to the third power, raise -y to the seventh power, and then add them

together

Ex.2Use the letter x to represent a number and write the following as an algebraic expression: a) A number decreased by 7, and then doubled

b) Add -1 to a number, and then take two thirds of the sum

c) Take one fourth of a number, and then subtract 5

d) The difference between a number and 8, then multiplied by 9

e) A number, first divided by 2, and then raised to the third power

f) The opposite of a number, then multiplied by 4

g) A quantity raised to the third power, and then increased by 6

h) The difference between a number and 4, then multiplied by y.

i) A number multiplied by the sum of the same number and 5

i) The opposite of a number, then raised to one hundred and twenty first power

k) Square a number, and then take the opposite of it

Ex.3 Let C be a variable representing the temperature in Celsius. Write the following phrase as an algebraic expression: Nine fifths of the Celsius temperature plus 32.

Ex.4 Let L be a variable representing the length of a rectangle, and W its width. Use L and W to write the following phrase as an algebraic expression: The sum of the length of a rectangle and its width, then multiplied by 2.

Ex.5 Let *m* represents mass, and *c* speed of light. Use *m* and *c* to write the following phrase as an algebraic expression: the product of mass and the square of speed of light.

In the following expressions circle the arithmetic operation, together with its operands, that Ex.6has to be performed first. Write the name of the operation next to your expression. For example, in 4+3x, multiplication of 3 and x has to be performed first, thus the answer is:

	4 + 3x	multiplication		
a) $a + b^5$		b) $(a+b)^5$	c) $-x^{8}$	d) $(-x)^8$
e) $\frac{a-b}{c}$		f) $a \div b \times c$	g) $4 - 7y$	h) $3 + a \div b$

Ex.7 There are two operations in the algebraic expression a + 3b: addition and multiplication. In order to evaluate a + 3b, we would have to perform them according to the presumed order of operations: First multiply 3 and b and then add a.

List, according to the presumed order of operations, operations that are in the following algebraic expressions:

a)
$$4x - y$$

b) $\frac{a+3}{x}$
c) $(x+3)y$
d) $\frac{s}{t} + 2$
e) $3x^2$
f) $(3x)^2$
g) $(a+c)^4$
h) $a+c^4$

Ex.8 Remove parentheses, if unnecessary. In cases when removing parentheses would change the order of operations (and thus parentheses cannot be removed), indicate so.

a) $(2-a)x$	b) $(c-3) - a$
c) $3a + (2 + x)$	d) $a - (c + b)$
e) $x \div (2ab)$	f) $\frac{(-c+d)}{a}$
g) $(a+2)^4$	h) $y(x)^{8}$
i) $(ab)^4$	j) $d(-c) - (d-c)$

Ex.9 Evaluate, if possible:

a) 2x+1, if $x = \frac{1}{2}$ b) 2a+1, if $a = \frac{1}{2}$ c) 2y+1, if $y = \frac{1}{2}$

d) Did you get the same answer for a, b, and c? Can you explain why it is so?

e) If
$$\frac{-4x^3 + x^2 + 2x}{3x - \frac{x}{2}} = \frac{3}{5}$$
 when $x = \frac{1}{2}$, evaluate $\frac{-4a^3 + a^2 + 2a}{3a - \frac{a}{2}}$ when $a = \frac{1}{2}$. You

should be able to arrive at your answer without performing any evaluation.

Ex.10 Let x = 3. Rewrite the expression replacing the variable with its value and evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a)
$$-2x-5$$

b) $-4x + x^2$
c) $\frac{x}{x-3}$
d) $(-x)^2$
e) $-x^2$
f) $\frac{3-x}{4+x}$

Ex.11 Let x = 4. Rewrite the expression replacing the variable with its value and evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a) -2^x b) $(-2)^x$ c) $(-x)^2$ d) $-x^2$ e) x^x

Ex.12 Let x = -1. Rewrite the expression replacing the variable with its value and evaluate, if possible a) -x + x(-x) b) -x - x(-x) c) (-x)(-x)(-x)

Ex.13 Substitute $A = -\frac{1}{2}$ and then evaluate the following expressions, if possible:

a)
$$\frac{1}{A}$$
 b) $\frac{1}{A} + A$ c) A^2 d) $(-A)^2$ e) $-(-A)^2$

Ex.14 Let x = -0.3. Rewrite each expression replacing the variable with its value and evaluate, if possible.

a)
$$x^2 - x$$
 b) $\frac{x}{0.1} - 2$ c) $\frac{0.3 - x}{x - 0.3}$ d) $1000x - 100x + 10x$

Ex.15 Evaluate, if possible: a) 2(l+w) if l=8 and w=5b) R(1-x) if R=100 and x=0.6

Ex.16 The expression $\frac{3-x}{y-5}$ cannot be evaluated for which of the following values of x and y?

Explain why.

a)
$$x = 3$$
, $y = -5$
b) $x = -3$, $y = 5$
c) $x = 3$, $y = 5$
d) $x = -3$, $y = -5$
e) $x = 3$, $y = 0$
f) $x = 0$, $y = 5$

Ex.17 If possible, evaluate when m = -2, n = 5. Indicate, if not possible. Before evaluating, rewrite the expressions substituting the numerical values of m and n:

a) 2m-3n b) 2m(-3n) c) 2(m-3n) d) (2m-3)n e) 2(m-3)n

Ex.18 If possible, evaluate when $m = -\frac{1}{8}$, $n = \frac{4}{5}$. Indicate, if not possible. Before evaluating, rewrite the expressions substituting the numerical values of *m* and *n*:

a) 8m - 10n b) 10mn c) 2(n - m) d) $-8m^2 + n$ e) $n \div (\frac{1}{8} + m)$ f) $n \div \frac{1}{8} + m$

Ex.19 If possible, evaluate when $A = \frac{1}{3}$, $B = -\frac{2}{3}$. Indicate, if not possible. Before evaluating, rewrite the expressions substituting the numerical values of A and B:

a)
$$2A^4$$
 b) B^4 c) $-B^4$ d) $\frac{A+B}{A-B}$ e) $\frac{A(-B)}{A \div B}$

Ex.20 Evaluate the following expressions: a^3 , 4^n , ab^2 , $(ab)^2$, $-a^n$, a^{n+m} if $a = -1, b = \frac{1}{3}, n = 3, m = 2$. If evaluation is not possible, clearly indicate so.

Ex.21 Let x = 2, y = -0.1, and z = -1. If possible, evaluate the following expressions: a) x(z + y) b) xz + y

Ex.22 Let a = 0.1, b = -0.2, c = -1. If possible (indicate if not possible), evaluate the following expressions. Before evaluating, rewrite the expressions substituting the numerical values of variables. a) a - bc b) a^{-c} c) b^{10a}

Ex.23 Find the value of $2a^2 - (2a)^2$ ifa) a = 1b) a = -1Ex.24 Find the value of 2A - B, ifa) A = -1, B = 3b) A = -2, B = -4c) A = 0.3, B = -0.7d) $A = \frac{2}{8}, B = -1$ e) $A = 1\frac{5}{6}, B = \frac{4}{5}$ f) $A = -\frac{3}{10}, B = -\frac{5}{7}$ Ex.25 Find the value of -(A + 3B), ifa) A = -1, B = -1c) A = 0.1, B = -0.2d) A = -2, B = -3d) $A = -2, B = -1\frac{2}{3}$

e)
$$A = \frac{2}{7}, B = -\frac{1}{6}$$
 f) $A = -4, B = -\frac{5}{9}$

Ex.26 Find the value of $\frac{0.1x}{y}$, if a) x = 2, y = 0.02 b) x = -200, y = 0.4 c) x = 0.1, y = -0.2

*Ex.*27 Evaluate the following expressions, if m = -1, n = 2, and p = -3. Before evaluating, rewrite the expressions substituting the numerical values of variables:

a) (m-1) - (n+p) b) m - n + 4p c) $3mn^{-p}$

*Ex.*28 Let m = -1, n = -2, p = -3, q = 4. If possible (indicate if not possible), evaluate the following expressions. Show your work:

a)
$$2m - (n - p) + q$$

b) mnp
c) $mn - p$
d) $\frac{q}{m + n - p}$
e) $m(n - p + q)$
f) $\frac{2m - n}{p - q}$

1. Rewrite the following expressions and circle the arithmetic operation together with its operands that has to be performed first. Write the name of the operation next to your expression. For example, in 4 + 3x, multiplication of 3 and x must be performed first, thus the answer is:

4 + (3x) multiplication

a) 2 + 3x

b) $a-b \div 2$

c)
$$\left(\frac{x}{2}\right)^3$$

d) $\frac{x^3}{2}$

2. Substitute m = 3, n = 2 in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a) 2 - m - 2n

b) 2 - (m - 2n)

c) 2 - m(-2n)

1. Rewrite the following expressions and circle the arithmetic operation together with its operands that has to be performed first. Write the name of the operation next to your expression. For example, in 4 + 3x, multiplication of 3 and x must be performed first, thus the answer is:

$$4 + (3x)$$
 multiplication

a)
$$4 \div a + y$$

b) $4 \div (a + y)$

c)
$$\left(\frac{a}{b}\right)^{90}$$

2. Evaluate, if possible (indicate, if not possible), when $x = -\frac{1}{2}$ and $y = \frac{2}{3}$. Before evaluating, rewrite the expression substituting the numerical values of x and y.

a) -2xy

b) -2(x+y)

c)
$$-\frac{2x}{y}$$
2C

1. Rewrite the following expressions and circle the arithmetic operation together with its operands that has to be performed first. Write the name of the operation next to your expression. For example, in 4 + 3x, multiplication of 3 and x has to be performed first, thus the answer is:

multiplication

a) xy + 6b) ax^3

c) $(ax)^{3}$

2. Substitute x = -4 in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a) $\frac{x-4}{-x+4}$

b) $\frac{x+1}{8-2x}$

c) $\frac{x+4}{16-x^2}$

1. List all arithmetic operations together with operands in the same order as the presumed order of operations indicates (for example, for the expression a + 3b we would write: "first multiply 3 and b and then add a").

a) $3 + x \cdot 2$

b) $(3+x) \cdot 2$

c) $x + y^7$

d) $(x + y)^7$

e) $x \div yz$

2. Substitute x = -2, y = 3 in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a) x^y

b) $-x^{y}$

c) $(-x)^{y}$

1. For each of the following expressions, list all arithmetic operations together with the operands in the same order as the presumed order of operations (for example, for the expression a + 3b we would write: "first multiply 3 and b, and then add a").:

a)
$$\frac{a+b}{x}$$

b) $\frac{a}{x}+b$
c) $a+\frac{b}{x}$
d) $a+b \div x$

e) $(a+b) \div x$

2. Substitute x = -2, y = 3, z = -8 in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a)
$$x + y - z$$

b) (x - y)z

c) x - (y - z)

1. Using the letter of your choice to represent a number, write each phrase as an algebraic expression:

- a) A quantity multiplied by -y, and then divided by 13
- b) Subtract a number from 7, and then take the opposite of it
- c) The sum of A and one-hundredth of a number, then raised to the fifth power
- d) The quotient of a number and 3, then increased by 7
- e) The sum of 1 and a number, then doubled

f) A number increased by 5, and then multiplied by the same number.

- 1. Write the following phrases using algebraic symbols:
- a) *x* raised to the third power, and then add y
- b) The sum of x and y, and then the result is raised to the third power
- c) One-third of a number is then subtracted from 5

2. Evaluate P(1+rt) if P = 500, r = 0.2, t = 5.

3. Evaluate
$$\frac{1}{R} + \frac{1}{S}$$
 if $R = \frac{1}{2}$ and $S = \frac{1}{3}$.

- 1. Write the following phrases as algebraic expressions:
- a) y raised to the 8th power, and then subtracted from z-t

b) z-t subtracted from y, then raised to the 8th power

2. Evaluate
$$\frac{9C}{5} + 32$$
, if $C = -5$.

3. Evaluate
$$\frac{bh}{2}$$
 if $b = 6$ and $h = \frac{1}{2}$.

1. Rewrite the following expressions removing all unnecessary parentheses. If removing parentheses is not possible, please, clearly indicate so.

a) a - (b + c)

b) $a(b \div c)$

2. Let F be a variable representing the temperature in Fahrenheit. Use this variable to write the following phrases as an algebraic expression: The temperature in Fahrenheit minus 22, then multiplied by five ninths

The temperature in Fahrenheit minus 32, then multiplied by five ninths.

- 3. Let a = -2. Evaluate, if possible:
- a) *a*
- b) *a*(-*a*)
- c) *a*(–*a*)
- d) -a(-a) a

1. Evaluate $\frac{x+1}{x-7}$ for each of the following values of x, when possible (if not possible, clearly indicate so):

- a) x = -7
- b) x = 7
- c) x = 0
- d) x = -1
- e) x = 1

2. Evaluate (x+6)(5-x)(4+x) for each of the following values of x, when possible (if not possible, clearly indicate so) :

a) x = -6

b) x = 5

c) x = -4

1. The expression $\frac{a}{c-8}$ cannot be evaluated for which of the following values of *a* and *c*? Explain why. a) a = 3, c = 0

b) a = 3, c = 8

c) a = 3, c = -8

- d) a = 0, c = -8
- e) a = 0, c = 0
- f) a = 0, c = 8

2. Let x = -2. Rewrite the expression replacing the variable with its value and evaluate, if possible. If evaluation is not possible, explain why it is not possible.

- a) $\frac{-x}{3}$
- b) $4x^2$
- c) $(4x)^2$

- 1. Let x = -3. Evaluate if possible. If not possible, clearly indicate so.
- a) (x-4)(x-1)
- b) x (4x 1)
- c) x(-4x-1)
- d) x(-4x)(-1)

2. Evaluate $\frac{-x-y}{(-x)(-y)}$ for each of the following values of x and y, when possible (if not possible, clearly indicate so) :

a) x = 3, y = 0

b) x = 0, y = -3

c) x = 3, y = -3

d) x = 0, y = 0

1. Let v = -1. Rewrite the expression replacing the variable with its value and evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a) $255^{(v+2)}$

b) *v*⁴

c) $-v^4$

d) $(-v)^4$

e) $255v^{(v+2)}$

f) $(255v)^{(v+2)}$

g) v²⁵⁵

1. If possible (indicate if not possible), evaluate the following expressions if $A = -\frac{2}{5}$, $B = -\frac{1}{4}$. Before evaluating, rewrite the expression using numerical values of variables.

a) 5A - B

b) *AB*−1

c) (-*A*)(-*B*)

d) $(A+B)^2$

e) $A^2 + B^2$

1. Let $x = -\frac{1}{3}$, $y = \frac{9}{10}$, $z = 1\frac{2}{5}$. If possible (indicate if not possible), evaluate the following expressions. Before evaluating, rewrite the expression using numerical values of variables.

a) – *xz*

b) z + 5x

c) z - 20y

d) 100*xy*

e) $z \div 7 + x$

f)
$$\frac{3x}{y}$$

1. Let x = -3, y = -2, $z = \frac{4}{3}$. Evaluate the following expressions if possible (indicate if not possible). Before evaluating, rewrite the expression using numerical values of variables.

a) z - x

b) -x + y + 15z

c) *xyz*

d) $-x^{3}$

e)
$$-y^{2}$$

1. Substitute x = -4, $y = \frac{5}{6}$ in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a) $\frac{1}{3} - xy$

b) (x-2)(6y-x)

c)
$$\frac{x+3}{y}$$

d) x + 3xy

e)
$$\frac{x+2}{6y}$$

1. Let a = 0.1, b = -0.2, c = -1. If possible (indicate if not possible), evaluate the following expressions. Before evaluating, rewrite the expression using numerical values of variables.

a)
$$\frac{a+b}{c}$$

b)
$$-\frac{b}{a}-c$$

c) a-b+c

d) a - (b + c)

e) ab+c

1. Evaluate the following expressions, if m = -1, n = 2, and p = -3. Before evaluating, rewrite the expression substituting the numerical values of variables.

a) -m-n+p

b) $(m+p)^{n}$

c) $-(m+p)^n$

2. Substitute $a = \frac{2}{13}$, $b = -\frac{5}{2}$ in the following expressions and then evaluate, if possible. If not possible, clearly indicate so.

a) 26a - b



c) $ab + \frac{5}{13}$

d) 6*b*-39*a*

- 1. Find the value of a + bc if
- a) a = -2, b = 3, c = -1

b)
$$a = \frac{1}{2}$$
, $b = -2$, $c = -4$

c)
$$a = \frac{2}{3}$$
, $b = 6$, $c = -\frac{1}{2}$

d)
$$a = -0.3$$
, $b = 0.1$, $c = 3.2$

- 1. Find the value -a+b-c if
- a) a = -2, b = 3, c = -4

b)
$$a = \frac{1}{8}$$
, $b = \frac{3}{8}$, $c = -\frac{7}{8}$

c)
$$a = \frac{3}{7}$$
, $b = -\frac{1}{14}$, $c = \frac{1}{2}$

Lesson 3

Topics:

Equivalent algebraic expressions, Review for Test 1.

Definition of equivalent algebraic expressions:

Suppose we wish to write as an algebraic expression "the sum of 2 and a number *x*". Should we write x+2 or 2+x? Because of the commutative property of addition, both answers are right. Both have the same meaning, although they appear to be different. We encounter a similar idea in arithmetic: fractions $\frac{2}{4}$ and $\frac{1}{2}$ are equivalent, which means that they represent the same number although they 'do not look the same'. Similarly, we would say that x+2 and 2+x are equivalent (we often say equal) and write x+2=2+x.

Equivalent AlgebraicTwo algebraic expressions are equivalent if when evaluated,Expressionthey have the same value for all replacements of variables

Suppose that two algebraic expressions are equivalent, like the two mentioned above: x + 2 and 2+x. What it means, according to the definition, is that if we choose any value of x, let's say x = 1, and evaluate x + 2 = 1 + 2 = 3, and then evaluate 2 + x = 2 + 1 = 3, the results must be the same. If we change the value of x, for example to 4, again two results are equal (x+2=4+2=6, and 2+x=2+4=6). No matter what the value of x, the two results are always going to be equal. Thus, to determine that two expressions are equivalent one would have to evaluate them for *all* possible sets of values of variables. Since we cannot check *all*, we cannot prove equivalence by performing evaluation (make sure that you understand that **even if we determine that two expressions assume the same value for many sets of values of variables, we still cannot claim that the two expressions are equivalent). To prove the equivalence of algebraic expressions, some general rules must be employed.**

Terms and factors:

In arithmetic, we often refer to numbers that are being added as terms, and to numbers multiplied as factors. For example, 3 and 4 are terms of addition 3+4=7, while 3 and 5 are called factors of 15, since $3 \times 5 = 15$.

The notion of terms and factors can be generalized:

Factors

Algebraic expressions that are multiplied are called factors.

Expression 4mn has factors 1, 4, *m*, *n* and a combination of these, like 4m, 4n, mn and of course, 4mn. In ab^2 , expressions *a*, b^2 are called factors but 1, *b*, b^2 , *ab*, ab^2 are also factors of this expression. Expressions 2x-3, x+4 are factors in (2x-3)(x+4).

TermsAlgebraic expressions that are added (or subtracted) are
called terms. Each sign, (+ or -), is a part of the term that
follows the sign.

In other words, the addition and subtraction signs break the expression into smaller parts, called terms, and so, in 3x + 2xy - y there are three terms: 3x, 2xy, -y. Notice that because y is preceded by a minus sign, the minus sign is a part of the term: -y. Expressions a, $\frac{5a}{d}$ are terms in $a + \frac{5a}{d}$. Some expressions have just one term. For example: both 3xy and x^3y have one term.

Algebra is an abstract generalization of arithmetic, where numbers are 'replaced' with variables. **The laws that are true for numbers hold also for algebraic** *expressions* (recall, algebraic expressions are merely symbolic representations of numbers). We will discuss some of the laws in light of equivalent expressions:

Commutative property of addition. Rearranging terms results in equivalent expression:

We know that 3+5 and 5+3 are both equal to the same number, 8. It is because the result of addition does not depend on the order of numbers that are being added. This property is called the commutative property of addition. Remember, subtraction does not have this property: $5-3 \neq 3-5$. But, if we view subtraction as the addition of the opposite number, we get: 5-3=5+(-3)=-3+5.

With the use of variables, we can express the above ideas in a general form (without the use of specific numbers). For any value of x and y,

Commutative Property of Addition

x + y = y + x

Also, since x - y = x + (-y) = -y + x, we have:

Consequence of Commutative Property of Addition

x - y = -y + x

Equivalently, we can say that, **changing the order of terms results in an equivalent expression** (expression that looks different, but 'means' the same).

For example,

Terms of a+3 are: a, 3. If we reverse the order of terms, we obtain: a+3=3+a.

Terms of $a^2 - b$ are: a^2 , -b Reversing their order gives us: $a^2 - b = -b + a^2$.

Let's consider an expression that consists of more than two terms and possibly is a mixture of addition and subtractions $(3-B+C, -x^2+2xy-y+x)$. Since subtraction is the same as adding the opposite, we can view the expression as repeated addition, and we are able to rearrange the order of terms. For example, the terms of 3-B+C are: 3, -B, and C. We can write them in any order: 3-B+C=-B+C+3 or 3-B+C=C+3-B (There are more possible rearrangements).

(There are more possible realizingenerics).

Commutative property of multiplication. Rearranging factors results in equivalent expression:

Multiplication, like addition, is commutative. the result of multiplication does not depend on the order: $3 \times 5 = 5 \times 3$. In general, we have:

Commutative Property of Multiplication

 $xy = x \cdot y = y \cdot x = yx$

This means that, the rearrangement of the order of factors results in an equivalent expression. For example,

 $2a = a \cdot 2$, $x^3 y = yx^3$ or (a+b)x = x(a+b).

Factors can be also rearranged if we have repeated multiplication (there are more than two factors). For example,

 $3ab = ba \cdot 3$, cdef = dcef = efdc = fedc (or any other order of factors of *c*, *d*, *e*, and *f*)

Applying rules of operations on fractions results in an equivalent expression:

The rules for addition and subtraction of fractions with common denominators are:

Kule for Addition and Subtraction of Fractions $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ $\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$ $c \neq 0$	
---	--

For example: $\frac{2+x}{7} = \frac{2}{7} + \frac{x}{7}$, $\frac{a-3}{x} = \frac{a}{x} - \frac{3}{x}$, or $\frac{2}{x-3} - \frac{y}{x-3} = \frac{2-y}{x-3}$.

The rules for multiplication and division of fractions:

Rule for Multiplication and Division of Fractions

$$\frac{ax}{y} = \frac{a}{y} \cdot x = a \cdot \frac{x}{y}, \qquad \qquad \frac{x}{y} = x \div y = x \cdot \frac{1}{y}$$
$$y \neq 0$$

Thus, we should immediately recognize, that for example,

$$\frac{x}{2} = \frac{1}{2}x$$
, $\frac{3x}{5} = \frac{3}{5}x$, or $(a+b)\frac{1}{3} = \frac{a+b}{3}$

Finally, you might recall that $-\frac{1}{2} = \frac{-1}{2} = \frac{1}{-2}$. Similarly, one may show (ask your instructor, if you would like to see how to do it) that:

Rule for Negative Signs in Fractions

 $-\frac{x}{y} = \frac{-x}{y} = \frac{x}{-y}$

Performing numerical operations results in equivalent expressions:

There are many operations one can perform on an algebraic expression to obtain an equivalent one. One of them is performing numerical operations according to the presumed order of operations.

For example, 2+3+x=5+x $2 \cdot 3x = 6x$

How to show that two expressions are not equivalent:

Two algebraic expressions are equivalent, if for all values of variables they assume the same value. Thus, if we can find just one set of values of variables for which the expressions do not assume the same value, it is enough to conclude that they are not equivalent.

As an illustration, we will demonstrate that x^2 is not equivalent to 2x.

To this end, we must find some value of x that when evaluated, the two expressions assume different values. We will use x = 5 (the choice of the value of x is arbitrary). We evaluate both algebraic expressions:

 $x^2 = 5^2 = 25$ and $2x = 2 \times 5 = 10$.

Since $25 \neq 10$, we conclude that 2x is not equivalent to x^2 .

Notice that there are other values of x for which 2x is not equal to x^2 , but since we only need one such value, we already proved that x^2 in not equivalent to 2x.

Examples and Problems with Solutions

Example 3.1 List all explicit factors of the following multiplication:

a) 5ab b) -3(a-1)x

Solution:

- a) The factors are: 5, a, b.
- b) The factors are; -3, (a-1), x.

Example 3.2 List all terms of the following expression:

a) 3 - y + z b) $-3x^2 + 4(z+1) - 2y$

Solution:

a) The terms are: 3, -y, z. Remember that signs are always a part of terms that follow: we list -y as a term

b) The terms are: $-3x^2$, 4(z+1), -2y. Notice the minus sign in $-3x^2$ and -2y, and that the expressions 4(x+1) should be viewed as one term.

Example 3.3 Using the fact that x - y = -y + x, rewrite the following expressions in their equivalent form. Use the equal sign to indicate that the resulting expressions are equivalent. a) 3a - b b) $-(b + c)^2 + 3a^2$

Solution:

a) According to x - y = -y + x, terms can be equivalently written in a different order. The terms are: 3a and -b. We get: 3a - b = -b + 3a

b) The terms of $-(b+c)^2 + 3a$ are $-(b+c)^2$ and 3a. If we reverse the order, we get: $-(b+c)^2 + 3a = 3a - (b+c)^2$

Example 3.4 Determine which of the following expressions are equal to 2x - 5y + 4z:

a) $2x + 4z - 5y$	b) $-5y + 4z + 2x$
c) $-5y + 2x + 4z$	d) $4z + 2x - 5y$

Solution:

All of them are. The terms of 2x - 5y + 4z are: 2x, -5y, 4z. As long as the sign that is in front of an expression is not altered, we can rearrange terms. The sign that goes before 2x is plus, and in all expressions (a)-(d) 2x is also preceded by a plus sign. 5y follows the minus sign and the same is true for all expressions (a)-(d). Finally, 4z is preceded by a plus sign in all these expressions.

Example 3.5 Rewrite the expression $y \cdot x \cdot \frac{a+1}{b}$ in two equivalent forms by multiplying its factors in a different order:

Solution:

We can rewrite the above expression in more than two equivalent forms. For example: $x \cdot y \cdot \frac{a+1}{b}$, $x \cdot \frac{a+1}{b} \cdot y$, $\frac{a+1}{b} \cdot yx$, $\frac{a+1}{b} \cdot x \cdot y$, or $y \cdot \frac{a+1}{b} \cdot x$. Any two would be the correct answer.

Example 3.6 Rewrite each of the following expressions as a sum or a difference of two expressions. Use: $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ or $\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$ (we assume that $c \neq 0$). Indicate with the equal sign that the resulting expressions are equivalent.

a)
$$\frac{2+s}{5}$$
 b) $\frac{a^2-2}{5+x}$

Solution:

a)
$$\frac{2+s}{5} = \frac{2}{5} + \frac{s}{5}$$

b) $\frac{a^2 - 2}{5+x} = \frac{a^2}{5+x} - \frac{2}{5+x}$

Example 3.7 Use the fact that $\frac{ax}{y} = \frac{a}{y}x$ to rewrite the following expressions as a product of a

numerical factor and an algebraic expression: a) $\frac{2b}{5}$ b) $\frac{x+b}{5}$

Solution:

a)
$$\frac{2b}{5} = \frac{2}{5}b$$

b) $\frac{x+b}{5} = \frac{1}{5}(x+b)$ (Notice the use of parentheses)

Example 3.8 Rewrite each of the following expressions in their equivalent form as a single fraction:

a)
$$\frac{6}{11}a$$
 b) $2\frac{x}{y}$ c) $\frac{1}{3}(a^2+1)$ d) $\frac{a}{x-2} + \frac{2b}{x-2}$ e) $-\frac{a}{4s} - \frac{1}{4s}a^2$

Solution:

a)
$$\frac{6}{11}a = \frac{6a}{11}$$

b) $2\frac{x}{y} = \frac{2x}{y}$
c) $\frac{1}{3}(a^2+1) = \frac{1(a^2+1)}{3} = \frac{a^2+1}{3}$ (write a^2+1 instead of $1(a^2+1)$)
d) $\frac{a}{x-2} + \frac{2b}{x-2} = \frac{a+2b}{x-2}$
e) $-\frac{a}{4s} - \frac{1}{4s}a^2 = \frac{-a}{4s} - \frac{a^2}{4s} = \frac{-a-a^2}{4s}$ First replace $-\frac{a}{4s}$ by $\frac{-a}{4s}$, and $\frac{1}{4s}a^2$ by $\frac{a^2}{4s}$

(equals can be substituted by equals). Then subtract, just as you subtract fractions.

Example 3.9 Explain why the following statements are true:

a)
$$v + \frac{1}{2} = \frac{1}{2} + v$$
 b) $a^2b = ba^2$ c) $\frac{-3}{x} = \frac{3}{-x}$

Solution:

a) $v + \frac{1}{2} = \frac{1}{2} + v$ is true because of the commutative property of addition. b) $a^2b = ba^2$ is true because of the commutative property of multiplication. c) $\frac{-3}{x} = \frac{3}{-x}$ true because of the rule of negative signs in fractions. Example 3.10 Replace Ψ with expressions such that the resulting statements are true. Use parentheses when needed: a) $2x - y + 5z = \Psi + 2x$ b) $-xyz = z \cdot \Psi$ Solution:

a)
$$2x - y + 5z = -y + 5z + 2x$$
; $\Psi = -y + 5z$
b) $-xyz = z(-x)y$; $\Psi = (-x)y$

Example 3.11 Determine which of the following expressions are equal to $\frac{2+b}{c-4}$:

b+2	2+b	2 b	$\frac{1}{(2+b)}$	2+ <i>b</i>
$\overline{c-4}$,	$\overline{4-c}$,	$\overline{c-4}$ $+$ $\overline{c-4}$,	$\frac{1}{c-4}(2+b),$	-4 + c

Solution:

The only expression that is not equivalent is $\frac{2+b}{4-c}$ (since $c-4 \neq 4-c$). To demonstrate that, let us set b=0, c=0 and evaluate: $\frac{2+b}{c-4} = \frac{2+0}{0-4} = -\frac{1}{2}$, $\frac{2+b}{4-c} = \frac{2+0}{4-0} = \frac{1}{2}$. Since $\frac{1}{2} \neq -\frac{1}{2}$, we conclude that $\frac{2+b}{c-4}$ is not equivalent to $\frac{2+b}{4-c}$. The expression $\frac{b+2}{c-4}$ is equivalent because of the commutative property of addition. $\frac{2}{c-4} + \frac{b}{c-4}$ is equivalent because of the rules for addition of fractions. $\frac{1}{c-4}(2+b)$ is equivalent because of the rules for multiplication of fractions. $\frac{2+b}{-4+c}$ is equivalent because we replaced c-4 by its equivalent expression: -4+c

<u>Example 3.12</u> When possible, perform a numerical operation to create an equivalent expression. If no numerical operation can be performed, clearly indicate so:

a) $-2 + x + 1$	b) $\frac{4x}{8}$	c) $12 + 3x$	d) $(2 \times 3)^m$
e) 2×3^m	f) $2x(-3)y$	g) $-(-x)$	h) $6 \cdot \frac{x}{3}$

1 ...

Solution:

a) -2 + x + 1 = -2 + 1 + x = -1 + x

b) $\frac{4x}{8} = \frac{x}{2}$ (Divide the numerator and denominator by 4 to reduce the fraction)

c) 12+3x Since multiplication of 3 and x has to be performed before addition, no numerical operation can be performed.

d) $(2 \times 3)^m = 6^m$

e) 2×3^m Since 3 must be first raised to *m*-th power, no numerical operation can be performed

f)
$$2x(-3)y = 2(-3)xy = -6xy$$

g) $-(-x) = (-1)(-1)x = x$
h) $6 \cdot \frac{x}{3} = 2x$ We can cancel 6 and 3.

Example 3.13 Show that the following two expressions are not equivalent:

a) $(2a)^3$ and $2a^3$ b) -(x-y) and -x-y

Solution:

a) We need to find one value of *a* such that when we evaluate $(2a)^3$ and $2a^3$, we obtain different results. We can use a = 1:

 $(2a)^3 = (2 \times 1)^3 = 2^3 = 8$ but $2a^3 = 2 \times 1^3 = 2 \times 1 = 2$. Since $8 \neq 2$, $(2a)^3$ and $2a^3$ are not equivalent.

b) Let us set x = 1, y = 1. Then -(x - y) = -(1 - 1) = 0 but -x - y = -1 - 1 = -2. Since $0 \neq -2$, the expressions -(x - y) and -x - y are not equivalent.

Exercises with answers (For answers see Appendix A)

Ex. 1 Write a word to complete each sentence:

In the expression $4x^2 \times 2y$, $4x^2$ and 2y are called ______ . In the expression $4x^2 + 2y$, $4x^2$ and 2y are called ______ .

Ex. 2 Rewrite the following expressions placing the multiplication sign ' \times ' whenever (according to the convention) it was omitted. Then, identify all explicit factors.

a) 2a b) 3(a+b) c) $-3x\frac{2}{y}$ d) 4(x+y)(b-c)

Ex. 3 List all terms of the following expressions:

a)
$$3+x$$
 b) $ab-cd$ c) $\frac{xy}{2}+2y^2-1$ d) $-(2-b)^2+\frac{x}{y}-z$

Ex.4 For each of the following expressions:

- List all its terms

- Rewrite in its equivalent form using commutative property of addition: x + y = y + x Use the equal sign to indicate that the resulting expressions are equivalent to the original ones (for example, the expression A + 9 should be rewritten as A + 9 = 9 + A).

a)
$$2m + z$$
 b) $2x^2 + y^3$ c) $c(d - f) + y^2$

Ex.5 Is 2+8 equal to 8+2? Is x + 8 equal to 8 + x? How about $\frac{2a}{b} + \frac{cd}{2}$ and $\frac{cd}{2} + \frac{2a}{b}$? Why?

Ex.6 a) Evaluate m - n and n - m when m = 2 and n = 3. Based on this evaluation, can you determine if the two expressions are equivalent? b) Is it true that m - n = -n + m?

- *Ex.7* For each of the following expressions:
 - List all of its terms

- Rewrite in its equivalent form using x - y = -y + x (notice, this it also means that: -y + x = x - y). Use the equal sign to indicate that the resulting expression is equivalent to the

original one (for example, the expression A - 9 should be rewritten as A - 9 = -9 + A).

a)
$$x-2$$
 b) $-3c+2$ c) $-(x-y)^2 + s$ d) $\frac{yx^2}{2} - \frac{(cd+f)^2}{t}$

Ex.8 List all terms, and then, by changing the order of these terms, create two new equivalent expressions for each of the following: a) $-x^2 + x - x^3$ b) $-a^2 - 2bc + \frac{3x}{2}$

Ex.9 For each of the following expressions (1)-(5) find an expression equivalent to it among expression (A)-(E). Rewrite each matched pair with the equal sign between them to indicate their equivalence. (1) s + t + u (A) t - u + s

(1) 5 1 1 1 10	() · · · · ·
(2) - t + s + u	(B) $-s - t + u$
(3) - u + s + t	(C) $t + s + u$
(4) $u - t - s$	(D) $s - u - t$
(5) $s - t - u$	(E) $s+u-t$

Ex.10 Rewrite each of the following expressions in its equivalent form using xy = yx. Use the equal sign to indicate that the resulting expression is equivalent to the original one (for example, the expression 9A should be rewritten as $9A = A \cdot 9$). Remember about parentheses: a) mn b) -5×7 c) -cd d) -c(a+d)

Ex.11 a) Rewrite the expression *vst* in its equivalent form by changing the order of its factors to create three new equivalent expressions. Indicate their equivalence by using the equal sign (for example, one of the answers might be vst = tsv). b) Repeat the above exercise for v(x - y)t.

Ex.12 a) Is *AB* equivalent to *BA*? How about -3AB and -3BA, -3AB and BA(-3)? Why?

b) Is -3AB equivalent to BA-3? Why? Support your answer by evaluating both expressions when A = 1 and B = 2.

Ex.13 According to the rules for adding and subtracting fractions, we have: $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ and $\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$ (assume $c \neq 0$) Rewrite each of the expression below as a sum or a difference of two expressions. Use equal signs to indicate that the resulting expressions are equivalent to the original ones (for example, the expression $\frac{2-t}{3}$ should be rewritten as $\frac{2-t}{3} = \frac{2}{3} - \frac{t}{3}$):

a)
$$\frac{2-5}{7}$$
 b) $\frac{a+6}{3}$ c) $\frac{a-2}{a+b}$ d) $\frac{ab+ca}{ab^2-c}$

Ex.14 Using the fact that $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ and $\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$ (assume $c \neq 0$), rewrite the following expressions in a form of a single fraction.

a)
$$\frac{m}{4} + \frac{n}{4}$$
 b) $\frac{7m}{4} - \frac{n^2}{4}$ c) $\frac{5m}{4c-2} - \frac{2n^2}{4c-2}$

Ex.15 In the expression $\frac{a}{d} + \frac{b}{d} - \frac{c}{d}$ replace $\frac{a}{d} + \frac{b}{d}$ with a single fraction. You now have an expression with two fractions. Replace these two fractions with a single one. As a result, you get: $\frac{a}{d} + \frac{b}{d} - \frac{c}{d} = \frac{a+b-c}{d}$. Applying the same idea, write the following expressions in a form of a single fraction. a) $\frac{4}{5} - \frac{7}{5} + \frac{2}{5}$ b) $\frac{7m}{4x} + \frac{n^2}{4x} - \frac{3}{4x}$ c) $\frac{m}{s-1} - \frac{3}{s-1} - \frac{t}{s-1}$

single fraction. a) $\frac{1}{5} - \frac{1}{5} + \frac{1}{5}$ b) $\frac{1}{4x} + \frac{1}{4x} - \frac{1}{4x}$ c) $\frac{1}{s-1} - \frac{1}{s-1} - \frac{1}{s-1}$

Ex.16 According to the rule for multiplication of fractions the following is true: $\frac{3x}{7} = \frac{3}{7}x$. We can say that the quotient $\frac{3x}{7}$ was written as a product of a numerical factor $\frac{3}{7}$ and an algebraic expression x. Write the following expressions as a product of a numerical factor and an algebraic expression:

a)
$$\frac{2x}{3}$$
 b) $\frac{2x^2}{3}$ c) $\frac{-2x^2}{3}$ d) $\frac{-2(a+2b)}{3}$ e) $\frac{x}{3}$ f) $\frac{a+2b}{3}$

Ex.17 According to the rule for multiplication of fractions $a\frac{x}{y} = \frac{ax}{y}$ (for example $2 \cdot \frac{x}{y} = \frac{2x}{y}$). According to this rule, rewrite each of the following expression as a single fraction. Remember about parentheses.

a)
$$3 \cdot \frac{m}{n}$$
 b) $3 \cdot \frac{-m}{n}$ c) $a\left(\frac{-b}{4}\right)$ d) $-a\left(\frac{-b}{4}\right)$ e) $(s-4)\frac{t}{n}$ f) $4 \cdot \frac{-a}{n-1}$

Ex.18 You know that $-\frac{x}{y} = \frac{-x}{y} = \frac{x}{-y}$. Find the opposite of each of the expressions below. By placing the minus sign differently, write your answer in three equivalent ways. Use parentheses when needed. a) $\frac{2a}{b}$ b) $\frac{2a+c}{c-2d}$

Ex.19 For each of the following pairs of expressions decide if they are equivalent or not (always assume that the denominator is different from zero):

a)
$$\frac{a+b}{c}$$
 and $\frac{b}{c} + \frac{a}{c}$ b) $\frac{a-b}{c}$ and $\frac{-b}{c} + \frac{a}{c}$ c) $\frac{a-b}{c}$ and $\frac{1}{c}(b-a)$
d) $\frac{a-b}{c}$ and $(-b+a)\frac{1}{c}$ e) $a\left(\frac{b}{c}\right)$ and $a\cdot\frac{1}{c}\cdot b$ f) $\frac{ab}{c}$ and $ba\frac{1}{c}$

Ex.20 Write the following expressions as a single fraction:

a)
$$\frac{2}{3}x + \frac{4}{3}y$$
 b) $\frac{2}{3}x - \frac{1}{3}y$ c) $2x \cdot \frac{1}{3} - 7 \cdot \frac{y}{3}$ d) $\frac{3}{t}x - \frac{1}{t}y$
e) $-x \cdot \frac{1}{3} - 4 \cdot \frac{y}{3}$ f) $-2\frac{x}{t} + 3\frac{y}{t}$ g) $-5 \cdot \frac{m}{k+t} - \frac{n}{k+t}$ h) $-\frac{1}{k+t} - 2\frac{n}{k+t}$

Ex.21 Is ab+2 equivalent to 2+ab? How about ab+2 and 2+ba? Why? How about (mn+4)(a+b) and (b+a)(4+nm)? Why?

Ex.22 Is
$$\left(\left(-\frac{1}{2}\right)^2 + \frac{3}{4}\right)x$$
 equivalent to x? Is $x\left(\left(-\frac{1}{2}\right)^2 + \frac{3}{4}\right)$ equivalent to x?

Ex.23 Create three different expressions equivalent to: a) ab + 2c b) $\frac{2-x}{4}$

c)
$$\frac{x}{y}$$

Ex.24 Determine which of the following expressions are equivalent to $\frac{5x}{6}$:

$$5 \cdot \frac{x}{6}, \quad \frac{5}{6}x, \qquad x \cdot \frac{5}{6}, \qquad \frac{5}{6x}, \qquad \frac{10x}{12}, \qquad 6 \cdot \frac{x}{36} \cdot 5, \qquad \frac{1}{6} \cdot 5x$$

Ex.25 Determine which of the following expressions are equivalent to m-n: n-m, m(-n), m-(n), -nm, -n+m, (-1)n-m

Ex.26 Determine which of the following expressions are equivalent to $x \div 2$: $\frac{x}{2}$, 2x, $x \cdot 2$, x^2 , $\frac{4x}{8}$, $\frac{-x}{-2}$

$$\frac{b}{6}, \frac{b}{6}, \frac{c}{6}(3a-b), \frac{b}{6}-b, \frac{b}{6}-\frac{b}{6}, \frac{b}{6}, \frac{b}{6}, \frac{b}{6}, \frac{c}{6}, \frac{c$$

Ex.28 Determine which of the following expressions are equivalent to 3+8a: 11*a*, 8a+3, $3+a\cdot8$, 11+a, $2\cdot\frac{3+8a}{2}$

Ex.29 Determine which of the following expressions are equivalent to -m $\frac{m}{-1}$, $-\frac{m}{1}$, m(-1), m-1, $-\frac{1}{m}$

Ex.30 Determine which of the following are equivalent to $-\frac{a}{b}$:

$$\frac{-a}{b}$$
, $-a \cdot \frac{1}{b}$, $-b \cdot \frac{1}{a}$, $-\frac{2a}{2b}$, $\frac{a}{-b}$, $\frac{-a}{-b}$

Ex.31 Determine which of the following are equivalent to $\frac{m}{3} - \frac{n}{3}$:

$$\frac{n}{3} - \frac{m}{3}, \qquad \frac{m-n}{3}, \qquad \frac{1}{3} \cdot m - \frac{1}{3}n, \qquad (m-n) \cdot \frac{1}{3}, \qquad \frac{-n+m}{3}$$

- *Ex.32* Determine which of the following are equivalent to -a-c+b+d: -a+b-c+d, -a+b-c-d, d+b-c-a, a+c-b-d
- *Ex.33* Determine which of the following expressions are equivalent to -a-b:

$$-b-a$$
, $+\left(\frac{-b}{1}\right)+\left(\frac{-a}{1}\right)$, $\frac{-b-a}{1}$, $-1a-b$, $(-b)(-a)$

Ex.34 Which of the following expressions are equivalent to $\frac{9C}{5} + 32$

$$\frac{5C}{9} + 32, \ 32 + \frac{9C}{5}, \quad \frac{9C}{5} + 4 \times 8, \quad \frac{180C}{100} + 32, \quad \frac{32C}{9} + 5, \quad \frac{-9C}{-5} - (-32)$$

Ex.35 Fill in the blanks to make a true statement:

a)
$$\frac{3x}{y} = x \cdot \underline{\qquad}$$

b) $\frac{3x}{y} = 3 \cdot \underline{\qquad}$
c) $\frac{3x}{y} = \frac{1}{y} \cdot \underline{\qquad}$
d) $\frac{a+b-c}{y} = \frac{1}{y} \cdot \underline{\qquad}$

Ex.36 Replace Ψ with expressions such that the resulting statement is true. Use parentheses when needed.

a)
$$a - 2b + c = c - 2b + \Psi$$
 b) $x = \frac{x}{\Psi} \cdot 4$ c) $\frac{xy}{4} = \frac{1}{4}x\Psi$ d) $a \div 2 = \frac{\Psi}{2}$
e) $\frac{x + y}{3} = \frac{x}{3} + \Psi$ f) $z - c = -c + \Psi$ g) $-xyz = yz\Psi$ h) $\frac{ab}{2} = a \cdot \Psi$

Ex.37 Perform all numerical operations that are possible. If none is possible, clearly indicate so.

a)
$$4 + a - 8$$

b) $4a(-8)$
c) $\frac{8a}{4}$
d) $8^2 x$
e) $8x^2$
f) $4 - 2x$
g) $(4 - 2)x$
h) $-(-2x)$
i) $(5 - 2)^m$
j) $2x^2\left(\frac{1}{2}\right)$
k) $-0.1x(10y^2)$
l) $5 - 2^m$
m) $10 \cdot \frac{x}{5}$
n) $2 \cdot (6 - 5)x$
o) $4 \cdot \frac{2}{5} - x$
p) $\frac{12y}{2xz}$
q) $-\frac{1}{2} - x - \frac{1}{2}$
r) $\frac{3bd}{9ac}$

Ex.38 Evaluate $x^2 + y^2$ and $(x + y)^2$, when x = -1, y = 2. Based on your results, is $x^2 + y^2$ equivalent to $(x + y)^2$?

Ex.39 Evaluate m-n+p and m-(n+p), when m=2, n=5, and p=1. Based on your results, is m-n+p equivalent to m-(n+p)?

Ex.40 Show that $(-x)^4$ is not equivalent to $-x^4$ by evaluating both expressions when x = -1.

*Ex.*41 a) Using x = 2, y = 2, and z = 2 show that x(y - z) is not equivalent to xy - z.

b) Now evaluate x(y-z) and xy-z when x = 1, y = 1, and z = 1. Do your results contradict the fact that x(y-z) and xy-z are not equivalent?

Ex.42 Is (x+2y)(1+3a) equivalent to x+2y(1+3a)? Explain your answer.

Ex.43 Evaluate $(-1)^m$ and -1^m when

- a) m = 1
- b) m = 3
- c) m = 5
- d) m = 7
- e) Based on the above, can you determine if $(-1)^m$ and -1^m are equivalent?

f) Evaluate $(-1)^m$ and -1^m when m = 2. Can you now determine if $(-1)^m$ and -1^m are equivalent?

Ex.44 Two algebraic expressions were evaluated for two different sets of values of variables. For the first set of values the first expression assumed value 3, and the second one assumed 4. For the second set of values the first expression assumed value 6, and the second one 6.

Which of the following is true?

- a) The two algebraic expressions are equivalent
- b) The two algebraic expressions are not equivalent
- c) No conclusion can be reached

Ex.45 Two algebraic expressions were evaluated for two different sets of values of variables. For the first set of values both expressions assumed the value 3, and for the second set of values both assumed 6. Which of the following is true?

- a) The two algebraic expressions are equivalent
- b) The two algebraic expressions are not equivalent
- c) No conclusion can be reached

1. How many terms are in the following expressions? List all of them.

a) 4 + y

b) $2 - c + x^2$

c) 2a - 4ab + 5a(b + c)

d) 3*xy*

2. Rewrite the following expressions placing the multiplication sign ' \times ' whenever (according to the convention) it was omitted. In each expression, identify all explicit factors

a) 2*x*

b) – 3*ab*

c)
$$2\frac{x-1}{t}$$

d) 5(b+5)(b-c)

- 1. For each of the following expressions:
 - List all its terms

- Rewrite in its equivalent form using commutative property of addition: x + y = y + x (this rule allows us to rearrange the order of terms). Use the equal sign to indicate that the resulting expression is equivalent to the original one (for example, the expression A + 9 should be rewritten as A + 9 = 9 + A).

a) 2x + 7

b)
$$x + \frac{7}{3}$$

c) $(a+b)^2 + x$

d) $(a+b)^2 + (c-d)^2$

2. List all terms of each of the following expressions, and then use the fact that we can change the order of those terms: x - y = -y + x (or equivalently -y + x = x - y), rewrite the expression. Use the equal sign to indicate that the resulting expression is equivalent to the original one.

a)
$$2x - 7$$

b)
$$x - \frac{7}{3}$$

c) $-2a + (4xy)^2$

d)
$$-\frac{1}{y-2}+2$$

1. Using the fact that x + y = y + x, rewrite each of the following expressions in its equivalent form. Use the equal sign to indicate that the resulting expression is equivalent to the original one (for example, the expression A + 9 should be rewritten as A + 9 = 9 + A):

a)
$$A^2 + 6B$$

b) $\frac{x}{y} + \frac{y}{x}$

c) $(2cb)^2 + 2(a-2d)^3$

2. Using the fact that x - y = -y + x (or equivalently -y + x = x - y), rewrite each of the following expressions in its equivalent form. Use the equal sign to indicate that the resulting expression is equivalent to the original one:

a) 3A - 4B

b)
$$3 - \frac{2x}{5}$$

c) $(a+b)^2 - x$

d) $(a+b)^2 - (c-d)^2$

3. Replace *A* with an expression such that the resulting statement is true.

 $2xy - a^2c = A + 2xy$

1. Using the fact that x - y = -y + x (or equivalently -y + x = x - y), rewrite each of the following expressions in its equivalent form. Use the equal sign to indicate that the resulting expression is equivalent to the original one:

a) $-a^2 + 2b$

b) $2b - (a - d)^2$

2. For each of the following expressions (1)-(5), find an expression equivalent to it among expressions (A)-(E). Write each matched pair with the equal sign between them to indicate their equivalence.

(1) 2s + 3v - w(A) 2s - 3v - w(2) -3v + 2s - w(B) w - 3v - 2s(3) w - 2s + 3v(C) 2s + w - 3v(4) w + 2s - 3v(D) 3v - w + 2s(5) -3v + w - 2s(E) 3v + w - 2s

- 2. Replace Γ with an expression such that the resulting statement is true.
- a) $x + y + z = \Gamma + x + y$
- b) $-x + y + z = \Gamma x$
- c) $ab-cd+2g = \Gamma + ab+2g$
- d) $-ab + cd + 2g = \Gamma + cd$
1. Rewrite each of the following expressions in its equivalent form using the commutative property of multiplication: xy = yx. Use the equal sign to indicate that the resulting expression is equivalent to the original one (for example, the expression 9*A* should be rewritten as $9A = A \cdot 9$). Remember about parentheses:

a)
$$-8 \times (-9)$$

- b) *a*(–*b*)
- c) -a(-b)
- d) b(a+1)
- e) -b(a+1)

f)
$$-b \cdot \frac{x+2}{3}$$

- 2. Replace Γ with an expression such that the resulting statement is true.
- a) $xyz = \Gamma \cdot x \cdot y$
- b) $-xyz = z \cdot \Gamma$

1. Write each of the following expressions in its equivalent form as one fraction:

a)
$$\frac{u}{3} + \frac{v^2}{3}$$

b)
$$\frac{s^2}{7} - \frac{w}{7}$$

c)
$$\frac{2}{3x} + \frac{b}{3x}$$

d)
$$\frac{-u}{3-x} - \frac{s}{3-x}$$

2. Write the following expressions in their equivalent form as a sum or difference of two expressions:

a)
$$\frac{2+x}{11}$$

b)
$$\frac{3x^2 - x}{4}$$

c)
$$\frac{a^2 - s^2}{t + 4}$$

Homework/Class Work/Quiz

1. According to the rule for multiplication of fractions $\frac{x}{y}a = a\frac{x}{y} = \frac{ax}{y}$ (for example $2 \cdot \frac{x}{y} = \frac{2x}{y}$). According to this rule, rewrite each of the following expressions as a single fraction. Remember about parentheses.

- a) $\frac{7}{3}m$
- b) $3 \cdot \frac{a}{4}$
- c) $\frac{7}{3-y} \cdot m$
- d) $\frac{7}{m} \cdot (3-y)$
- e) $\frac{-m}{n} \cdot 2$
- f) $-7 \cdot \frac{m}{4}$

g)
$$-7 \cdot \frac{1}{m+2}$$

h)
$$(m-7)\frac{1}{m+2}$$

- 1. Replace Ω with an expression such that the resulting statement is true:
- a) $\frac{abc}{3} = \frac{ab}{3} \cdot \Omega$
- b) $\frac{abc}{3} = a \cdot \Omega$
- c) $\frac{abc}{3} = \frac{1}{3} \cdot \Omega$

d)
$$\frac{abc}{3} = abc \div \Omega$$

- 2. Rewrite each expression in its equivalent form as a single fraction,
- for example, $\frac{1}{4}x = \frac{x}{4}$.

a)
$$\frac{4}{7}y$$

b)
$$z \cdot \frac{7}{9}$$

c)
$$\frac{1}{6}(y^2 + z)$$

d) $(y^2 + 7 - x) \cdot \frac{1}{5}$

- 1. Replace A with an expression such that the resulting statement is true.
- a) $\frac{2a}{9} = \frac{2}{9}A$
- b) $\frac{2a}{9} = A \cdot \frac{a}{9}$
- c) $\frac{2a}{9} = \frac{1}{9}A$
- d) $\frac{2(x^2 + y)}{9} = \frac{2}{9}A$
- e) $\frac{z^2}{9} = \frac{1}{9}A$

f)
$$\frac{z^2 + z}{9} = A(z^2 + z)$$

2. Determine which of the following expressions is equivalent to $\frac{2xy}{9}$

$$\frac{2}{9}xy, \qquad 2\cdot\frac{xy}{9}, \qquad 2xy\cdot\frac{1}{9}, \qquad \frac{1}{9}xy\cdot 2, \qquad \frac{xy}{9}\cdot 2, \qquad \frac{2x}{9}\cdot y$$

1. Using the fact that $-\frac{x}{y} = \frac{-x}{y} = \frac{x}{-y}$ rewrite each of the following expressions twice, each time placing the minus in a different way. Use parentheses when needed:

a)
$$\frac{-2}{3}$$

b) $-\frac{2x}{z}$
c) $\frac{2}{-(a+b)}$

d)
$$\frac{-xy}{c-d}$$

2. Recall that -(-2) = -1(-1)2 = 2. Separate the following numbers into two groups: one, consisting of numbers equal to $\frac{1}{2}$, the other one, consisting of numbers equal to $-\frac{1}{2}$: $\frac{-1}{2}$, $\frac{-(-1)}{2}$, $\frac{1}{-2}$, $-\left(-\frac{1}{2}\right)$, $\frac{-1}{-2}$, $\frac{-1}{-(-2)}$, $-\left(\frac{-1}{-2}\right)$

3. Recall that -(-x) = -1(-1)x = x. Separate the following expressions into two groups: one group consisting of expressions equivalent to $\frac{1}{x}$, another one consisting of expressions equivalent to $-\frac{1}{x}$:

$$\frac{-1}{x}, \quad \frac{-(-1)}{x}, \quad \frac{1}{-x}, \quad -\left(-\frac{1}{x}\right), \quad \frac{-1}{-x}, \quad \frac{-1}{-(-x)}, \quad -\left(\frac{-1}{-x}\right)$$

- 1. Write the following expressions in the form of a single fraction:
- a) $\frac{3}{5}x$
- b) $9 \cdot \frac{s}{t}$
- 2. Write the following expressions as a single fraction:

a)
$$\frac{7}{5}s + \frac{4}{5}t$$

b) $\frac{2}{m}s - \frac{1}{m} \cdot t$
c) $\frac{-2}{m}s + \frac{1}{m} \cdot t$
d) $2 \cdot \frac{s}{m} - 3\frac{t}{m}$
e) $-\frac{7}{m} + 2\frac{t}{m}$
f) $\frac{1}{m+5}s - \frac{1}{m+5}t$
g) $2\frac{s}{m+5} - \frac{4}{m+5}t$

Homework/Class Work/Quiz

1. The correct answer to the question is -m + n. Susan's answer is n - m. Is Susan also correct?

2. Determine which of the following expressions are equal to 2x

 $x \cdot 2$ $\frac{3x}{6}$ -(-2x) 4-2x 5x(-3)

3. Determine which of the following expressions are equal to -abc

b(-a)c $\frac{cba}{-1}$ a-bc $-\frac{bca}{1}$ ab-c ab(-c)

4. Write $\frac{-3a+4b}{x}$ in five different ways.

1. Determine which of the following expressions are equivalent to $\frac{7x}{14}$

 $\frac{x}{2}$ $\frac{70x}{140}$ 2x $\frac{-7x}{-14}$ $\frac{2+5x}{14}$

2. Determine which of the following expressions are equivalent to 3x

 $\frac{3x}{1}$ $\frac{1}{3x}$ (1+2)x 1+2x $x(\frac{1}{2}+\frac{5}{2})$

- 3. Determine which of the following expressions are equal to -x + 2y 7z
- a) -x-7z+2y
- b) 2y 7z x
- c) -7z x + 2y
- d) x 2y + 7z
- e) (-7z)(-x)(+2y)
- f) (-7z x) + 2y

3N

1. Students were to write an answer to the following problem:

Using algebraic symbols, write an opposite number to $\frac{s}{t}$.

Student A gave the answer: $-\frac{s}{t}$ Student B gave the answer: $\frac{-s}{t}$ Student C gave the answer: $\frac{s}{-t}$ Who was right? Why?

2. Replace Ψ with numbers so that the resulting statement is true.

a)
$$x = 2 \cdot \frac{x}{\Psi}$$

b)
$$z = -\Psi \cdot \frac{z}{245}$$

c) $-x = \Psi x$

1. Determine which of the following expressions are equivalent to $\frac{4}{7}h - \frac{3k}{7}$

 $\frac{4h}{7} - \frac{3k}{7} \qquad -\frac{4h}{7} + \frac{3k}{7} \qquad \frac{4h - 3k}{7} \qquad \frac{4}{7}h - \frac{3}{7}k \qquad \frac{1}{7}(4h) - \frac{1}{7}(3k)$

2. Determine which of the following expressions are equal to $\frac{xy+z}{w}$:

z + yx	$\frac{xy}{z}$	$(rv+z) \div \frac{1}{2}$	z + yx	$\frac{1}{2}(vr+z)$
W	w ' w	$(xy + z) \cdot W$	W	W

3. Which of the following expressions are equivalent to $\frac{h(a+b)}{2}$?

$$\frac{(a+b)h}{2}$$
 $\frac{1}{2}h(a+b)$ $\frac{h}{2} + \frac{a+b}{2}$ $\frac{h(b+a)}{2}$ $\frac{(b+a)h}{2}$

Homework/Class Work/Quiz

1. Perform all numerical operations that are possible. If no numerical operation can be performed, clearly say so.

a) 2 + 5y

b) $2 \cdot 5y$

c) $\frac{4x}{16}$

d) 3x(-2)y

e) -1 + x - 2

f) (4+3)(-2a)

1. Perform all numerical operations that are possible. If no numerical operation can be performed, clearly say so.

a) 4 - 3a + 2

b) a^{4+2}

c)
$$\frac{3-y}{3}$$

d) -4+3(2a)

e)
$$\frac{x}{3} + 2$$

f) (1-2)a(-1+3)

1. Perform all numerical operations that are possible. If no numerical operation can be performed, clearly say so.

a) 5(-2*x*)

b) 5 - 2x

c) $-4x^2(-3)(-2)$

d) $-4x^{2}(-3)-2$

e) $\frac{5x}{15}$

f) $4 \cdot 3^m$

1. Perform all possible numerical operations to create equivalent expressions. If no numerical operation can be performed, clearly say so.

a)
$$(-1)x(5)y$$

b)
$$\frac{15(x+y)}{5}$$

c) $(2+\frac{2}{7}\times7)^{m+5}$
d) $-(-4w)$
e) $(-1+4-7)a$
f) $(\frac{1}{3}+\frac{2}{5})^m$

g)
$$-\frac{1}{3}a \cdot 3$$

h) $-x \cdot 3$

1. The correct answer to a problem is $\frac{vt^2}{2}$. John's answer is $\frac{1}{2}vt^2$. Is John right? How about Mary whose answer is: $\frac{t^2}{2} \cdot v$?

2. Is x + 3 = 3 + x? Why?

Is $3x = x \cdot 3$? Why?

How about x - 3 and 3 - x?

3. Perform all possible numerical operations to create equivalent expressions. If no numerical operation can be performed, clearly say so.

a)
$$\frac{-3x}{3}$$

b) 2 + (2 - 3)b

1. Is -(-A) equivalent to -A. Why?

2. When possible, perform a numerical operation to create an equivalent expression. If no numerical operation can be performed, clearly say so.

a) $-3xy \cdot 5$

b) -3+5x

c)
$$\frac{1}{3} + a - \frac{5}{7}$$

3. Determine which of the following expressions are equal to $\frac{x}{2}$:

$$x \cdot \frac{1}{2}, \quad \frac{2x}{4}, \quad \frac{1}{2}x, \quad \frac{x}{2+0}, \quad \frac{2}{x}, \quad \frac{x}{1+1}$$

1. Fill in the blanks to make a true statement:

 $\frac{m}{5} = \underline{\qquad} \cdot m = \frac{1}{5} \cdot \underline{\qquad} = m \cdot \underline{\qquad}$

2. Determine which of the following expressions are equal to $\frac{4+a}{2}$

$$\frac{a+4}{2}$$
 $2+\frac{a}{2}$ $2+a$ $\frac{4}{2}+\frac{a}{2}$ $\frac{1}{2}(a+4)$

3. If performing a numerical operation is possible, perform it. If not, clearly indicate so.

a) $10 \cdot 4^{m}$

b) 4 - 2b(-1)

c)
$$\frac{-3-2}{-3-2x}$$

1. Evaluate -2(x + y) and -2x + y. when x = 1, y = 1. Based on the result, are the two expressions equivalent?

2. Create four different expressions equivalent to 2 + 3x.

3. Determine which of the following expressions are equal to x:

 $1 \times x$, $\frac{x}{1}$, $\frac{0}{2} + x$, $\frac{-x}{-1}$, -2 + 3x, -(-x)

4. Fill in blanks to make the following statements true:

$$\frac{5x}{9} = x \cdot ___= 5 \cdot ___= \frac{1}{9} \cdot ___$$

1. Evaluate x^{y} and y^{x} when x = 2 and y = 3. Based on your calculations, is x^{y} equivalent to y^{x} ?

2. Deterr	nine which of	the following ex	pressions are ed	representation qual to $\frac{2a}{3}$
$\frac{2}{3}a$,	$a\cdot\frac{2}{3}$,	$2\cdot\frac{a}{3}$,	$\frac{3a}{2}$,	$2a \cdot \frac{1}{3}$

- 3. Replace Ψ so that that the resulting statement is true.
- a) $abc = bc\Psi$

b) $abc = 3 \cdot \frac{abc}{\Psi}$

1. Use x = 2, y = 3 and z = 1 to show that x - (y + z) is not equivalent to x - y + z.

2. Use m = 2 to show that 2^m is not equivalent to m^2 .

3. Use n = 1, m = 3 to show that $(n - m)^2$ is not equivalent to $n^2 - m^2$.

4. Using a = 1, b = 2, and c = 3 show that ab + c is not equivalent to a + bc.

Homework/Class Work/Quiz

1. Evaluate $x^2 - 1$ and $-x^2 + 1$ when a) x = 1

b) x = -1

c) Based on the above, can you determine if $x^2 - 1$ and $-x^2 + 1$ are equivalent?

d) Evaluate $x^2 - 1$ and $-x^2 + 1$ when m = 2. Can you now determine if $x^2 - 1$ and $-x^2 + 1$ are equivalent?

2. Two algebraic expressions were evaluated for a given set of variables (the same set for each algebraic expression). The first expression assumed value 5, the second one 3. Which of the following is true?

- a) The two algebraic expressions are equivalent
- c) No conclusion can be reached
- b) The two algebraic expressions are not equivalent

3. Two algebraic expressions were evaluated for a given set of variables (the same set for each algebraic expression). Both expressions assumed value 5. Which of the following is true?

- a) The two algebraic expressions are equivalent
- b) The two algebraic expressions are not equivalent
- c) No conclusion can be reached

Lesson 4

Topics:

Test1; Operations on power expressions with positive integer exponents.

Exponential notation

Exponential notation is used to write repeated factors in a compact way. Exponents are another way of writing multiplication. For example,

$$2^5 = \underbrace{2 \times 2 \times 2 \times 2 \times 2}_{5-times}, \quad 3^2 = \underbrace{3 \times 3}_{t-times}.$$

We will extend this idea to algebraic expressions.

Exponential Expression

The expression of the form a^n is called an exponential expression. It is defined as follows:

$$a^{0} = 1,$$

$$a^{1} = a,$$

$$a^{2} = a \times a,$$

$$a^{n} = \underbrace{a \times a \times \dots \times a}_{n \text{ times}},$$
 for $n = 1, 2, 3, \dots$

(Notice there are n *a*'s in the product). *a* is called the base, *n* is called an exponent (or power).

Notice that, according to the definition any expression raised to the zero-th power is equal one. Also, a **variable that appears to have no exponent is raised to the first power:**

 $a^1 = a$

Recall that a^n is read as "*a* to the n-th power". In the case of a^2 , often, instead of "*a* to the second power" we read it as "*a* squared", a^3 is often read as "*a* cubed".

You should also remember that:

The exponent pertains only to "the closest" number or variable:

In $2b^3$, only *b* is raised to the third power,

In $-x^n$, only x is raised the *n*-th power (*n* pertains only to x not to -x)

To apply the exponent to the entire expression we must place parentheses around the expression. The exponent is then placed outside the parentheses:

In $(2b)^3$, 2b is raised to the third power,

In $(-x)^n$, -x is raised to the *n*-th power

The convention can be interpreted as the consequence of the order of operations. For example, in $(2x)^2$ parentheses indicate that multiplication should be performed first, and only then the result is raised to the second power. This means that the entire 2x is raised to the second power. Without parentheses, we first exponentiate, and then multiply by 2, so only x is squared. Notice also, that without this convention, we would have no means to distinguish between, let's say, $2x^2$ and $(2x)^2$, or $(a+b)^3$ and $a+b^3$.

Terminology: numerical coefficient

Numerical Coefficient In a product of a number, variables or algebraic expressions, the numerical factor is called a numerical coefficient.

For example, in 2x, $5(a+b)^m$, $-4a^5$, the numerical coefficients (often called coefficients) are 2, 5, -4 respectively. If there is no number in front of a product, it is implied that the coefficient is 1. If there is a negative sign, it is implied that the coefficient is -1. For instance, the coefficient of x^2y is equal to 1. The coefficient of -a is -1.

Laws of exponents

added

Consider:
$$a^3 \cdot a^2 = (a^3) \cdot (a^2) = (a \cdot a \cdot a) \cdot (a \cdot a) = a \cdot a \cdot a \cdot a \cdot a = a^5$$

We have 3 *a*'s and another 2 *a*'s for a total of $3+2=5$ *a*'s (to multiply a^3 and a^2 we exponents). The idea can be generalized to obtain:

Product Rule for Exponents

$$a^m a^n = a^{m+n}$$

When multiplying exponential expressions with like bases, we add the exponents and keep the common base.

If we have:

$$\frac{a^{5}}{a^{3}} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a \cdot a} = \frac{\cancel{a \cdot \cancel{a} \cdot \cancel{a} \cdot a} \cdot a \cdot a}{\cancel{a \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a}}} = a \cdot a = a^{2}, \qquad a \neq 0$$

Out of 5 repeated *a*'s in the numerator, we canceled 3 *a*'s for a total 5-3=2 *a*'s (to divide a^5 by a^3 we subtracted exponents). In general one can prove that:

Quotient Rule for Exponents

$$\frac{a^m}{a^n}=a^{m-n}, \quad a\neq 0$$

When dividing exponential expressions with like bases, subtract the exponents and keep the common base.

If we have: $(a^2)^3 = (a^2) \cdot (a^2) \cdot (a^2) = (a \cdot a) \cdot (a \cdot a) \cdot (a \cdot a) = a \cdot a \cdot a \cdot a \cdot a \cdot a = a^6$ We can think of this as 3 groups of 2 for a total of $2 \times 3 = 6$ *a*'s. In general:

Power Rule for Exponents

 $(a^m)^n = a^{mn}$

When raising an exponential expression to another power, keep the same base, and multiply the exponents.

Now, consider:

$$(ab)^3 = (ab) \cdot (ab) \cdot (ab) = (a \cdot a \cdot a) \cdot (b \cdot b \cdot b) = a^3 b^3$$

This can be extended to:

Product to Powers Rule for Exponents:

 $(ab)^n = a^n b^n$

An exponent outside the parentheses applies to all parts of a product inside the parentheses and thus to raise a product to a power, one can equivalently raise each factor to that power.

Finally:

$$\left(\frac{a}{b}\right)^3 = \frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b} = \frac{a \cdot a \cdot a}{b \cdot b \cdot b} = \frac{a^3}{b^3}, \qquad b \neq 0$$

In general:

Quotient to Powers Rule for Exponents

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad b \neq 0$$

An exponent outside the parentheses applies to all parts of a quotient inside the parentheses. Thus, to raise a quotient to a power, one can equivalently raise both numerator and denominator to that power.

For your convenience, we will display all the rules together:

Laws of Exponents: 1. $a^{0} = 1$ 2. $a^{1} = a$ 3. $a^{m}a^{n} = a^{m+n}$ 4. $\frac{a^{m}}{a^{n}} = a^{m-n}$, $a \neq 0$ 5. $(a^{m})^{n} = a^{mn}$ 6. $(ab)^{n} = a^{n}b^{n}$ 7. $\left(\frac{a}{b}\right)^{n} = \frac{a^{n}}{b^{n}}$, $b \neq 0$

Examples and Problems with Solutions

Example 4.1 In the following expressions, identify bases, exponents and numerical coefficients: a) $3a^{27}$ b) $\left(\frac{yz}{z}\right)^9$ c) $-x^4$

Solution:

a) base: a; exponent: 27; numerical coefficient: 3
b) base: ^{yz}/_z; exponent: 9, numerical coefficient: 1
c) base: x, exponent 4; numerical coefficient: -1

<u>Example 4.2</u> Rewrite using exponential notation whenever it is possible:

a) 7mmm b) -aaaa - aa c) b(c+2d)bb(2d+c)bb d) $\frac{y}{x} \cdot y \cdot y$

Solution:

a)
$$7\underbrace{mmm}_{3-tomes} = 7m^3$$

b) $-\underbrace{aaaa}_{4-times} - \underbrace{aa}_{2-times} = -a^4 - a^2$
c) Notice that $c + 2d = 2d + c$, thus
 $b(c+2d)bb(2d+c)bb = \underbrace{bbbbb}_{5-times} \underbrace{(c+2d)(c+2d)}_{2-times} = b^5(c+2d)^2$
d) Notice that $\frac{y}{x} \cdot y \cdot y$ is equivalent to $\frac{yyy}{x}$, thus $\frac{y}{x} \cdot y \cdot y \cdot = \frac{yyy}{x} = \frac{y^3}{x}$

Example 4.3 Evaluate a) $(2x)^0$ b) $2x^0$ c) $4 \times (8^{25} \times 3^7)$ Solutions:

a) Remember that any expression raised to the zero-th power is equal to 1, and that because of parentheses the zero power pertains to the entire expression 2x, thus $(2x)^0 = 1$

b) This time only x is raised to the zero-th power, hence $2x^0 = 2 \times 1 = 2$ c) The expression $8^{25} \times 3^7$ is raised to the zero-th power, so: $4 \times (8^{25} \times 3^7)^0 = 4 \times 1 = 4$

Example 4.4 Expand, that is write without exponential notation: a) $(5A)^3$ b) $5A^3$ Solution:

a) The exponent pertains to the entire expression 5A: $(5A)^3 = 5A \cdot 5A \cdot 5A$

b) The exponent pertains only to A: $5A^3 = 5A \cdot A \cdot A$.

Example 4.5 Perform the indicated operations and simplify:

a)
$$3x(-2)(-4x)$$

b) $x^7 x^3$
c) $\frac{x^7}{x^3}$
d) $(x^5)^2$
e) $3x(2x^6)^3$
f) $\frac{y^4(x^2)^3}{vx}y$

Solution:

a)
$$3x(-2)(-4x) = 3(-2)(-4)xx = 24x^{2}$$

b) $x^{7}x^{3} = x^{7+3} = x^{10}$
c) $\frac{x^{7}}{x^{3}} = x^{7-3} = x^{4}$
d) $(x^{5})^{2} = x^{5\times 2} = x^{10}$
e) $3x(2x^{6})^{3} = 3x \cdot 2^{3} \cdot (x^{6})^{3} = 3 \cdot 2^{3} \cdot x \cdot x^{6\times 3} = 3 \cdot 8 \cdot x \cdot x^{18} = 24x^{1+18} = 24x^{19}$
f) $\frac{y^{4}(x^{2})^{3}}{yx}y = \frac{y^{4}x^{2\times 3}y}{yx} = \frac{y^{4+1}x^{6}}{yx} = \frac{y^{5}x^{6}}{yx} = y^{5-1}x^{6-1} = y^{4}x^{5}$

Example 4.6 Remove parentheses and write as a single exponential expression. Identify the a) $(-v)^3$ b) $(-2v)^3 2v^2$ numerical coefficient of the final expression: Solution:

a)
$$(-y)^3 = (-1y)^3 = (-1)^3 y^3 = -1y^3 = -y^3$$
; the coefficient: -1
b) $(-2y)^3 2y^2 = (-2)^3 y^3 2y^2 = -8 \times 2y^3 y^2 = -16y^{3+2} = -16y^5$; the coefficient: 16

Example 4.7 Write as an algebraic expression using parentheses where appropriate, then remove the parentheses and simplify:

a) The product of -x and y^2 , then raised to the sixth power

b) The quotient of $4a^2b^3$ and ab^2 , then raised to the second power Solution: \.

a)
$$((-x)(y^2))^6 = (-1x)^6 (y^2)^6 = (-1)^6 x^6 y^{2\times 6} = 1x^6 y^{12} = x^6 y^{12}$$

b) $\left(\frac{4a^2b^3}{ab^2}\right)^2 = (4a^{2-1}b^{3-2})^2 = (4a^1b^1)^2 = (4ab)^2 = 4^2a^2b^2 = 16a^2b^2$

Example 4.8 Simplify the expression $\frac{(3x^5)^2}{x^3x^4}$ and then evaluate, when $x = -\frac{1}{2}$. Saluti

$$\frac{(3x^5)^2}{x^3x^4} = \frac{3^2(x^5)^2}{x^{3+4}} = \frac{9x^{5\times 2}}{x^7} = \frac{9x^{10}}{x^7} = 9x^{10-7} = 9x^3$$

To evaluate, we substitute $x = -\frac{1}{2}$: $9x^3 = 9\left(-\frac{1}{2}\right)^3 = 9\left(\frac{-1}{2}\right)^3 = 9\cdot\frac{(-1)^3}{2^3} = 9\left(\frac{-1}{8}\right) = -\frac{9}{8}$.

Example 4.9 Which of the following expressions are equivalent to $4x^2y^7$? $4y^7x^2$, $(2x)^2y^7$, $4x^2(y^5)^2$, $4xy^7x$, $4x^2y^3y^4$ Solution:

 $4x^2(y^5)^2 = 4x^2y^{5\times 2} = 4x^2y^{10}$, and hence $4x^2(y^5)^2$ is not equivalent. All others are equivalent: $4y^7x^2$ is equivalent because only the order of factors y^7 and x^2 has been changed.

$$(2x)^{2}y^{7} = 2^{2}x^{2}y^{7} = 4x^{2}y^{7}, \quad 4xy^{7}x = 4xxy^{7} = 4x^{2}y^{7}, \quad 4x^{2}y^{3}y^{4} = 4x^{2}y^{3+4} = 4x^{2}y^{7}.$$

Example 4.10 Find the numerical value of Ω such that the following statements are true:

a)
$$4^6 = 2^{\Omega}$$
 b) $36^3 \times 6^7 = 6^{\Omega}$

Solution:

a) We must express 4^6 as an exponential expression with the base 2 (to match it to 2^{Ω}). Since $4 = 2^2$, we get $4^6 = (2^2)^6 = 2^{12}$. As a result $2^{12} = 2^{\Omega}$, hence $\Omega = 12$.

b) We must express $36^3 \times 6^7$ as an exponential expression with the base 6. Notice that $36 = 6^2$, and thus $36^3 \times 6^7 = (6^2)^3 6^7 = 6^6 6^7 = 6^{13}$; $\Omega = 13$.

<u>Example 4.11</u> Evaluate: a) $\frac{9^{81}}{9^{80}}$ b) $\frac{4^7 \times 4^6}{4^{11}}$

Solution:

a)
$$\frac{9^{81}}{9^{80}} = 9^{81-80} = 9$$

b) $\frac{4^7 \times 4^6}{4^{11}} = \frac{4^{7+6}}{4^{11}} = \frac{4^{13}}{4^{11}} = 4^{13-11} = 4^2 = 16$.

Common mistakes and misconceptions

Mistake 4.1

There is a difference between $-x^2$ and $(-x)^2$. In $-x^2$ only x is squared, in $(-x)^2$, -x is squared: $(-x)^2 = (-x)(-x) = x^2$. Just like $-3^2 = -9$, while $(-3)^2 = 9$.

Mistake 4.2

In the expression $x^7 y^3$, since the bases are not the same, DO NOT add exponents.

Mistake 4.3

Although it is true that $(ab)^2 = a^2b^2$, $(a+b)^2 \neq a^2 + b^2$ We recognize that $(7x)^2 = 49x^2$, however, $(7+x) \neq 49 + x^2$.

Mistake 4.4

Please remember: $x^2x^5x = x^2x^5x^1 = x^{2+5+1} = x^8$ (not $x^{2+5} = x^7$). In other words, if one of the factors does not have an explicit exponent, it means it is raised to the first power, and thus one has to be added.

Exercises with Answers (For answers see Appendix A)

Ex.1. In Lesson 4 the following theorems were given: The Product Rule for Exponents, The Quotient rule for Exponents, The Power Rule for Exponents. Were those theorems actually proved? Why or why not?

is called

<i>Ex.2.</i> the	In the expression $3x^m$, 3 is called the or and x is called the	_ , <i>m</i>
Ex.3	Fill in the blanks:a) An expression x raised to the power is equal to itself.b) An expression x raised to the power is equal to 1.	
Ex.4.	a) In the expression ab^{n} the exponent pertains to b) In the expression $(ab)^{n}$ the exponent pertains to c) In the expression $c(de)^{n}$ the exponent pertains to d) In the expression $(-a)^{n}$ the exponent pertains to e) In the expression $-a^{n}$ the exponent pertains to f) In the expression $\left(\frac{2x}{y}\right)^{m}$ the exponent pertains to	

Ex.5. Write the following statements as algebraic expressions using parentheses where appropriate, then rewrite the expression without parentheses. Perform all numerical operations whenever possible.

- a) The quotient of a and 2, then raised to the fourth power
- b) Two thirds of a, then raised to the third power
- c) *c* cubed, and then divided by 7
- d) The product of a and 5, then raised to the second power
- e) Raise *a* to the second power, and then multiply the result by 8.
- f) The opposite of x, then raised to the fifth power.
- g) Raise x to the tenth power, and then take the opposite of the result.

Ex.6 In each of the following expressions, identify the base, exponent and numerical coefficient:

a)
$$3x^4$$
 b) $-x^m$ c) $\frac{2x^3}{3}$ d) $-(a-bc)^2$
e) $\left(\frac{x}{y}\right)^m$ f) $\frac{(x+y)^7}{4}$ g) $\frac{3}{4}\left(\frac{3x+z}{w}\right)^7$ h) $-\frac{(ab)^5}{2}$

*Ex.*7 Write using exponential notation whenever it is possible:

a)
$$6 \times 6 \times 6 \times 6 \times 6$$

b) $zzzz$
c) $3a \cdot 3a \cdot 3a \cdot a$
f) $xxy - yyx$
g) $(a+b)(a+b)(a+b)$
h) $(2t^3)(2t^3)(2t^3)(2t^3)$
i) $m+nn+m$
j) $\frac{kkk}{n} \cdot k$
k) $\frac{-z-z-z}{zzzz}$
l) $\frac{(-z)(-z)(-z)}{z+z+z}$
m) $x \cdot \frac{x}{2} \cdot x \cdot x$
n) $(3-x)(-x+3)(3-x)$
o) $\left(\frac{2a}{b}\right)\left(2\frac{a}{b}\right)\left(\frac{2a}{b}\right)$

p)
$$\left(-\frac{3}{x}\right)\left(\frac{-3}{x}\right)\left(\frac{3}{-x}\right)$$
 q) $\frac{1}{(w+2v)(2v+w)(2v+w)}$ r) $\left(\frac{x}{m}-\frac{y}{m}\right)\left(\frac{x-y}{m}\right)$
s) $(m+n)(m+n)m+n$ t) $(m+p-n)(p+m-n)(n+m-p)$

Ex.8 Write the following expressions without using exponential notation:

a) $(-4)^5$ b) -4^5 c) $(-m)^3$ d) $-m^3$ e) $(2a)^3$ f) $2a^3$ g) $(a+b)^2$ h) $a+b^2$

Ex.9	Simplify by raising to the indicated power:			
	a) $(3x)^0$	b) $3x^{0}$	c) $3^{0}x$	d) $a(b+c)^{0}$
	e) abc^0	f) $(abc)^{0}$	g) $ab + c^0$	h) $a^{0}b^{0}c^{0}$

Ex.10 Rewrite the following expressions without unnecessary parentheses. If parentheses are needed, clearly indicate so.

a) $(x)^{3}$	b) $(-x)^4$	c) $-(x)^{7}$	d) $a + (2b)^3$
e) $(a+2b)^3$	f) $a + 2(b)^3$	g) $a(bc)^m$	h) $\left(\frac{2x}{y}\right)^4$

Ex.11 Evaluate a) $2x^3$ b) $(2x)^3$ when x = 100. Explain why you did not get the same result.

Ex.12 Write as an algebraic expression using parentheses where appropriate, then remove the parentheses and simplify:

a) The product of -m and -2m²
b) The quotient of 3x⁵ and 18x³
c) The expression 2a⁵ raised to the third power.

Ex.13 .Write as a single exponential expression:

a)
$$(-4x)^2$$
 b) $n^3 \cdot n^{20}$ c) $\frac{a^4}{2a^2}$ d) $2m^4m^5$
e) $-4x^2(-2x)$ f) $x^4 \cdot \frac{3}{x}$ g) $\frac{1}{b} \cdot b^2$ h) b^7b
i) $\frac{9a^{20}}{3a^4}$ j) $x^3x^4x^5$ k) $(s^7)^2$ l) $\left(\frac{1}{v}\right)^2 v^7$
m) $-5(t^3)^4$ n) $\frac{0.5s^{12}}{0.1s}$ o) $6a^{11} \cdot a^3 \cdot a^{18}$ p) $(2x^7)^3$

Ex.14 Simplify the following expressions, and then evaluate when a = 2.

a)
$$\frac{a^7}{a^6}$$
 b) $2a^2a^2$ c) $(2a)(-\frac{1}{2}a^2)$ d) $\left(\frac{a^2}{3}\right)^2$

Ex.15 Simplify the following expressions, and then evaluate when m = -1.

a)
$$m^3 m^5$$
 b) $\frac{m^3}{-2m^2}$ c) $\frac{(m^2)^{14}}{3}$ d) $2m^7 m^{50}$

Ex.16 Simplify the following expression $\frac{-x^{10}}{x^8}$, and then evaluate when

a)
$$x = 7$$
 b) $x = -7$ c) $x = -\frac{2}{3}$ d) $x = -0.07$

Ex.17 Remove parentheses and write as a single exponential expression. Identify the numerical coefficient of the final expression:

a)
$$(-B)^5$$
 b) $(-B)^8$ c) $(-B)^5 B^3$ d) $\frac{2B^4}{6B^3}$ e) $(3B)^2 3B^2$ f) $-\frac{2(3B)^2}{3B}$

Ex.18 Perform the indicated operations and simplify:

a)
$$(3x)(-2)(4x)$$
 b) $(3x^{2})(4x^{5})(-2)$ c) $\frac{3x^{3}}{(3x)^{2}}$ d) $(3aa^{2})^{2}$
e) $(-4x)^{2}(-2x)$ f) $\frac{a^{4}}{(2a)^{2}}$ g) $\frac{(4x)^{3}}{-x}$ h) $\frac{(-a)^{3}}{2a^{2}}$
i) $-2a \cdot \frac{a^{7}}{(-2a)^{2}}$ j) $\frac{-2x^{3}x^{5}}{x^{4}}$ k) $\frac{(-x^{6})(3x^{2})}{xx^{4}}$ l) $\frac{a^{4}}{(2a)^{2}} \cdot 2a^{2}$

Ex.19 Write as an algebraic expression using parentheses where appropriate, then remove the parentheses and simplify:

a) The product of -3x and x^2 , then raised to the third power

b) The quotient of $4a^{12}$ and a^2 , then raised to the second power

c) $-a^3$ raised to seventh power, then multiplied by a

d) xy^7 raised to the fifth power , then divided by xy^3

e) $3ab^3$ squared, and then divided by the product of a^2 and b.

Ex.20 Perform the indicated operations and simplify:

a)
$$3a(b^2)^2$$
 b) $3(ab^2)^2$ c) $(x^5y^3)^2$ d) $\left(\frac{x^2y}{x}\right)^3$
e) $4(m-n)^2(m-n)^3$ f) $\frac{a^{13}b^2c^4}{a^5c^2b}$ g) $-\left(\frac{a^3}{4b^2}\right)^2$ h) $\left(\frac{(a+b)^8}{(a+b)}\right)^3$
i) $\frac{(2x)^3y^2}{x} \cdot y$ j) $x^2y(-x^3)y^4$ k) $(4x-y)^0(4x-y)^3$ l) $\frac{s^2x}{(4s)^2} \cdot x^2$

Ex.21 Simplify the following expressions, and then evaluate when m = -2 and n = 1. a) $\frac{m^2 n^5}{mn}$ b) $(5mn^3)^2$ c) $\frac{(m+n)^4}{(m+n)^2}$ d) $(2m-n)^0$

Ex.22 Circle all expressions that are equivalent to $\frac{2}{5}y^2$:

$$\frac{4}{25}y^2, \qquad \frac{2y^2}{5}, \qquad y \cdot \frac{2}{5} \cdot y, \qquad \left(\frac{2}{5}y\right)^2, \qquad \frac{2}{5}yy$$

Ex.23 Circle all expressions that are equivalent to $\left(\frac{a}{3}\right)^{20}$:

$$\frac{a^{20}}{3}, \qquad \frac{a^{20}}{3^{20}}, \qquad \frac{(a^{12})^8}{3^{20}}, \qquad \frac{a^{12}a^8}{3^{20}}, \qquad \frac{(a^4)^5}{3^{20}}$$

Ex.24 Circle all expressions that are equivalent to $4x^3y^2$: $4y^2x^3$, $(2y)^2x^3$, $(-2)y^2(-2)x^3$, $(4xy)^2x$, $4(xy)^2x$

Ex.25 Circle all expressions that are equivalent to $2a^6b^3c^2$: $2a^6c^2b^3$, $2a^2b^3c^2a^3$, $2(abc)^{11}$, $\frac{7c^2b^3a^6}{14}$, $2(a^3c)^2b^3$

Ex.26 Replace Ψ with a number so the following are equal: a) $81^4 = 9^{\Psi}$ b) $7^{\Psi} = 49^5$ c) $16^{100} = 2^{\Psi}$ d) $\left(\frac{1}{4}\right)^7 = \left(\frac{1}{2}\right)^{\Psi}$ e) $27^4 = 3^{\Psi}$ f) $0.2^{\Psi} = (0.04)^5$ g) $\left(\frac{1}{8}\right)^{10} = \left(\frac{1}{2}\right)^{\Psi}$ h) $\frac{16^5}{16^3} = 4^{\Psi}$ i) $36^{15} \cdot 6^4 = 6^{\Psi}$

Ex.27 Evaluate the following expressions:

a)
$$\frac{15^{248}}{15^{247}}$$

b) $\frac{3^{21} \cdot 3^{10}}{3^{29}}$
c) $\frac{(12^3)^6}{-12 \times 12^{16}}$
d) $\frac{0.5^{18}}{0.25^8}$
e) $-\frac{64^3}{4^7}$
f) $\frac{4^{15} \cdot 2^9}{2^{38}}$

Homework/Class Work/Quiz

1. In the expression $-7a^n$, -7 is called the ______, *n* is called the ______, *n* is called the ______.

2. In the following expressions, identify exponents, bases, and numerical coefficients:

	Numerical coefficient	Base	Exponent
a) $3a^{15}$			

- b) $(yx)^{3}$
- c) $-x^{2}$
- 3. Write using exponential notation:
- a) $10 \times 10 \times 10 \times 10$
- b) $x \cdot x \cdot x$
- c) $x \cdot y \cdot x \cdot y \cdot x$
- 4. Write the following expression without using the exponential notation:
- a) 3⁵
- b) *s*⁴

4B

1. In the following expressions, identify exponents, bases, and numerical coefficients:

	Numerical coefficient	Base	Exponent
a) $-3(ab)^{m}$			
b) $(y-x)^{m+2}$			

c)
$$\frac{x^4}{7}$$

2. Whenever possible, write using exponential notation:

a)
$$\frac{aaa}{4bbbb}$$

b)
$$(-x)(-x)(-x)$$

c) $\frac{2}{a} \times \frac{2}{a} \times \frac{2}{a} \times \frac{2}{a} \times \frac{2}{a} \times \frac{2}{a} \times 2$

1. Whenever possible, write using exponential notation:

a) $x \cdot x - x \cdot x \cdot x \cdot x$

b) $-x - x \cdot x \cdot x \cdot x$

c)
$$(-x)(-x) - x \cdot x \cdot x \cdot x$$

- d) (-x)(-x)(-x) x
- 2. Evaluate: a) $(34567y)^0$

b)
$$34567y^0$$

3. Evaluate: a) $(-x)^2$ when x = 1

b) $-x^2$ when x = 1

1. Whenever possible, write using exponential notation:

a)
$$(a+b)(b+a)$$

b) $3 \times 3 \times 3 \times (a-2b) \cdot (-2b+a)$

c)
$$\frac{xyxyx}{-b-b-b}$$

d)
$$\left(-\frac{m}{n}\right)\left(\frac{-m}{n}\right)\left(\frac{m}{-n}\right)$$

e)
$$\left(\frac{a}{3} + \frac{b}{3}\right)\left(\frac{a+b}{3}\right)$$

f) $(a^3)(3a^3)(a^3)(3a^3)(3a^3)$

g)
$$\frac{2v \cdot 2v \cdot 2v}{5u \cdot 5u} \cdot 2v$$
Homework/Class Work/Quiz

- 1. Fill in the blanks:
- a) In -2^m , the expression that is raised to the *m*-th power is ______.

b) In $-x^m$, the expression that is raised to the *m*-th power is ______.

c) In $-\frac{2^m}{x}$, the expression that is raised to the *m*-th power is _____.

d) In
$$-\left(\frac{2}{x}\right)^m$$
, the expression that is raised to the *m*-th power is _____

e) In
$$\left(-\frac{2}{x}\right)^m$$
, the expression that is raised to the *m*-th power is ______.

- 2. Write the following phrases as algebraic expressions, and then evaluate them when x = 5.
- a) x squared, and then the result is doubled
- b) *x* doubled, and then the results squared

1. Replace Σ with a number to make the following statements true.

a)
$$x^{\Sigma} = 1$$

b) $x^{\Sigma} = x$

2. Rewrite the following expressions without parentheses that are not needed. If parentheses are needed, then simply recopy the original problem without any changes.

a)
$$3(x)^5$$

- b) $(-x)^4$
- c) $(a+b)^M$
- d) $\left(\frac{c}{d}\right)^8$
- e) $(a)(b)^n$
- f) $(ab)^n$
- 3. Write the following expression without using the exponential notation:
- a) $-x^{3}$
- b) $(-x)^{3}$

1. Remove parentheses and write as a single exponential expression. Identify the numerical coefficient of the final expression.

a) $(-x)^5$

b) $\left(\frac{3a}{7}\right)^2$

c)
$$\left(\frac{1}{10}x\right)^4$$

d)
$$\left(\frac{-a}{4}\right)^3$$

e) $4(2x)^3$

2. Circle all expressions that are equivalent to $-\left(\frac{x}{3}\right)^2$

$$\frac{x^2}{3}$$
, $-\frac{x^2}{3}$, $\frac{x^2}{9}$, $-\left(\frac{1}{3}x\right)^2$, $-\frac{1}{9}x^2$, $-\frac{x^2}{3^2}$, $-\frac{x^2}{9}$,

Homework/Class Work/Quiz

Fill in the blanks, each time using one of the following words: add, subtract, multiply, divide.
 a) To ______ exponential expressions with the same bases one needs to ______ the exponents.

b) To ______ exponential expressions with the same bases one needs to ______ their exponents

c) To raise an exponential expression to another power one needs to ______ exponents.

2. Perform the indicated operations and simplify:

a) $a^2 a^3$

b) $m^5 m^3 m$

c)
$$\frac{m^7}{m^5}$$

d) $(m^4)^3$

e)
$$m^{3} \cdot \frac{1}{m^{2}}$$

1. Write the following expressions without using exponential notation:

a) $-5s^{3}$

b)
$$\frac{m^3}{n}$$

c)
$$-\left(\frac{m}{n}\right)^3$$

d)
$$-a^2 - a^3$$

2. Simplify the following expressions and then evaluate when x = 2

a)
$$\frac{3x^3}{6x}$$

b) $-x(2x)^{0}$

1. Write the following statements as algebraic expressions using parentheses where appropriate. Then remove parentheses and perform all numerical operations possible:

a) The quotient of 5 and b, then raised to the third power

b) Two sixth of a, then squared

- c) The opposite of c, then raised to the seventh power
- 2. Perform the indicated operations and simplify:
- a) $-aa^2$

b) $(2a)(-3a^4)$

c) $2(a^7)^2$

d)
$$\frac{a^{7}}{5a^{2}}$$

e)
$$\frac{6a^5}{9a^3}$$

 $(ab)^0$ ab^0

2. Remove parentheses and write as a single exponential expression. Perform all possible numerical operations:

a) $(a^2)^3$

b) $(2a^2)^3$

c) $2(a^2)^3$

d)
$$(-a^2)^3$$

3. Circle all expressions that are equivalent to $-x^6$

$$(-x)^{6}, -(x^{2})^{3}, -x^{2}x^{3}, -x^{2}x^{4}, \frac{x^{6}}{-1}, -(x^{4})^{2}$$

Homework/Class Work/Quiz

1. Write as a single exponential expression. Identify a numerical coefficient of each of the resulting expressions:

a) $a^{3}(-a^{4})$

b) $(3m)(-4m)m^2$

c) $(-2x^3)x$

d)
$$\frac{(2a)^2}{3}$$

e)
$$\left(\frac{1}{3}a^2\right)\frac{2}{5}a$$

1. Remove parentheses and write as a single exponential expression. Identify the numerical coefficient of the final expression.

a)
$$\frac{(4x)^3}{(4x)^2}$$

b)
$$\frac{(4x)^3}{4x^2}$$

c)
$$\frac{4x^3}{4x^2}$$

2. Write as an algebraic expression using parentheses where appropriate, then remove the parentheses and simplify:

- a) The product of 3x and $-4x^2$
- b) $3y^5$ raised to the third power
- c) The quotient of $-4x^7$ and 8x

Homework/Class Work/Quiz

1. Remove parentheses and write as a single exponential expression. Perform all possible numerical operations:

a) $(2a)^3(2a)^2$

b) $2a^3 2a^2$

c) $2a^{3}(-2a)(2a)^{2}$

d) $(2a)2a^{2}(-2)(-a)$

e)
$$\frac{a^3(2a)}{a^2}$$

f)
$$\frac{-2a^6}{2a^2a^4}$$

- 1. Write the following expressions without using exponential notation:
- a) $a b^4$
- b) $(a-b)^4$
- 2. Simplify the following expressions and then evaluate when $a = -\frac{1}{4}$
- a) $-8a\left(\frac{a}{2}\right)$

b)
$$\left(\frac{4a}{6}\right)\left(\frac{3a^2}{2}\right)$$

c)
$$\frac{10a^2}{15a}$$

d)
$$-8a^3\left(\frac{2}{a}\right)$$

- 1. Perform the indicated operations and simplify:
- a) $(-3a^2)^2 4a$

b) $(-3a^2 \cdot 4a)^2$

c)
$$\frac{(-3a^2)^2}{4a}$$

d)
$$\left(\frac{-3a^2}{4a}\right)^2$$

2. Simplify
$$\frac{(-a)^3(-a)^8}{a^9}$$
 and then evaluate, when
a) $a = -\frac{2}{5}$

b) *a* = 1.1

- 1. Write as a single exponential expression:
- a) $2(-x)(-2x)^2$

b) $a^{3} \frac{1}{5a}$

c)
$$(-a)(-a)(-a)^0$$



e) $x^7 (-3x^8)^2$

- 1. Write as a single exponential expression:
- a) $(-2x^2)(-3x^3)$

b)
$$-\frac{1}{6}a^2(6a)^2$$

c)
$$3x^2(-x)\left(\frac{x^4}{6}\right)$$

d)
$$-x(-x^2)(-2)(-x)^4$$

e)
$$\frac{5x^4(2x)^3}{-2x}$$

1. Perform the indicated operations and simplify:

a)
$$\left(\frac{a^2}{b^3}\right) \cdot b^3$$

b)
$$b^4 \cdot \left(-\frac{a^2}{b^3}\right)$$

c)
$$\left(\frac{a^2}{b^3}\right)^4 b^4$$

2. Circle all expressions that are equivalent to $2x^3y^4$:

$$\frac{x^3y^4}{2}, \qquad 2xy^4x^2, \qquad 2y^4x^3, \qquad 2x(xy^2)^2, \qquad 2xxy^4x, \qquad 4-2\cdot x^3y^4$$

3. Write as an algebraic expression using parentheses where appropriate, then remove the parentheses and simplify:

a) Take the opposite of $4x^2y^4$, and then raise the result to the third power

b) Raise $4x^2y^4$ to the third power, and then take the opposite of the result

1. Circle all expressions that are equivalent to $-xz^5y^2$:

$$-y^2xz^5$$
, $-xyyz^5$, $-1z^5y^2x$, $(-x)y^2z^5$, $-(z^2)^2y^2xz$, $-z^5(xy)^2$

2. Perform the indicated operations and simplify:

a)
$$\frac{-7(a^3b)^2}{14a^5b}$$

b)
$$\frac{3(a+b)^2(a+b)^3}{a+b}$$

c)
$$\frac{1}{8(1-x)^2}(1-x)^3$$

d)
$$(w+v)^2 \cdot \frac{(w+v)^3}{w+v}$$

4U

a)
$$\frac{(x+y)(y+x)^3}{(x+y)^2}$$

b)
$$-xy(3x-4y^3)^0$$

2. Evaluate:

a)
$$\frac{6^{25}}{6^{24}}$$

b)
$$\frac{(0.3)^{49}(0.3)^{51}}{(0.3)^{98}}$$

- 1. Replace Ψ with a number so the following are equal:
- a) $4^2 = 2^{\Psi}$

b) $3^{\Psi} = 9^{22}$

c) $27^{15} = 3^{\Psi}$

d) $25^{10} = 5^{\Psi}$

e)
$$\left(\frac{1}{16}\right)^7 = \frac{1}{4^{\Psi}}$$

- 1. Replace Ψ with a number so the following are equal:
- a) $4^5 \cdot 2^3 = 2^{\Psi}$

b)
$$\frac{9^7}{3^5} = 3^{\Psi}$$

c) $5^{10} \cdot 25^3 = 5^{\Psi}$

d)
$$(0.09)^3 (0.09)^7 = 0.3^{\Psi}$$

2. Evaluate:

a)
$$\frac{11^{100}}{11^{99}}$$

b)
$$\frac{(0.7)^5 (0.7)^{16}}{(0.7)^{19}}$$

c)
$$\frac{13^{21} \cdot 13^{19}}{(13^2)^{19}}$$

Topics:

Multiplication of algebraic expressions; The Distributive Law; Factorization of a common factor; Factorization of -1; Simplification of algebraic fractions with the use of factorization.

This section is about the Distributive Law, the law that relates addition with multiplication.

The Distributive Law:

If 5 girls and 7 boys get 2 cookies each, then the girls get $2 \times 5 = 10$ cookies, the boys get $2 \times 7 = 14$ cookies, so the total number of cookies they all receive is:

 $2 \times 5 + 2 \times 7 = 10 + 14 = 24$

Alternatively, one can find the number of cookies by adding the number of girls and boys first: 5 + 7 = 12. Since each of the children gets 2 cookies, together they get:

$$2 \times (5+7) = 2 \times 12 = 24$$

This property is called The Distributive Law. In general:

The Distributive Law:

a(b+c) = (b+c)a = ab + aca(b-c) = (b-c)a = ab - ac

To remove parentheses from the expression of the form a(b+c) or a(b-c), multiply the factor outside parentheses by each term inside parentheses.

The Distributive Law applied to the product of two sums:

We can apply the Distributive Law to remove parentheses in the product of two sums: (x + y)(b + c). We need to look at this expression as a sum of two terms in parentheses: (b + c), multiplied by a number (x + y). In other words, since (x + y) 'plays the role of a' in the Distributive Law, we may replace a in the Distributive Law with (x + y):

$$(x+y)(b+c) = (x+y)b + (x+y)c$$

If we now apply The Distributive Law to $(x+y) \cdot b$ and $(x+y) \cdot c$, we obtain:
 $(x+y)(b+c) = (x+y)b + (x+y)c = xb + yb + xc + yc$
Thus the following is true:

Thus the following is true:

The Distributive Law For Product of Two Sums

(x+y)(b+c) = xb + yb + xc + yc

To multiply two sums, you need to multiply each term of the first sum by each term of the second one and then add all the products.

Please recall, that terms can be rearranged so, equivalently, we can write:

$$(x+y)(b+c) = xb + xc + yb + yc.$$

Any other order of terms would be also correct.

Factorization:

The Distributive Law lets us change an expression from a product to a sum. When we do that 'in reverse', the process is called factorization.

Factorization Changing the sum of two or more terms to the product is called factorization.

When factoring algebraic expressions, we merely rewrite them in a different form. The expression obtained in the process is equivalent to the original one.

Factorization of a common factor:

Consider: 3A + Ax. The expression has two terms: 3A and Ax. Each has A as its factor. A is **a** common factor of both terms. We will factor A from 3A + Ax:

$$3A + Ax = A \cdot \left(\frac{3A}{A} + \frac{Ax}{A}\right) = A(3+x)$$

The first step follows from the Distributive Law. To convince yourself, multiply each term inside parentheses by $A: A \cdot \left(\frac{3A}{A} + \frac{Ax}{A}\right) = A \cdot \frac{3A}{A} + A \cdot \frac{Ax}{A} = 3A + Ax$. The second step is obtained by canceling A's. 3A + Ax was originally written as a sum of two terms. By factoring A from 3A + Ax, we are now able to express it in factored form (that is, as a product of two expressions, rather than a sum): 3A + Ax = A(3 + x).

Notice that, after factorization, you can always check your answer by multiplying factors.

Factorization of -1

Consider:
$$a-b$$
. We can factor -1 :

$$a - b = (-1) \cdot \left(\frac{a}{-1} - \frac{b}{-1}\right) = (-1)(-a + b) = -(-a + b) = -(b - a)$$

Factorization of -1 causes the signs in front of each term in the original expression to change (in our example *a* changes to -a, -b changes to *b*). Thus, a-b = -(b-a) as we see above.

Simplification of algebraic fractions:

Recall the operation of simplifying fractions:

$$\frac{15}{25} = \frac{5 \times 3}{5 \times 5} = \frac{\cancel{5} \times 3}{\cancel{5} \times 5} = \frac{3}{5}$$

To simplify this fraction we divide the numerator and denominator by their common factor 5. We 'cancel 5'. We can always divide the numerator and denominator by the same non-zero expression. This rule, true for numerical fractions, is also true for algebraic fractions, that is fractions whose numerator and denominator are not necessarily numbers but any algebraic expressions (like for example: $\frac{3x}{x-1}$, $\frac{a^2}{a(1-a)}$, or $\frac{xy+x^4}{2x}$). To simplify an algebraic fraction divide the numerator and denominator by all of their common factors. For example:

Consider

$$\frac{3a(b+c)}{ad}, \qquad ad \neq 0$$

Since a is the common factor of the numerator and the denominator, we divide both, the numerator and the denominator, by a. We "cancel a":

$$\frac{3a(b+c)}{ad} = \frac{3a(b+c)}{ad} = \frac{3(b+c)}{d}$$

The following rule must always be followed:

In algebraic fractions, only factors can be cancelled, but not terms.

For example, in the expression: $\frac{a+x}{a}$, $a \neq 0$, *a* cannot be cancelled. Although *a* in the denominator can be viewed as a factor $(1 \cdot a = a)$, in the numerator *a* is used as a term and thus we cannot 'cancel it'. (Just like we cannot cancel 3 in $\frac{3+1}{3}$: $\frac{3+1}{3} \neq 1$)

Application of factorization of a common factor to simplification of algebraic fractions:

Factorization can often be used to simplify algebraic fractions: In the expression:

$$\frac{3a-ay}{ax} , \qquad ax \neq 0$$

a is a factor of the denominator but not of the numerator. Notice, however, that a is factor of each term in the numerator. Thus, we can factor a from the numerator, and then cancel it:

$$\frac{3a + ay}{ax} = \frac{a(3 + y)}{ax} = \frac{a(3 + y)}{ax} = \frac{a(3 + y)}{ax} = \frac{3 + y}{x}$$

<u>Application of factorization of -1 to simplification of algebraic fractions:</u>

Consider:

$$\frac{-x+y}{x-y} , \qquad x-y \neq 0$$

There are no common factors that could be canceled, but we should notice that signs in the numerator are exactly the opposite of signs in the denominator (x follows a minus sign in the numerator but a plus sign in the denominator; y is preceded by a plus sign in the numerator but a minus sign in the denominator). Factoring -1 either in the numerator (or in the denominator; the choice is arbitrary) will reverse the signs and allow the simplification:

$$\frac{-x+y}{x-y} = \frac{-1(x-y)}{x-y} = \frac{-1(x-y)}{x-y} = -1$$

More Examples and Problems with Solutions

Example 5.1 Use the Distributive Law to remove parentheses in the following expression: a(b+c+d).

Solution:

The expression inside the parentheses should be treated not as a sum of three terms but as a sum of two: (b+c) and d, that is b+c+d = (b+c)+d. We can then apply the Distributive Law: a(b+c+d) = a((b+c)+d) = a(b+c)+ad

If we apply the Distributive Law once again, this time to a(b+c), we get:

a(b+c+d) = a((b+c)+d) = a(b+c) + ad = ab + ac + ad.

Thus a(b+c+d) = ab + ac + ad.

Example 5.2 Remove parentheses:

a)
$$(a-b)(2a+3)$$

b) $\left(3x-\frac{1}{2}\right)2x$
c) $-(x+3y-z)$
d) $(b+c-d+e)a$
e) $(a+b)(c+d)(e-f)$

Solution:

a) Use the Distributive Law for Product of Two Sums: $(a-b)(2a+3) = a(2a) + a(3) - b(2a) - b(3) = 2a^2 + 3a - 2ab - 3b$

b) Apply the Distributive Law (remember that $\left(3x - \frac{1}{2}\right)2x = 2x\left(3x - \frac{1}{2}\right)$):

$$(3x - \frac{1}{2})2x = 3x(2x) - \frac{1}{2}(2x) = 6x^2 - x$$

c) Use the result from Exercise 5.1:
$$-(x + 3y - z) = (-1)(x + 3y - z) = -1x - 1(3y) - 1(-z) = -x - 3y + z$$

d) Like in Example 5.1, each term inside parentheses must be multiplied by *a*.

(b+c-d+e)a = ba + ca - da + ea

e) Multiply
$$(a + b)(c + d)$$
, then the result must be multiplied by $(e - f)$
 $(a + b)(c + d)(e - f) = (ac + ad + bc + bd)(e - f) = ace + ade + bce + bde - acf - adf - bcf - bdf$

Example 5.3 Factor x^2 from the following expression: $3x^5 - 2x^3 + x^2$ Solution:

$$3x^{5} - 2x^{3} + x^{2} = x^{2} \left(\frac{3x^{5}}{x^{2}} - \frac{2x^{3}}{x^{2}} + \frac{x^{2}}{x^{2}} \right) = x^{2} (3x^{3} - 2x + 1)$$

Example 5.4 Factor xy^2 from the following expression: $4x^2y^2 + xy^4 - 3x^3y^3$ Solution:

$$4x^{2}y^{2} + xy^{4} - 3x^{3}y^{3} = xy^{2} \left(\frac{4x^{2}y^{2}}{xy^{2}} + \frac{xy^{4}}{xy^{2}} - \frac{3x^{3}y^{3}}{xy^{2}} \right) = xy^{2} (4x + y^{2} - 3x^{2}y)$$

Example 5.5 Factor (a + 2b) from the following expression: 4x(a + 2b) - (a + 2b). Solution: 1

$$4x(a+2b) - (a+2b) = (a+2b)\left(\frac{4x(a+2b)}{(a+2b)} - \frac{a+2b}{a+2b}\right) = (a+2b)(4x-1)$$

Please, notice that $\frac{a+2b}{a+2b} = 1$ and thus 1 appears as one of the terms inside parentheses.

<u>Example 5.6</u> Factor $\frac{1}{4}$ from the following expression: $\frac{1}{4}x - \frac{3}{4}y$.

Solution:

$$\frac{1}{4}x - \frac{3}{4}y = \frac{1}{4} \left(\frac{\frac{1}{4}x}{\frac{1}{4}} - \frac{\frac{3}{4}y}{\frac{1}{4}} \right) = \frac{1}{4} \left(\frac{1}{4} \cdot \frac{4}{1} \cdot x - \frac{3}{4} \cdot \frac{4}{1} \cdot y \right) = \frac{1}{4} (x - 3y)$$

<u>Example 5.7</u> Factor 5 from the following expression: x + 5. Solution:

$$x + 5 = 5\left(\frac{x}{5} + \frac{5}{5}\right) = 5\left(\frac{x}{5} + 1\right)$$

Example 5.8 Factor -1 from the following expression: 2x - y + z. Solution:

2x - y + z = -1(-2x + y - z) = -(-2x + y - z)

Example 5.9 Simplify the following expressions, if possible. If not possible, explain why it is not possible:

a)
$$\frac{a+2ab}{a}$$
 b) $\frac{xy}{x^2y-3xy^2}$ c) $\frac{v-4z}{4z}$ d) $\frac{a+4d}{-4d-a}$

Solution:

a) One needs to find all common factors of the denominator and of all terms of the numerator. The common factor is a. We will factor a in the numerator and then cancel it with a in the denominator:

$$\frac{a+2ab}{a} = \frac{a(1+2b)}{a} = 1+2b$$

b) The common factor is xy. Factor xy in the denominator and cancel:

$$\frac{xy}{x^2y - 3xy^2} = \frac{xy}{xy(x - 3y)} = \frac{1}{x - 3y}$$

c) It cannot be simplified. In the numerator, 4z is used only as a term, not as a factor.

d) One needs to notice 'the reversed signs' (all terms in the numerators follow a plus sign, while all terms in the denominator follow a minus sign). This requires factorization of -1 (either in the numerator or denominator). We will factor -1 in the denominator:

$$\frac{a+4d}{-4d-a} = \frac{a+4d}{-1(4d+a)} = \frac{a+4d}{-1(a+4d)} = -1$$

Common mistakes and misconceptions

Mistake 5.1

When factoring 3 from 3x + 3y + 3: $3x + 3y + 3 \neq 3(x + y)$ Instead, 3x + 3y + 3 = 3(x + y + 1).

Mistake 5.2

 $\frac{x+1}{x} \neq 1$. Remember that: only factors can be cancelled, not terms.

Exercises with Answers (For answers see Appendix A)

In all exercises of this lesson, we assume that denominators are different from zero.

Ex.1 The Distributive Law states that c(a + b) = ca + cb. Explain how we can use it to remove parentheses from the following expression

(a+b)c

Ex.2 The Distributive Law applied to the product of two sums states: (x + y)(b + c) = xb + yb + xc + yc

Can we instead state it as : (x + y)(b + c) = xb + xc + yb + yc? Why? Reformulate the Distributive Law in four different ways.

Ex.3 Write each of the following statements as an algebraic expression using parentheses where appropriate, and then remove parentheses:

a) The product of $x^2 - y$ and 5 b) The opposite of -4x + 1

c) The product a and -c+1d) The product of 2y and -a + 2b + df) The opposite of $x - x^2 + 2x^4$ e) The product of x-1 and y^3+2 Write the following expressions in five different equivalent ways: Ex.4a) 3(a+b)b) (2 - y)zEx.5Circle all expressions that are equivalent to m(n + p): mn + mp(n+p)mpm + nmmn + pmp + nm*Ex.6* Circle all expressions that are equivalent to -(x - y + z): (x-y+z)(-1)(x-y+z)-1(-1)(x - y + z)-1(x - y + z)-x-y+z-x+y+z-x+y-z-(x+z-y)*Ex*.7 Remove parentheses from the following expressions: a) 2(L+W)b) R(1-x)c) P(1+rt)d) $(R^2 - r^2)s$ f) $(x^2 - 7z)x$ e) c(2a + c)Remove parentheses from the following expressions: Ex.8a) $\frac{5}{6}(F-18)$ c) $(x^2 - 2x)x^4$ b) $3y(6y^4 + 8y^3)$ d) $-\frac{2}{3}(3c-33d)$ e) $a(a^2 + ab + ab^2)$ f) $(2x^3y + \frac{3}{7} - xy^4)xy$ g) (x-y)(z+w) h) $(a-\frac{2}{5}b)(10-a^3)$ i) (a+b-g)(c-d)

j) $(x^3 + x^2 + x + 1)(y - 1)$ k) $\frac{1}{4}(4x - 8)(2y + 3)$ l) (2a - 1)(1 - b)(c - 2)

Ex.9 a) After factoring a common factor from a two term expression, how many terms should you have inside parentheses?

b) After factoring a common factor from a three term expression, how many terms should you have inside parentheses?

c) After factoring a common factor from an m-term expression, how many terms should you have inside parentheses?

- Ex.10 Factor
 - a) 5 from the expression: 5x + 5yc) 2 from the 2lw + 2lh + 2whe) c from the expression: $3c - 2c^2$ g) y from the expression: $-8xy^6 + y$ b) 7 from the expression: 7 - 49ad) -11 from the expression: -11t + 44f) x from the expression: $4x^3 - 5x^2 + 5x$

Ex.11 Factor *xy* from the following expressions
a)
$$2xy - a^2xy$$
 b) $-x^2y + xy^2$ c) $axy + xby - yx$

Ex.12 Factor -1 from:

b)
$$-a+b+1$$
 c) $a-\frac{x+y-2}{2}$

Ex.13 Factor

a) 5*a* from the following expression: $10a - 15a^2$ b) 11*t* from the following expression $-\frac{11}{2}t^2 + 44t$ c) 5 x^3 from the following expression: $15x^5 + 5x^3$ d) $-4y^5$ from the following expression: $-8xy^6 + 4y^5$ e) 2*xy* from the following expression: $2xy - 2xy^2 + 4x^2y^2$ f) $-3x^2y$ from the following expression: $-3x^2y + 9x^3y^2$ g) a^3b^3 from the following expression: $a^3b^4 + 5b^3a^7 - a^3b^3$ h) 17*xy* from the following expression: $17x^5y^3 + 34x^3y^2 + 51xy$ i) $-7ab^2$ from the following expression: $-35ab^3 - 14a^2b^4 + 21ab^2$ j) a^3b^3 from the following expression: $a^3b^4 + 5b^3a^7 - a^3b^3$ k) $-4ac^3$ from the following expression: $-16ac^3 + 8ac^7 - 12ac^8d$

Ex.14 Factor

a)
$$\frac{2}{3}$$
 from the following expression: $\frac{2}{3}x^2y - \frac{4}{3}z$
b) $-\frac{1}{5}$ from the following expression: $-\frac{1}{5}x - \frac{1}{25}$

Ex.15 Factor
$$a + b$$
 from the following expressions:
a) $6(a+b) - x(a+b)$ b) $4(a+b) - 3(a+b)^2$

Ex.16 Factor

a) x-2y from the following expression: -2(x-2y)z + (x-2y)z²
b) c-d from the following expression: 3a(c-d)² - (a+b)⁶(c-d)
c) (c+d)² from the following expression: (c+d)² - 4a(c+d)³
d) (b³ + c)³ from the following expression: (b³ + c)³ - 6(b³ + c)⁶ + 8(c+b³)⁴
e) (cd)² from the following expression: 8(cd)² - ac²d³

Ex.17 Factor *a* from the following expression: $a - a^2 + 3$

Ex.18 From the expression: 4-x, factor the following: a) 2 b) x c) 2x d) 4x

Ex.19 List all terms of the denominator and numerator of the following algebraic fraction. For each such term list all its explicit factors. Find all factors that are common to all terms. If you were asked to simplify the fraction, what would be the expression by which you would divide the numerator and denominator to simplify it?

a)
$$\frac{t}{2t-ty}$$
 b) $\frac{x+xy}{2x-ax}$ c) $\frac{3ab}{ab-a}$

Ex.20 In the expression: $\frac{7x}{x-5}$, can x be viewed as a factor of the denominator? Can x be viewed as a factor of the numerator? Can we "cancel x". If not, why? If yes, what is the resulting expression?

Ex.21 In the expression: $\frac{7x}{x^2y}$, can x be viewed as a factor of the denominator? Can x be viewed as a factor of the numerator? Can we "cancel x". If not, why? If yes, what is the resulting expression?

Ex.22 In the expression: $\frac{(a-b)x}{a-b}$, can a-b be viewed as a factor of the denominator? Can a-b be viewed as a factor of the numerator? Can we "cancel a-b". If not, why? If yes, what is the resulting expression?

Ex.23 In the expression: $\frac{a-b+x}{a-b}$, can a-b be viewed as a factor of the denominator? Can a-b be viewed as a factor of the numerator? Can we "cancel a-b". If not, why? If yes, what is the resulting expression?

Ex.24 Simplify, if possible. If not possible, clearly say so. Also, name the expression by which you divide the numerator and denominator.

a)
$$\frac{3xy}{9yx}$$
 b) $\frac{-a^2}{b^2a^2}$ c) $\frac{a+b}{7(a+b)}$ d) $\frac{2abc}{8ab}$
e) $\frac{15x(a-b)}{25x}$ f) $\frac{a^2(b-c)}{2a}$ g) $\frac{5xy^4}{20y^3}$ h) $\frac{4x(-5x^2)}{x}$
i) $\frac{2}{2x+2y}$ j) $\frac{xy+xz}{3x}$ k) $\frac{4x-5x^2}{x}$ l) $\frac{2x+2y}{2}$
m) $\frac{3x}{3x-9x^2}$ n) $\frac{a^3b-4b^4a^4}{a}$ o) $\frac{2x+3}{2x}$ p) $\frac{bc(b+e)}{b}$

Ex.25 Simplify, if possible. If not possible, clearly say so.

a)
$$\frac{a-3b}{-3b+a}$$

b) $\frac{x}{2x-x^2}$
c) $\frac{2a+b+c}{2a+b}$
d) $\frac{xy+2z}{y+2z}$
e) $\frac{12x-4}{3x-1}$
f) $\frac{-4x+12y+8z}{4}$
g) $\frac{xy-xy^2+x^2y}{3yx}$
h) $\frac{u+2v-s}{s-2v-u}$
i) $\frac{3(u-v)^2}{u-v}$
j) $\frac{3x^4y-6x^2y^2z}{xy}$

Recall that the area of a rectangle is equal to the product of its length and width. Let's denote a rectangle's length by x and its width is y. We have:



a) Find the area of a rectangle whose length is 5 inches (x = 5), and width is 4 inches (y = 4).

b) Find the area of a rectangle whose length is *a* inches (x = a), and width is *c* inches (y = c).

c) Consider a rectangle of length a + b, and width is c:



The area of the above rectangle is c(a+b). Explain why.

d) This rectangle can be split into two smaller rectangles as follows:



The area of the above rectangles is: *ca* of the first one, and *cb* of the second one

Since the area of the rectangle before splitting is equal to the sum of areas of two rectangles after splitting, we have:

$$c(a+b) = ca + cb$$

The above is a known law. What is the name of this law?

- 1. Rewrite the following expressions without parentheses:
- a) 2(x + y)

b) -2(x+y)

c) -(x+y)

d) $(x+2y)\cdot 4$

e) $(\frac{x}{2} - \frac{y}{4}) \cdot 4$

a) 5(a+3c)

b) -4a(c-5d)

c)
$$\left(\frac{1}{5}x+10y\right)\cdot 15$$

d)
$$0.1a(b - 30c + 10d)$$

- e) -(4x-6y)
- 2. Circle all expressions that are equivalent to $5(x + \frac{y}{5})$:
- a) 5x + y
- b) $(x + \frac{y}{5})5$ c) $5(\frac{y}{5} + x)$ d) $5x + \frac{y}{5}$

1. Write the following statements as algebraic expressions using parentheses where appropriate and then remove parentheses:

a) The product of 2x and 3-x

b) The product of
$$a + 2b + \frac{1}{4}c$$
 and 4.

- c) The product of $3x^2 2x + 5$ and -2
- d) The opposite number of 3x y

2. Write
$$\frac{2}{7}(14-7a)$$
 in four different equivalent ways.

- 3. Circle all expressions that are equivalent to -4(x+3y)
- a) -4x 12y
- b) -12y 4x
- c) (x+3y)(-4)
- d) (x+3y) 4
- e) -4x + 3y

1. Write the following statements as algebraic expressions using parentheses where appropriate and then remove parentheses:

a) The product of:
$$-a - \frac{b}{3}$$
 and -12

b) The product of 0.2 and 2 - x + 0.3y.

c) The product of
$$\frac{2}{5}$$
 and $x^3 - \frac{10}{11}$

- d) The opposite number to $-4(a+b) \frac{c}{7}$
- 2. Underline all expressions that are equivalent to -3(a-2b):
- a) -3a 6b
- b) -3a + 6b
- c) (a-2b)(-3)
- d) (2b-a)(-3)
- e) 6b 3a

- 1. Remove parentheses and perform the indicated operations:
- a) $x^2(a+2x)$
- b) $(-x + y^2)y$

c)
$$(2b + \frac{1}{2}a^3)2ab$$

d)
$$3x^2y \cdot (2x - y^3)$$

e) (m+n)(x+y)

f)
$$(1+y)(8z^3-2)$$

g)
$$(3x^3 - 9)(\frac{1}{9} + xy)$$

1. Write the following statements as algebraic expressions using parentheses where appropriate and then remove parentheses:

- a) The product of a+4 and 2-b
- b) The product of -x+2 and x^3-1

- 2. Remove parentheses:
- a) $(3a^2 b)(a + b^3)$
- b) $(2x y)(3x^3 1)$

c) (0.2 + 8a)(0.1a - 10)

d)
$$\left(\frac{2}{3}-c\right)\left(\frac{1}{4}+\frac{d}{6}\right)$$

- 1. Remove parentheses:
- a) -a(b+2c-4d+5f)

b) -a(b+2c)x

c)
$$-2x^5z(-2x+y^4-3xyz)$$

d)
$$(3x^2 - 2y + z^3)(x - 3y)$$

e) -(3x - xy - 2)(1 - z)
1. List all terms in each of the following expressions. After factoring a common factor from each of them, how many terms should you have inside parentheses?

a) ax + 2x

b)
$$-xy^4 - x^2y^3(y-1)$$

c)
$$-3xa - 4x^8a^2 - yx + a^7x^3$$

d)
$$4(v-w) - y(v-w)$$

- 2. Factor -1 in each of the following:
- a) *a*+*b*
- b) *a*−*b*
- c) -a+b

d) -a-b

1. Factor 2 from the following expression:

2l + 2w

2. Factor -3 from the following expression:

-3x + 6y

3. Factor 2x from the following expression:

-2x+4ax

4. Factor -2x from the following expression:

 $4x^2 - 2x$

1. From the expression $4xy - 8xy^2 + 12x^3y^5$, factor the following:

a) -1

b) 4

c) -4

d) *x*

e) 4*xy*

- 1. Factor 2 from the following expression:
- $2x^2 + 4y^4$
- 2. Factor x^2 from the following expression:
- $2x^2 3x^4$
- 3. Factor *xy* from the following expression:
- $-5xy+4yx^3z$

- 4. Factor $2x^2y$ from the following expression:
- $6x^2y + 14y^7x^3$

- 5. Factor 3*ab* from the following expression:
- $6ab^5 3ba$

1. Factor a^2 from the following expression:

$$a^{3} - 4a^{2}x$$

2. Factor z^3 from the following expression:

$$3z^5 + z^4 - 2z^3$$

- 3. Factor $\frac{1}{3}$ from the following expression: $\frac{1}{3} + \frac{1}{9}x - \frac{5}{18}y$
- 4. Factor a^2b from the following expression:

$$a^3b - a^2b + 4a^7b^2$$

5. Factor $3abc^2$ from the following expression:

 $3abc^2-12a^6bc^5$

1. Factor -1:

$$2d - \frac{a-b}{2}$$

2. Factor -1:

$$\frac{a-b}{2} + 2d$$

3. Factor $-\frac{1}{2}$ $-\frac{q}{2} - \frac{xy}{4}$

- 4. Factor $\frac{5}{9}$ from the following expression: $\frac{5}{9}F - \frac{160}{9}$
- 5. Factor $3z^2y^3$ from the following expression:

 $3z^4y^6 - 9y^4z^2$

6. Factor $x^9 yz^3$ from the following expression:

$$2x^{10}yz^3 - 7x^{11}y^3z^4 + 2x^{12}yz^4$$

1. Factor x from the following expression:

```
ax + bx
```

2. Factor (x - y) from the following expression:

a(x-y) + b(x-y)

3. Factor x^2 from the following expression:

$$2x^3 - x^2$$

4. Factor $(x - y)^2$ from the following expression:

$$2(x-y)^3 - (x-y)^2$$

5. Factor $(3-a)^2$ from the following expression:

$$(3-a)^3 y - 3x^2(3-a)^4$$

1. From 3 + a factor:

a) -1

b) 3

c) a

2. From $r^2 + rR + R^2$ factor:

a) *r*

b) *R*

c) *rR*

- 1. Consider $\frac{x+1}{x+a}$
 - a) Can x be viewed as a factor of the denominator?
 - b) Can x be viewed as a factor of the numerator?
 - c) Can we "cancel x". If not, why? If yes, what is the resulting expression?

- 2. Consider $\frac{x+1}{a(x+1)}$
 - a) Can x+1 be viewed as a factor of the denominator?
 - b) Can x+1 be viewed as a factor of the nominator?
 - c) Can we "cancel x+1". If not, why? If yes, what is the resulting expression?

- 1. Consider $\frac{a^2}{1+a^2}$
 - a) Can a^2 be viewed as a factor of the denominator?
 - b) Can a^2 be viewed as a factor of the numerator?
 - c) Can we 'cancel a^2 '. If not, why? If yes, what is the resulting expression?

2. Simplify by dividing numerator and denominator by the same expression. Also, name the expression by which you divide the numerator and the denominator:

a)
$$\frac{3x}{x^2}$$

b)
$$\frac{4x}{8y}$$

c)
$$\frac{3xy}{9x^2y}$$

Homework/Class Work/Quiz

1. Simplify, when possible, by dividing numerator and denominator by the same expression. Name the expression by which you divide the numerator and the denominator. If the simplification is not possible, indicate so.

a)
$$\frac{3mn}{m^2}$$

b)
$$\frac{-16x^2y}{8y+1}$$

c)
$$\frac{3x+3}{9x^2y}$$

d)
$$\frac{6a+a^3}{a^5}$$

e)
$$\frac{uv}{uv - uvt^3}$$

5T

1. Simplify, if possible. If simplification is not possible, clearly say so. Also, name the expression by which you divide the numerator and denominator.

a)
$$\frac{3x - xy}{x}$$

b)
$$\frac{22a-b}{11c}$$

c)
$$\frac{ab-a}{8ab-4a^2}$$

d)
$$\frac{3x^2y - yx^2}{x^2yz}$$

e)
$$\frac{h-2t}{2t-h}$$

a)
$$\frac{4-a}{1-a}$$

b)
$$\frac{3a^2b}{3a-9ba}$$

c)
$$\frac{A^4 + A^5}{A^3 - A^7}$$

d)
$$\frac{-ab+ab^2}{2ab^2-2ab}$$

e)
$$\frac{p-q-r}{q-p+r}$$

1. Simplify, if possible. If simplification is not possible, clearly indicate so.

a)
$$\frac{x(a+1) - y(a+1)}{2(a+1)}$$

b)
$$\frac{(a+b)(2c-d)}{4(a+b)^2}$$

c)
$$\frac{2x-y}{2(x-y)}$$

d)
$$\frac{2q-s+t}{t-s+2q}$$

Lesson 6

Topics:

Addition and subtraction of algebraic expressions; Review for Test 2.

In this lesson we will learn how and when to perform the operations of addition and subtraction of algebraic expressions.

<u>Like terms:</u>

Consider $3x - 4y^2 + z$. The expression consists of three terms: 3x, $-4y^2$, z. Each term can be viewed as a product of a numerical and non-numerical factor (recall that numerical factors are also called numerical coefficients). And so,

- 3x has a numerical factor 3, non-numerical factor x
- $-4y^2$ has a numerical factor -4, non-numerical factor y^2
- z has a numerical factor 1, non-numerical factor z

Like Terms Like terms are terms that have non-numerical factors equal.

What is meant by non-numerical factors being equal is that they are equal (equivalent) as algebraic expressions. That does not mean that they 'look identical'. You will notice that in the second example below, xy and yx are equivalent even though they do not 'look identical'. Examples:

3a and 7a are like terms because their non-numerical factors, both *a*'s, are equivalent. 4xy and -3yx are like terms because *xy* and *yx* are equivalent.

 x^2 and x^3 are not like terms since x^2 is not equivalent to x^3 .

Another way of looking at like terms is that two terms are alike, if they are can be written as expressions that have the same variables with the same exponents. For example, $5x^3y^2$ and $4yx^3y$ are like terms, because $4yx^3y$ can be written as $4x^3y^2$, and thus both expressions consist of variable x in the third power and y in the second one. It should be stressed once again, that being like terms does not mean 'looking identical'.

Adding and subtracting like terms:

If we are to add or subtract objects they must have the same units. For example, we can add dollars to dollars and pounds to pounds, but we cannot add dollars to pounds. If we have 3 oranges and 5 oranges, together we have 8 oranges, but if we have 3 oranges and 5 plums, we cannot add them together. The same idea applies to addition and subtraction of algebraic expressions: Like and only like terms can be added or subtracted.

We apply the Distributive Law:

3a + 5a = (3 + 5)a = 8a(Or, 3 oranges and 5 oranges equal 8 oranges)

To add (or subtract) like terms, add (or subtract) their numerical coefficients and keep non-numerical factors the same.

For example:

3x - 8x = (3 - 8)x = -5x

Similarly,

$$\frac{1}{7}xy + \frac{3}{7}yx = \left(\frac{1}{7} + \frac{3}{7}\right)xy = \frac{4}{7}xy$$
$$0.3y^2 - 0.1y^2 = 0.2y^2$$

The process of adding and subtracting like terms is called **combining like terms** or **collecting like terms**.

Simplification of expressions with two or more terms by collecting like terms:

Let's examine the following expression: 3x + 4y + 2x - 6y. There are two "groups" of like terms in this expression: x's and y's. By changing the order of terms, we can group all like terms together, collect them, and as a result obtain a simplified expression:

$$3x + 4y + 2x - 6y = 3x + 2x + 4y - 6y = (3 + 2)x + (4 - 6)y = 5x - 2y$$

Simplification of expressions requiring the removal of parentheses:

the same.

Collecting like terms often has to be preceded by removing parentheses. This is always done according to the Distributive Law:

a + 3(a - 2) = a + 3a + 3(-2) = a + 3a - 6 = 4a - 6	We multiply 3 by each term in parentheses, and then collect all like terms.
3x - (5x - 1) = 3x - 5x + 1 = -2x + 1	Parentheses following a minus sign are removed by reversing the sign of each term inside parentheses (operation equivalent to multiplication by -1).
4m + (-2 + m) = 4m - 2 + m = 4m + m - 2 = 5m - 2	Parentheses following a plus sign are redundant, and thus can be dropped. Each term inside parentheses stays

The Distributive Law, together with the ability to collect like terms, allows us to derive two important formulas:

Consider $(a+b)^2$. Our goal is to remove parentheses:

 $(a+b)^2 =$ use the definition of the square of the expression (a+b)(a+b) = apply the Distributive Law for the product of two sums $a^2 + ab + ba + b^2 =$ collect like terms $a^2 + 2ab + b^2$

Thus the following is true:

The Square of the Sum of Two Expressions.

 $(a+b)^2 = a^2 + 2ab + b^2$

Notice that this means that the square of the sum of two terms is not equal to the sum of their squares. In other words: $(a+b)^2 \neq a^2 + b^2$.

Using exactly the same technique, one can prove that:

The Square of the Difference of Two Expressions.

 $(a-b)^2 = a^2 - 2ab + b^2$

Again, this means that the square of the difference of two terms is not equal to the difference of their squares. In other words: $(a-b)^2 \neq a^2 - b^2$.

Examples and Problems with Solutions

Example 6.1 Circle all expressions that are like $3x^3y^2$:

 $-4y^2x^3$ $\frac{yx^3y}{2}$ $0.2(xy)^2y$ $-2x^2y^3$

Solution:

We should circle: $-4y^2x^3$ (because $-4y^2x^3 = -4x^3y^2$) and $\frac{yx^3y}{2}$ (because $\frac{yx^3y}{2} = \frac{x^3y^2}{2} = \frac{1}{2}x^3y^2$).

Example 6.2 If possible, collect like terms. If not possible, clearly say so.

a)
$$-3x + 4x$$
 b) $\frac{1}{2}a^2 - \frac{3}{7}a^2$ c) $0.5mn - 0.7m$ d) $\frac{ab}{3} - \frac{1}{6}ba$

Solution:

a)
$$-3x + 4x = (-3 + 4)x = x$$

b) $\frac{1}{2}a^2 - \frac{3}{7}a^2 = \left(\frac{1}{2} - \frac{3}{7}\right)a^2 = \left(\frac{7}{14} - \frac{6}{14}\right)a^2 = \frac{1}{14}a^2$
c) Since terms *mn* and *m* are not like terms, terms cannot be combined.
d) $\frac{ab}{3} - \frac{1}{6}ba = \frac{1}{3}ab - \frac{1}{6}ab = \left(\frac{1}{3} - \frac{1}{6}\right)ab = \left(\frac{2}{6} - \frac{1}{6}\right)ab = \frac{1}{6}ab$

Example 6.3 Simplify by collecting like terms:

a) 3x + a - 4x - aSolution:

> a) 3x + a - 4x - a = 3x - 4x + a - a = (3 - 4)x + a - a = -xb) 7y + 5 - 12y + y - 6 = 7y - 12y + y + 5 - 6 = (7 - 12 + 1)y - 1 = -4y - 1

<u>Example 6.4</u> Write each of the following expressions using algebraic symbols, remove parentheses, if necessary, and collect like terms:

b) 7v + 5 - 12v + v - 6

a) add 3x-2 and -4x+2

b) subtract 3x-2 from -4x+2

c) multiply 3x - 2 and -4x + 2

Solution:

a) This should be written as: 3x-2+(-4x+2). We remove parentheses and collect like terms (x's separately, numbers separately) to get:

3x - 2 + (-4x + 2) = 3x - 2 - 4x + 2 = -x

b) This should be written as: -4x + 2 - (3x - 2) (Notice that "subtract from" causes us to reverse the order of the expressions). Removing parentheses following a minus sign reverses signs of *all* terms inside parentheses (3x 'becomes' -3x, 2 'becomes' -2). After removing parentheses we collect like terms (x's separately, numbers separately) to get:

-4x + 2 - (3x - 2) = -4x + 2 - 3x + 2 = -7x + 4

c) This should be written as: (3x-2)(-4x+2). To remove parentheses we apply the Distributive Law:

$$(3x-2)(-4x+2) = 3x(-4x) + 3x(2) - 2(-4x) - 2(2) = -12x^{2} + 6x + 8x - 4 = -12x^{2} + 14x - 4$$

<u>Example 6.4</u> Remove parentheses and simplify by collecting like terms:

a)
$$3(x^5-2)-(4-x^5)$$

b)
$$(2-x)^2$$

Solution:

a)
$$3(x^5 - 2) - (4 - x^5) = 3x^5 + (3)(-2) - 4 + x^5 = 3x^5 + x^5 - 6 - 4 = 4x^5 - 10$$

b) $(2 - x)^2 = (2 - x)(2 - x) = 2 \times 2 + 2(-x) - x(2) - x(-x) = 4 - 2x - 2x + x^2 = 4 - 4x + x^2$

Common mistakes and misconceptions

Mistake 6.1

2 apples + 3 apples \neq 5(apples)². Similarly, $2x + 3x \neq 5x^2$. Instead, 2x + 3x = 5x (although $2x(3x) = 6x^2$).

Mistake 6.2

 $3x-(a+b)\neq 3x-a+b$

Minus sign pertains to the whole expression (a + b), and all signs must be 'changed to the opposite', not only the first one: 3x - (a + b) = 3x - a - b

Mistake 6.3

We discussed it (see Mistake 4.3), but let us repeat once again: Although it is true that $(ab)^2 = a^2b^2$, $(a+b)^2 \neq a^2 + b^2$

Mistake 6.4

When collecting like terms in $\frac{x}{3} + \frac{2x}{3}$ you CANNOT multiply the expression by 3 to get rid of fractions: $\frac{x}{3} + \frac{2x}{3} \neq x + 2x$ (it would be like saying that $\frac{1}{3} + \frac{2}{3} = 1 + 2$).

Exercises with Answers (For answers see Appendix A, page A1)

Ex.1 Are x and
$$-x$$
 like terms?

Ex.2 Are any of the following like terms: $7x^2y$, $7xy^2$, and $7(xy)^2$?

Ex.3 Which of the following rows consists of all like terms? a) $-\frac{1}{2}x$, $4x^2$, $0.5x^2$, $-\frac{1}{6}x^2$, 3xb) -3xy, 8yx, -0.6xy, 2xy, $-\frac{3}{7}xy$ c) 5x, $5x^2$, $5x^3$, $5x^4$, 5d) -2a, -5a, 7a, 29a, a, -11a

Ex.4 Circle all terms that are like
$$6x^2y$$

 $6x^2y^2 - 2x^2y$ 5xy $0.3yx^2$

Ex.5 Circle all terms that are like
$$-\frac{3}{7}a^2b$$

 $5(ab)^2$ $2a^2b$ $-\frac{3}{7}ab^2$ $\frac{1}{7}ba^2$

Ex.6 Circle all expressions that are like
$$x^2y$$

 yx^2 xyx $2xy$ y^2x xxy

*Ex.*7 Circle all expressions that are like a^5b^3 $a^2b^5a^3 - b^2a^5b$ $(ab)^3b^3$ $2(ab)^3a^2$ a^3b^5

Ex.8 If possible, add (or subtract) the following expressions. If not possible, clearly say so. a) 4x-2x b) $a^2 - a$ c) y - y d) a + abe) st + ts f) $ac^2 + cac$ g) $ac^2 + 6ca^2$ h) 7mhv - hmvi) $2xy^3z^5 + y^3z^5x$ j) $3m^3n + 6m^2nm$ k) $7xy^3z^5 - xy^4z^5$ l) $9a^2b^2 - 7(ab)^2$

Ex.9 Collect like terms by first factoring x.

a)
$$3x-4x$$

b) $\frac{1}{3}x-\frac{2}{7}x$
c) $\frac{2}{11}x-\frac{3x}{22}$
d) $0.3x-0.5x$
e) $-\frac{7}{9}x-\frac{2x}{5}$
f) $\frac{x}{5}-\frac{2}{3}x+\frac{3x}{10}$

Ex.10 If possible, collect like terms. If not possible, clearly say so.

a) 7x-8xb) 2a-7ac) $\frac{1}{2}x+\frac{1}{2}x$ d) $\frac{1}{2}x-\frac{3}{2}x$ e) ab-8baf) 4-7xg) $5c^3-10c^4$ h) $-0.4x^2y-0.8yx^2$

Ex.11 Rewrite by grouping all like terms together. Then collect like terms. (for example 3x - 7y + 2x - 2y = 3x + 2x - 7y - 2y = 5x - 9y). a) -3x + 8y - 8x - 2x b) -5a - 3b - 2b - 7a c) -2ab - 4ba + 2 + 3ab - 1

Ex.12 Simplify by collecting all like terms:

a)
$$3j-4j+2j$$

b) $2+a-\frac{2}{3}a$
c) $0.2z-0.5z+z$
d) $2-7m-4$
e) $-x^3 + x^4 - x^3$
f) $2x + y + x - 2y$
g) $-\frac{3x}{2} + \frac{1}{2}x - 1$
h) $-3y + \frac{1}{2}x - x + 4y$
i) $\frac{cd}{3} + \frac{1}{3}dc - d + c$
j) $a^2b^3 - 4b^2a^3 - 6a^2b^3$
k) $\frac{3}{38}ab - \frac{7}{19}ba + 2\frac{3}{4} + 3\frac{1}{8}$

Ex.13 Collect like terms, and only then evaluate $-3x + 2y - \frac{2}{3}y + 4x - \frac{1}{3}y$ when

a)
$$x = -1$$
, $y = -3$
b) $x = -2$, $y = \frac{1}{5}$
c) $x = 0.5$, $y = -3.2$

Ex.14 Evaluate $\frac{2}{5}x + \frac{1}{2}x + \frac{x}{10}$ when a) x = 4 b) x = -10 c) $x = \frac{2}{3}$. If you have not done this yet, simplify the expression by collecting like terms. Compare the

obtained expression with the results of the evaluations.

Ex.15 Remove parentheses, and then collect like terms a) (6a-2)+12a b) -2(t-5)+4t c) y-3(-y+2)

d)
$$a - 0.1(2 - 3a)$$

e) $\frac{2x}{5} - (2x - 1)$
f) $(q + 6)(-8) - 21q$
g) $-(x - 1) - x$
h) $4a + 7a + 2(8a + 1)$
i) $3a - 9b - (4a - \frac{4}{5}b)$
j) $2x - 5y - 4(3x + y)$
k) $-6(\frac{2}{3}d - \frac{1}{2}a) - a - d$
l) $3a^2 - 1 - (4a^2 + 2)$
m) $-3xy + 7yx - (xy + 3)$
n) $-3cb + abc - 2(a + bc)$
o) $\frac{1}{3}a - b - \frac{1}{3}(a - 2b)$
p) $(6x^4 + 3x^3 - 1) - (3x^3 - 3x^2 - 3)$
q) $(1 + 4x + 6x^2 + 7x^3) + (5 - 4x + 6x^2 - 7x^3)$

Ex.16 Write as an algebraic expression, rewrite without parentheses and collect like terms, if possible;

a) The sum of 3x - 1 and -4x + 2b) The difference of $-4a^3 + 2a$ and $4a^3 - 2$ c) The difference of -a + 2 + 3b and -2b + ad) Subtract -2xy from 3yxe) The product of $a - \frac{1}{3}$ and $\frac{2}{5} - 2a$ f) The product of $2x^2 - y$ and $3y - x^2$ g) The sum of -mnk, 4nmk, and -3mnh) The sum of 3x and 2, then raised to the second power i) The difference of 2a and b, then raised to the second power j) The product of 2 and -4x + 1, then added to 5x + 2

Ex.17 Remove parentheses, and then collect like terms

a)
$$(x^{4} + 2)^{2}$$

b) $(3x - 1)^{2}$
c) $(a - 2)(a + 4)$
d) $(3b - c)(b + \frac{1}{2}c)$
e) $\frac{1}{3}(3c - x) + 2(5x - c)$
f) $(1 + x - 2x^{2})(2 - x)$
g) $(a^{3} + b)(a^{2} - 2b)$
h) $(\frac{2}{3} - 2x)^{2}$
i) $-(a - 3b + c) - (c - a)$

Ex.18 Remove parentheses, collect like terms, and only then evaluate -a - 2(a+b) - b if

a)
$$a = -1$$
, $b = -2$ b) $a = \frac{3}{14}$, $b = \frac{2}{7}$ c) $a = -2.4$, $b = 0.6$

Ex.19 Remove parentheses, collect like terms, and then evaluate $4x^2 - xy - 2(yx - 1 + 2x^2) - 2$ when

a) x = -1, y = -3 b) x = -2, $y = \frac{1}{6}$ c) x = 0.5, y = -0.2

Homework/Class Work/Quiz

Recall that the area of a rectangle is equal to the product of its length and width. If we denote a rectangle's length by x and its width is y. We have:

Area of a rectangle =xy

A square is a rectangle that has all its sides equal. Thus, x = y, and we have:

Area of a rectangle = x^2

a) Find the area of a square with the side x = 3.

b) Find the area of a square with the side x = s.

c) Consider a square with the side a + b. Can you see that its area is $(a + b)^2$.



We can cut this square as shown below.



Find the area of each of four rectangles (two of them are actually squares). Write the sum of the areas of four rectangles as an algebraic expression and simplify it by collecting like terms.

d) The area of the square before cutting, $(a+b)^2$, is equal to the sum of areas of those four rectangles, that is to the expression obtained in (c). We can write:

$$(a+b)^2 = a^2 + 2ab + b^2$$

Do you recognize this formula? What is it?

1. Are the following like terms?

$$-2x^3y \qquad -2xy^3 \qquad -2(xy)^3$$

- 2. Are the following like terms?
- $3a^2 \qquad -3a^2 \qquad 3^2a^2$
- 3. In which of the following rows are all the terms like terms?

a)
$$-\frac{3}{5}x$$
, $-6x^2$, $0.2x^2$, $-\frac{1}{7}x^2$, $8x$
b) $-3ab$, $9ba$, $-0.3ab$, $-8ab$, $-\frac{2}{5}ba$
c) $3x$, $3x^2$, $3x^3$, $3x^4$, 3
d) $-4y$, $-7y$, $8y$, $9y$, y , $-y$

4. Circle all terms that are like $-a^2b^3$

$$6(ab)^3 - 2b^3a^2 7ab^3a 0.3a^2b^3$$

1. Circle all terms that are like $4m^2n^5$

 $-n^5m^2$ $7 \cdot m \ n^5m$ $\frac{m^2n^5}{2}$ $-2(mn)^2n^3$

2. If possible, add (or subtract) the following expressions. If not possible, clearly say so.

a)
$$-2x + 3x$$

b) 5*a*-7*a*

c) -xy - yx

d) $4a - 7a^2$

1. Circle all terms that are like $\frac{5x}{y}$

 $-\frac{x}{y} \qquad \frac{5y}{x} \qquad \frac{25x^2}{y^2} \qquad -2x\frac{1}{y} \qquad \frac{4}{y} \cdot x$

2. If possible, add (or subtract) the following expressions. If not possible, clearly say so.

a) -2x - 7x

b) -4ca - 7cb

c) $-xy^2 + 4y^2x$

d) -x-x

1. If possible, add (or subtract) the following expressions. If not possible, clearly say so.

- a) abc + bca
- b) $\frac{xy}{z} \frac{y}{z}x$
- c) $a^4 + (a^2)^2$
- d) $ab^{3} (ab)^{3}$
- 2. Simplify the following expressions by collecting like terms:
- a) 3x 8x
- b) 4a 7a 2a
- c) $-5x^2 + 8x^2 7x^2$

d)
$$\frac{1}{2}x + \frac{1}{2}x$$

e)
$$-\frac{1}{2}x - \frac{1}{2}x$$

- 1. If possible, add (or subtract) the following expressions. If not possible, clearly say so.
- a) xy + 3yx

b) $x^2y + 3yx$

c) $x^2y + 3yx^2$

d) $xyx + 3yx^2$

2. Simplify the following expressions by collecting like terms:

a)
$$\frac{2x}{3} - \frac{2}{3}x$$

b) $-\frac{7}{5}a^2 - \frac{2}{5}a^2$
c) $-\frac{2}{3}vt + \frac{5}{6}tv$

d)
$$\frac{7}{8}a - \frac{4}{5}a$$

1. Simplify by collecting like terms:

a) -x + 4y - 7x - 2

b) 3a + b - 7a + b

c)
$$xy - xy^2 + 2 - 4xy - 5xy^2$$

d)
$$\frac{a^2}{7} - \frac{2}{3}a^2 - 7a - 9 + 2a - 1$$

- 1. Simplify by collecting like terms:
- a) -m+n-3m+m

b)
$$3y+1-7y+2x+8x-5$$

c)
$$-\frac{1}{6}-b^{3}c-\frac{b^{3}c}{6}$$

d)
$$0.3n + m + k - 0.4n - k + m$$

2. Collect like terms, and only then evaluate $2 + \frac{R}{4} - \frac{R}{3} - 2 + \frac{13R}{12}$ when R = 0.123.

1. Remove parentheses and collect like terms:

a) 3x - 2 + (-4x + 1)

b) 3x - 2 - (-4x + 1)

c) 3x - 2 + 5(-4x + 1)

d) 3x - 2 - 5(-4x + 1)

6J

a) -(x-2) + (-x-2)

b) -(x-2)-(-x-2)

2. Students were asked to simplify the following expression: -2(a-b) + 3a - 3b. The following answers were given:

Student A: a-bStudent B: b-aStudent C: -b+aStudent D: -a+b

List all students who gave the right answer.

- 1. Remove parentheses and collect like terms:
- a) -2acb+3a-(-4abc+1-a)

b)
$$-x^2y - 2 - \frac{2}{5}(10yx^2 + 15)$$

c)
$$\frac{3}{4}x - \frac{2}{5}y - \left(\frac{4}{3}y - x + 2\right)$$

d)
$$-(-0.5x^2) + (-0.7x + 0.2x^2)$$

1. Remove parentheses and collect like terms:

a)
$$x^3 - 2x^2 - 2(-4x^2 + 7x^3)$$

b)
$$-(2b^2a+3a^2)-(-4ab^2+2-a^2)$$

c)
$$-(-2xy-2)-3(5yx^2+4)$$

d)
$$\frac{3}{5}a - \frac{1}{7}b - \left(a - \frac{4}{3}b - \frac{1}{3}\right)$$

a) Add
$$-A+2$$
 and $\frac{A}{3}$

b) Subtract xyz - v from -3yzx - 7v

c) Subtract
$$\frac{-x}{3} + \frac{2y}{3}$$
 from $\frac{2x}{4} - \frac{1}{3}y$

d) Add 3x - 2 + y, -2x + 1 and -y

Homework/Class Work/Quiz

a) Subtract -4x+5 from 3x

b) Subtract ab-c+de from 3ba-7ed

c) Add
$$-2x + 4a - 1$$
 and $\frac{x}{3} + \frac{2}{3}a$

d) Multiply -3x+2 and 5

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1.a) Students were asked to remove parentheses and collect like terms of the following expression: $(A+B)^2$. One student claimed that the answer was $A^2 + B^2$, the other one that it was $A^2 + 2AB + B^2$. Was either of them right?

b) If you were asked to remove parentheses and collect like terms of $(A - B)^2$, what would be your answer?

2. Remove parentheses and collect like terms:

a) $(c+2a)^2$

b)
$$\left(\frac{1}{2}-x\right)^2$$

c) $(0.2+Z)^2$
a) (a+3)(a-2)

b)
$$(1+x)(2-\frac{x}{2})$$

c)
$$(3x-2y)(x-y)$$

d) $(6+m)^2$

- 1. Remove parentheses and then collect like terms:
- a) -(-a) (b+a)

b) $(2y - 3x)^2$

c) -4(a-b+2)+(-3)

3. Remove parentheses, collect like terms:

$$\frac{2}{3}x - \left(\frac{4}{5}x + y\right)$$

After collecting like terms, evaluate the expression when: a) x = -15, y = -3

b)
$$x = 3, y = -\frac{1}{5}$$

a) $(ab-b^3)^2$

b) $(x + 2yx)^2$

$$c)\left(\frac{3}{5}-y^2\right)^2$$

3. Remove parentheses, collect like terms:

$$-\frac{2}{7}xy - \left(\frac{4}{5}yx - 1\right)$$

After collecting like terms, evaluate the expression when:

a) x = -35, y = -2

b) $x = \frac{1}{2}, y = -\frac{70}{19}$

1. Remove parentheses and collect like terms:

a)
$$-\frac{1}{3}(3x+6z)-(-2-z)$$

b)
$$-5(-\frac{3}{5}x-8y)-\frac{2}{3}y$$

c)
$$(a-10b)^2 + 20ba$$

d)
$$(2-x)^2 + (2+x)^2$$

1. Write the following statements as algebraic expressions, and then rewrite without them using parentheses. Collect like terms:

a) Subtract $4x^2$ from y and then raise the result to the second power.

b) Add $\frac{b}{3}$ and y^2 , and then raise the quantity to the second power

c) Subtract 4x from $2y^3$, and then multiply the result by $\frac{xy}{4}$

d) Write the opposite of -3x + 4y, and then add -2x + y to the result.

Lesson 7

Topics:

Test 2; Evaluation of more complicated algebraic expressions; Substitution of not only numbers but also algebraic expressions.

Recall:

If two quantities are equal, you can always substitute one for the other. "Equals can be substituted for equals"

Substitution of numbers for entire parts of expressions:

According to the principle **Equals can be substituted for equals,** not only can we substitute numbers for variables, but also for entire 'parts of expressions':

If, for example, we know that a + b = 2, we can evaluate $(a + b)^2 - 3(a + b)$ by substituting the value 2 for a + b:

 $(a+b)^2 - 3(a+b) = 2^2 - 3 \times 2 = 4 - 6 = -2$

Replacing parts of algebraic expressions with other equivalent expressions:

If we know that x = -3a and y = 5a, then we can **express** x + y in terms of a, i.e. write it as an expression that depends *only* on the variable a. Equals can be substituted for equals, and so we replace x with -3a, y with 5a:

x + y = -3a + 5a = 2a

This exhibits the direct relationship between x + y and a, and in turn, allows us to find the value of x + y any time we know the value of a (see Example 7.3).

Using equivalent forms of an algebraic expression for its evaluation:

If we are asked to evaluate 'a complicated' algebraic expression, we may be able to simplify the expression first, and then use its simplified form to perform the evaluation. Notice, that what we do is replace ('substitute for') the original algebraic expression with its equivalent ('equal') form. The principle **Equals can be substituted for equals** is used.

Sometimes, the *only* way to perform the evaluation is to first replace the expression with a certain equivalent form of it: Suppose, for example, that we know that x = 5, and y + z = 4. Is it possible to evaluate xy + xz? We will be able to do that if we can find an equivalent form of xy + xz, a form that would 'match' the information we have. If we factor x, we obtain:

$$xy + xz = x(y + z)$$

Now, we can replace x with 5, and y + z with 4: xy + xz = x(y + z) = 5(4) = 20

We are asked to evaluate 3(4x-3) - 6(2x-3), when x = 1. If we simplify the expression, we get: 3(4x-3) - 6(2x-3) = 12x - 9 - 12x + 18 = 12x - 12x - 9 + 18 = 9. The fact that 3(4x-3) - 6(2x-3) = 9 means that *no matter what the value of x is, the expression is always equal to 9*. We could plug in 4, 9.9, $\frac{34}{87}$, or any other number for x and the result of the evaluation would always be the same: 9. In particular, when x = 1, 3(4x-3) - 6(2x-3) = 9. Even if we replace x with an algebraic expression (for instance, a) or a very complicated looking expression (like (t + z)(u - w) or any other messy looking expression), the resulting expression is *still* equal to 9.

Examples and Problems with Solutions

Example 7.1 Evaluate the following expressions, if a-b=-1 and c=2

a)
$$a - b + c$$
 b) $a - c - b$ c) $\frac{3a}{c} - \frac{3b}{c}$ d) $2c - (b - a)$

Solution:

a) a-b+c = (a-b)+c = -1+2 = 1

b) We change the order of terms: a-c-b = a-b-c = -1-2 = -3

c) We write the expression as one fraction, and then factor 3 in the numerator:

 $\frac{3a}{c} - \frac{3b}{c} = \frac{3a - 3b}{c} = \frac{3(a - b)}{c} = \frac{3(-1)}{2} = -\frac{3}{2}$

d) We remove parentheses, and then change the order of terms: 2c - (b - a) = 2c - b + a = a - b + 2c = -1 + 2(2) = -1 + 4 = 3

<u>Example 7.2</u> Evaluate the following expressions, if $\frac{m}{n} = 4$:

a)
$$\frac{m^2}{n^2}$$
 b) $\frac{n}{m}$

Solution:

a)
$$\frac{m^2}{n^2} = \left(\frac{m}{n}\right)^2 = 4^2 = 16$$

b) $\frac{n}{m} = \frac{1}{\frac{m}{n}} = \frac{1}{4}$ Notice that $\frac{n}{m} = \frac{1}{\frac{m}{n}}$, because $\frac{1}{\frac{m}{n}} = 1 \div \frac{m}{n} = 1 \times \frac{n}{m} = \frac{n}{m}$

Example 7.3 The area of a rectangle is equal to the product of its length L, and width W: LW. Consider a rectangle with the length and width equal to each other. Recall that this type of a rectangle is called a square. Let a denote the side of the square, we have W = a, L = a. Express LW in terms of a to obtain the expression representing the area of the square. Find the area of a square, whose side is 2 inches long. Solution: We replace both W and L with a: $LW = aa = a^2$. Thus, we conclude, that the area of a square is equal to the square of its side. If a = 2 (side is 2 inches long), then $a^2 = 2^2 = 4$ and the area of the square is 4 in².

Example 7.4 If x = -z express the following expressions in terms of z: a) x^2 b) -xSolution: a) $x^2 = (-z)^2 = z^2$ b) -x = -(-z) = zExample 7.5 Express $(m+n)(m^2 - mn + n^2)$ in terms of a) x, if m = 2, and n = 3x. b) m, if n = m. Solution: a) We replace m with 2, and n with x, and simplify: $(m+n)(m^2 - mn + n^2) = (2 + 3x)(2^2 - 2(3x) + (3x)^2) = (2 + 3x)(4 - 6x + 3^2)$

 $(m+n)(m^{2} - mn + n^{2}) = (2+3x)(2^{2} - 2(3x) + (3x)^{2}) = (2+3x)(4-6x+3^{2}x^{2}) = (2+3x)(4-6x+9x^{2})$ b) We replace *n* with *m*, and simplify: $(m+n)(m^{2} - mn + n^{2}) = (m+m)(m^{2} - mm + m^{2}) = 2m(m^{2} - m^{2} + m^{2}) = 2mm^{2} = 2m^{3}$

Example 7.6 If $A^2 = v$ and $B^2 = 2v$, express (A+B)(A-B) in terms of v: Solution:

We must find some equivalent form of the above expression expressed only in terms of A^2 and B^2 . We will remove parentheses by multiplying:

 $(A+B)(A-B) = A^2 - AB + BA - B^2 = A^2 - B^2$. Now, we can replace A^2 with v, B^2 with 2v to get: $(A+B)(A-B) = A^2 - B^2 = v - 2v = -v$.

Example 7.7

a) Show that $(x-2)^2 - x^2 + 4x = 4$ b) Evaluate $(x-2)^2 - x^2 + 4x = 4$, if $x = -\frac{3478}{46299}$. c) Evaluate $(x-2)^2 - x^2 + 4x = 4$, if $x = (a+b)^2$. d) Find the value of $[(x-2)^2 - x^2 + 4x]^2$ Solution: a) $(x-2)^2 - x^2 + 4x = (x-2)(x-2) - x^2 + 4x = x^2 - 2x - 2x + 4 - x^2 + 4x = 4$ b) We have proved in (a) that $(x-2)^2 - x^2 + 4x$ is always equal to 4. Thus, if $x = -\frac{3478}{46299}$, $(x-2)^2 - x^2 + 4x = 4$. c) No matter what we substitute for x, $(x-2)^2 - x^2 + 4x$ is always equal to 4.

d) To find the value of $[(x-2)^2 - x^2 + 4x]^2$ replace $(x-2)^2 - x^2 + 4x$ with 4, to obtain: $[(x-2)^2 - x^2 + 4x]^2 = 4^2 = 16$

Exercises with Answers (For answers see Appendix A)

Ex.1 If
$$x^2 = 10$$
, evaluate the following:
a) $2x^2$ b) $\left(\frac{1}{30}x^2\right)^3$ c) $-3x^2 - 2$
Ex.2 If $A^5 = -3$, evaluate the following a) $-2A^5$ b) $-(2A^5)^2$ c) $-2-2A^5$
Ex.3 Evaluate the following expressions, if $\frac{a+b}{c} = -2$
a) $\frac{a+b}{c} - 2$ b) $-\left(\frac{a+b}{c}\right)^2$ c) $\left(-\frac{a+b}{c}\right)^2$
Ex.4 If $x + y = -2$, evaluate a) $7x + 7y$ b) $\frac{x}{7} + \frac{y}{7}$ c) $-x - y$ d) $(-x - y)^2$
Ex.5 If $\frac{1}{A} = -\frac{2}{7}$, evaluate a) $1 + A$ b) $\frac{-1}{A}$ c) $\frac{1}{-A}$ d) $\frac{1}{A^2}$
Ex.6 Evaluate the following expressions, if $2a + b - c = -7$:
a) $-c + b + 2a$ b) $-2a - b + c$ c) $-(-2a - b + c)$ d) $2a - (c - b)$ c) $a + b - c + a$
Ex.7 Evaluate the following expressions, if $GHJ = -1$:
a) $-JHG$ b) $G^3(HJ)^3$ c) $\frac{G^2}{G^3HJ}$
Ex.8 Evaluate the following expressions, if $\frac{a}{b} = 3$
a) $-\frac{a}{b}$ b) $a + b$ c) $\frac{b}{a}$ d) $a \cdot \frac{2}{b}$ e) $\frac{a^2a}{b^3}$
Ex.9 Evaluate the following expressions, if $\frac{xy}{z} = -\frac{1}{3}$
a) $\frac{yx}{z}$ b) $\frac{xy}{-z}$ c) $-\frac{x^2y^2}{z^2}$ d) $\frac{1}{z} \cdot xy$ e) $\frac{z}{xy}$
Ex.10 Evaluate the following expressions if
a) $a + b + c - d$ b) $2(a - d + c + b)$ c) $9(a + b)^2 - d + c$
Ex.11 Evaluate the following expressions if $x^2y = -0.2$, $x - z = 0.6$
a) $\frac{x - z}{x^2y}$ b) $-(x - z) - x^2y$ c) $y(x - z)x^2$

Ex.12 Evaluate the following expressions, if xy = -1, zt = -3: a) -4xtzy b) -xy + zt c) $\frac{xy - zt}{xy}$ *Ex.13* Evaluate the following expressions, if $x^3 y^4 z^6 = -\frac{2}{3}$.

a) $-z^6 y^4 x^3$ b) $x^3 (y^2 z^3)^2$ c) $-\frac{x^3 y^4}{3} \cdot z^6$

Ex.14 Evaluate the following expressions, if $x^2 + y^2 = 0.1$

a) $\frac{x^2}{0.2} + \frac{y^2}{0.2}$ b) $\frac{1}{2}x^2 + \frac{1}{2}y^2$ c) $\frac{1}{x^2 + y^2}$

Ex.15 Rewrite the expression $\frac{3a}{2}$ in terms of x, if it is given that a = 2x. Simplify.

Ex.16 Let C = -3x, $D = \frac{x}{2}$. Express the following expression in terms of x. Simplify.

a) *CD* b) *C* – *D* c) $3C^2D$ d) $\frac{C-2D}{4}$

Ex.17 Rewrite the expression a^2 in terms of x for each of the following. Write your answer without parentheses: a) a = x b) a = 5x c) a = -x d) a = -5x e) a = x + 1 f) $a = x^3$

Ex.18 Rewrite the expression
$$a^2 - 2ab + b^2$$
 in terms of x, if
a) $a = 1, b = x$ b) $a = 3x, b = 2x$ c) $a = \frac{x}{2}, b = -x$ d) $a = b = x$

Ex.19 Let $P = 3x^2 - 2x + 1$, $Q = -6x^2 + x - 2$, and $R = -3x^2 - 1$. Express the following in terms of x. Remove parentheses and simplify: a) P + R b) 2R - Q c) 2P + Q

Ex.20 Express the following expression in terms of s, if $x = s^3$, y = 2s. Simplify, if possible:

a) 2x - y b) $x^2 y$ c) $2x + y^3$ d) $\frac{xy^2}{4}$

Ex.21 Express *ab* in terms of *x*, if $a = \frac{5x}{126}$, $b = \frac{126}{5}$. Simplify.

- *Ex.*22 Express the expression $m^3n 2m^2n^2$ in terms of *s*, if a) m = s, n = -2s b) $m = 2s^2$, $n = s^4$ c) n = m = s
- *Ex.23* Express the expression $(m-n)^2 + (m+n)^2$ in terms of a) s, if m = 2s, n = 3s b) m, if n = m c) m, if n = -m

Ex.24 The following is true: $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ a) Express $(a+b)^4$ in terms of b, if a = bb) Express $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ in terms of b, if a = b. Simplify. c) Compare the results of (a) and (b) d) Express $(a+b)^4$ in terms of b, if a = -be) Express $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ in terms of b, if a = -b. Simplify. f) Compare the results of (d) and (e)

*Ex.*25 Rewrite the expression a - 2b + 3c + 4d in terms of x, if it is given that a + 3c = 5x and 4d - 2b = -x. Simplify.

Ex.26 Express the following expression in terms of m, if ab = m

a)
$$-3ba$$
 b) $\frac{a}{2}b$ c) $(6^{a})^{b}$ d) $a^{3}b^{3}$ e) $(-2b)(-\frac{1}{2}a)$

Ex.27 Express the following expression in terms of y, if $\frac{x}{z} = -y$, $\frac{t}{z} = 2y$. Simplify:

a)
$$\frac{x}{z} - \frac{t}{z}$$
 b) $\left(\frac{x}{z}\right)^2 - \left(\frac{t}{z}\right)^2$ c) $\frac{x+t}{z}$ d) $\frac{1}{z}x - \frac{2}{z}t$

Ex.28 Rewrite the expression (a-b)(a+b) in terms of x, if it is given that a = 5x and b = 2 - x. Simplify.

Ex.29 Simplify the following expression $\frac{3x+9z}{x+3z}$, and then evaluate when x = 2, z = -3. *Ex.30* The following is true (you are not asked to check it): $\left(a + \frac{1}{a}\right)^2 - \left(a - \frac{1}{a}\right)^2 = 4$ a) What is the value of the above expression, if a = 0.003? b) What is the value of the above expression, if a = x + y?

c) What is the value of
$$\left(x + \frac{1}{x}\right)^2 - \left(x - \frac{1}{x}\right)^2$$
?
d) What is the value of $\left(x^7 + \frac{1}{x^7}\right)^2 - \left(x^7 - \frac{1}{x^7}\right)^2$?
e) What is $-\left[\left(a + \frac{1}{a}\right)^2 - \left(a - \frac{1}{a}\right)^2\right]$ equal to?

Ex.31 One can check that $\frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-a)(x-c)}{(b-a)(b-c)} + \frac{(x-b)(x-c)}{(a-b)(a-c)} = 1$ (Now are not asked to sheak it)

(You are not asked to check it).

a) Evaluate the above expression, if a = 1, b = 2, c = 3, x = 4b) Evaluate the above expression, if $a = \frac{1}{2}, b = \frac{1}{3}, c = \frac{1}{4}, x = -\frac{2}{5}$. c) What is $1 - \left[\frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-a)(x-c)}{(b-a)(b-c)} + \frac{(x-b)(x-c)}{(a-b)(a-c)}\right]$ equal to?

- 1. Evaluate the following expressions, if $x + y = \frac{1}{3}$
- a) 3(x + y)

b) $\frac{x+y}{7}$

- c) -(y+x)+2
- d) $(x+y)^2$
- e) $2(x+y)^3$
- 2. Evaluate the following expressions, if $x^2 y = \frac{3}{2}$.
- a) $-4x^2y$

b) – *xyx*

.

1. If $z^2 = 3$, evaluate
a) $-z^2$
b) $z^2 z^2$
c) 2 <i>z</i> ×5 <i>z</i>
d) $(-z^2)^2$
e) $-(z^2)^2$
2. Evaluate $(x+y)z$, if
a) $x + y = 0.2$, $z = -3$

b)
$$xz = -\frac{5}{9}$$
, $yz = -\frac{2}{3}$

Homework/Class Work/Quiz

1. Evaluate the following expressions, if abc = -1:

a)
$$-\frac{3}{abc}$$

b)
$$\frac{cba}{2}$$

c)
$$a(-2)c(-3)b$$

d)
$$(ab)^2 \cdot c^2$$

2. Evaluate
$$\frac{v}{u}$$
, if
a) $\frac{u}{v} = 3$

b)
$$\frac{u}{v} = \frac{4}{9}$$

c)
$$\frac{u}{v} = 0.2$$

Homework/Class Work/Quiz

1. Evaluate the following expressions, if $ab = \frac{3}{4}$, and $bc = -\frac{1}{9}$:

a) ab + bc

b) bacb

- 2. Evaluate the following expressions, if $A + B = \frac{3}{4}$ and $C = -\frac{2}{7}$
- a) $\frac{A+B}{C}$
- b) A C + B
- c) 2A + 2B + C

d) AC + BC

1. Evaluate the following expressions, if $\frac{a+b}{c} = -2$

a)
$$-\left(\frac{a+b}{c}\right)^2 - 3\left(\frac{a+b}{c}\right)$$

b)
$$\frac{a}{c} + \frac{b}{c}$$

c)
$$-\frac{1}{c} \cdot (a+b)$$

- 2. Evaluate the following expressions, if 2m n = -7.
- a) -n+2m
- b) 4m 2n

c) n-2m

1. If
$$a^2 + b^2 = 4$$
, $ab = -\frac{1}{4}$, evaluate:
a) $(a^2 + b^2)ab$

b) $\frac{b^2 + a^2}{ab}$

2. Evaluate the following expressions, if m + n = 3 and x = -1

a) x^{m+n}

b) $(x-1)^{m+n-1}$

c) mx + nx

1. Evaluate the following expressions, if $a^2b^8c = 0.3$

a)
$$\frac{a^2b^8c}{0.01}$$

b) $-10(ca^2b^8)^2$

c) $c(ab^4)^2$

d)
$$\frac{a^2b^8}{2} \cdot (-c)$$

1. Express – P in terms of m, when the following is true. Write your final answer without parentheses:

a) P = -m

b) $P = m^2 - m$

2. Express A^3 in terms of x, when the following is true. Write your answer without parentheses. Simplify:

a) A = 2x

b) A = -x

c)
$$A = x^7$$

d) $A = -x^{3}$

Homework/Class Work/Quiz

2. Write the expression $a^2 + 4a$ in terms of x , if a = x - 2 . Remove parentheses and simplify:

3. Express the following expression in terms of *a* and *b*. Remove parentheses and simplify:

$$\frac{x-y}{x+y} \qquad \text{if} \quad x=a+b, \quad y=a-b$$

4. Express the following expression in terms of x. Simplify the new expression:

$$\frac{x-y}{x+y} \qquad \text{if} \quad y=x$$

5. Express the following expression in terms of *s*. Simplify the new expression:

$$\frac{x-y}{x+y} \qquad \qquad \text{if} \quad y=x=-s^2$$

1. Let P = -x + 2, Q = 4x - 1. Express the following in terms of x. Write your final answer without parentheses, in a simplified form,:

a) P + Q

b) P-Q

c) PQ

d) $P^2 + Q^2$

2. Simplify the expression $\frac{-xy}{2}$ if $y = 2x^2$

1. Express $x^2 - xy + y^2$ in terms of v, if the following is true. Simplify the new expression.

a) x = y = v

b)
$$x = \frac{v}{2}, y = 4v - 2$$

2. Let $P = -x^3$ and $R = 2x^2$. Express the following expressions in terms of x. Simplify. a) PR^2

b) $\frac{P}{R}$

c) $-(PR)^3$

- 1. Rewrite the expression $a^2 b^2$ in terms of
- a) x, if a = 5x and b = 1. Write the new expression without parentheses. Simplify.
- b) x, if a = 5x 2 and b = -x. Write the new expression without parentheses. Simplify.
- c) a, if a = b. Simplify.
- d) *a*, if a = -b. Simplify.

- 2. Express the expression $(mn)^2 mn^2$ in terms of
- a) s, if m = s, n = -2s. Simplify.

b) *m*, if n = m. Simplify.

1. Express $4A^2 - AB$ in terms of x and y, if the following is true. Write your answer without parentheses. Simplify.

a) A = x, B = -y

b) $A = 2x, B = 3y^2$

c)
$$A = \frac{x}{2}, B = 2y - 4x$$

2. Write the expression (a-b)(a+b) in terms of x, if the following is true. Simplify your answer:

a) a = 5x and b = -x

b) a-b=5x and a+b=-x

1. Let A = -x + 2, B = 4x and $C = \frac{x}{3} - 2$. Express the following in terms of x. Write the expressions without parentheses. Simplify:

a) A+B-C

b) -AB-C

2. Express $\frac{3x+9y}{3}$ in terms of *Y*, if x+3y = Y. Give your answer in a simplified form.

3. Knowing that x + y = t, express the following in terms of t. Simplify your answer:

a)
$$\frac{x}{3} + y + \frac{y}{3} + x$$

b)
$$\frac{x+2+y}{x+y}$$

- 1. Rewrite the expression a^2b^3 in terms of
- a) x, if $a = x^3$, $b = x^4$. Write the new expression without parentheses. Simplify.
- b) x, if $a = x 2x^2$, b = -x. Write the new expression without parentheses. Simplify.
- c) a, if a = b. Simplify.

- d) *b*, if $b = -a^2$. Simplify.
- 2. Express the following expression in terms of m, if a = m 4. Simplify your answer:
- a) (a-3)(2a+6)

b) $2a - 3a^2$

- 1. If a + 3b = m, express the following expressions in terms of m. Simplify, if possible.
- a) $7^a 7^b 7^b 7^b$

b)
$$\frac{1}{7}a + \frac{3}{7}b$$

c) -3b-a

d)
$$(a+2+3b)^{a+3b}$$

e)
$$\frac{a}{3b+a} + \frac{3b+1}{3b+a}$$

$$-8x + \frac{3}{2}x + 4x + \frac{5}{2}x = 0$$

Evaluate the above expression, if

- a) x = -1
- b) $x = -\frac{45}{57}$
- c) x = a
- d) $x = (a + 2b 3c)^{56}$
- 2. Show that if u = 3x z then $\frac{z 3x}{u} = -1$.

3. Evaluate $\frac{m-n+k}{k+m-n}$, if m = 0.3, n = 0.7, and k = -0.8 (Hint: try to first simplify the expression).

1. The following is true: $\left(\frac{1}{a} + \frac{1}{b}\right)\frac{ab}{a+b} = 1$ (you are not expected to check it).

a) What is the value of
$$\left(\frac{1}{x} + \frac{1}{y}\right)\frac{xy}{x+y}$$
?

b) What is the value of
$$\left(\frac{1}{3} + \frac{1}{5}\right)\frac{3\cdot 5}{3+5}$$
?

c) Evaluate
$$\left(\frac{1}{a} + \frac{1}{b}\right) \frac{ab}{a+b}$$
, if $a = \frac{2}{3}$ and $b = -\frac{5}{7}$.

d) Evaluate
$$\left(\frac{1}{a} + \frac{1}{b}\right)\frac{ab}{a+b}$$
, if $a = 2x-1$ and $b = 3x+2$.

e) Evaluate
$$-2\left(\frac{1}{a}+\frac{1}{b}\right)\frac{ab}{a+b}$$
.

Lesson 8

Topics:

Generalities on equations; Solving linear equations in one unknown.

This lesson introduces an important algebraic concept: equations.

Equations:

EquationA mathematical statement consisting of two expressions
separated by the equal sign is called an equation.

The following are examples of equations:

2+3=5, 3x=5, x+3=y.

We will refer to expressions on the left of the equal sign as **the left-hand side of the equation** and to the expressions on the right of the equal sign side as **the right-hand side of the equation**. In x+3 = y, x+3 is the left-hand side of the equation, and y is the right-hand side of the equation. We can always reverse sides of the equation. For instance, instead of x+3 = y, we can write y = x+3. Both statements have exactly the same meaning.

The difference between equations and algebraic expressions:

Notice the difference between an equation and algebraic expression. Equations are two algebraic expressions separated by the equal sign. There is always the left-hand side and the right-hand side of each equation. Algebraic expressions are different from equations. For example: x - y = 3x + 2 is an equation, but x - y, 3x + 2 are simply algebraic expressions.

A solution, the solution set, and what it means to solve an equation:

Equations are often used to state the equality of two expressions containing one or more variables (often called **unknowns**).

The solution setThe value of the variable(s) for which the equation is true
(that is, a value for which the left hand side of the
equation is equal to the right hand side) is called a
solution. The solution set of the equation is the set of all
solutions of the equation.

For example, x = 7 is a solution of the equation x + 2 = 9 (when we replace x with 7 the righthand side is equal to the left-hand side of the equation: 7 + 2 = 9). Later on, we will learn that x = 7 is the only solution of this equation, and thus, since there are no other solutions, x = 7 is also **the solution set** of x + 2 = 9. But, there are equations that have more than one solution. Consider the equation x + y = 8. Notice that since there are two variables in this equation, x and y, each solution will consist of two numbers: the value of x and the value of y. One can check that x = 3, y = 5 is a solution of x + y = 8 (substitute 3 for x and 5 for y in the equation to see that these values make the equation true: 3 + 5 = 8). It is not the solution set, because this is not the *only* solution of this equation, for example x = 4, y = 4 is another one. The solution set of an equation consist of all solutions.

To Solve an EquationTo solve an equation is to find the set of all solutions of
the equation or prove that it does not have a solution.

Operations that can be performed on any equation without changing the set of solutions:

The following operations can be done to any equation without changing its solution:

Any quantity can be added to,	or subtracted	from bot	h sides	of an	equation
For example:		If $x = y$	then :	x + 2 =	= y + 2
		If $x = y$	then :	x - 2 =	= y - 2

Both sides of an equation can be multiplied,	or divided by any nonzero quantity.
For example:	If $x = y$ then $x \times 2 = y \times 2$
	If $x = y$ then $\frac{x}{2} = \frac{y}{2}$

It is important to remember, that any time we perform any operation on one side of an equation, exactly the same operation must be performed on the other side. We always perform the same operation **on both sides.**

Solving linear equations in one variable:

There are a lot of different types of equations, and depending on the type of equation, different solving techniques are involved. We will learn how to solve linear equations in one variable (the formal definition of a linear equation will be provided later).

To solve a linear equation we perform the operations of adding, subtracting, multiplying or dividing both sides of an equation by suitable quantities with the goal of isolating the variable (often x) on one side, and 'bringing all numbers to the other side' of the equation. The numerical value obtained on the other side is the solution.

We will learn how to implement the above idea by solving several equations:

Solve the following equation: x + 2 = 7.

We need to isolate x, and to this end, we must somehow 'get rid of 2' on the left-hand side of the equation. 2 is added to x, so if we subtract 2 (opposite operation to addition), we will get zero, and 2 will no longer be on the left-hand side. But if we subtract 2 from the left-hand side of the equation, the same operation must be performed on the right-side. Thus we need to subtract 2 from **both sides**. We could say that we are 'bringing 2 to the other side'

x+2=7 x+2-2=7-2 x=5The solution of x+2=7 is x=5.

• Solve the following equation: 3x = 6

3x = 6

 $\frac{3x}{3} = \frac{6}{3}$

To isolate x, 'bring 3 to the other side'. 3 is multiplied by x, thus we need to divide (opposite operation to multiplication) **both sides** by 3.

Simplify.

Simplify.

Subtract 2 from both sides.

x = 2The solution of 3x = 6 is x = 2

• Solve the following equation: $\frac{x}{4} = 9$ To isolate x multiply **both sides** by 4 (x is divided by 4; the opposite operation to multiplication is division). $4 \cdot \frac{x}{4} = 9 \cdot 4$ Simplify. x = 36The solution of $\frac{x}{4} = 9$ is x = 36 .

• Solve the following equation: 3x + 2 = 5

The goal is to isolate x on one side by 'undoing' the operations that were performed on x. Unknown x was multiplied by 3, and then 2 was added. To 'undo' that, first subtract 2, and then divide by 3:

3x + 2 = 5	Subtract 2 from both sides of the equation.
3x + 2 - 2 = 5 - 2	Simplify.
3x = 3	Divide both sides by 3.
$\frac{3x}{3} = \frac{3}{3}$	Simplify.
x = 1	
n = 1 = 1 = 1	

The solution of 3x + 2 = 5 is x = 1

• Solve the following equation: 2x - 9 = 3 - 4x

2x-9=3-4x If x's appear on both sides, 'bring all x's on one side'. 4x is subtracted, thus we need to add (opposite operation) 4x to **both sides**.

2x - 9 + 4x = 3 - 4x + 4x	Collect like terms (on both sides separately!)
6x - 9 = 3	Now, the equation is in the form we already know
	how to solve. We will continue solving it using
	methods introduced earlier: To isolate x , we first
	'bring 9 to the other side' by adding it to both sides .
6x - 9 + 9 = 3 + 9	Simplify.
6x = 12	Divide both sides by 6.
$\frac{6x}{6} = \frac{12}{6}$	Simplify.
x = 2	
The solution of $2x - 9 = 3 - 4x$ is $x = 2$.	

• Solve the following equation: 3(x-2) = 9.

3(x-2) = 9	If parentheses are involved on one or both sides of
	the equation, we first remove parentheses by
	applying the Distributive Law.
3x - 6 = 9	The equation is in the form we already know how to
	solve: add 6 to both sides . The operation of adding
	6 to -6 can easily be performed mentally, without
	recording it and this is how we will be doing it from
	now on. You are encouraged to do the same.
3x = 9 + 6	Divide each side by 3.
$x = \frac{15}{3}$	Simplify.
x = 5	
The solution of $3(x-2) = 9$ is $x = 5$.	

Checking solution of a linear equations:

Notice that it is possible **to check your solution:** Replace the variable in the original equation with the value of the solution and verify that the left-hand side of the equation is equal to the right-hand side. For example,

To check that x = 5 is indeed a solution of the equation 3(x-2) = 9, substitute 5 for x in the equation:

$$3(5-2) = 9$$
$$3 \times 3 = 9$$
$$9 = 9$$

Since the left hand side is equal to the right hand side of the equation, 5 is indeed a solution.

Linear equations with no solution or solution consisting of all real numbers:

All of the above equations had **exactly one solution**. However, there are some linear equations that have no solutions. The solution set of other equations may consist of all real numbers. We

will see below an example of an equation that has **no solution** followed by an example whose solution consists of **all real numbers**.

• Solve the following equation: 2x - 1 = 2x + 5.

2x-1=2x+5 Subtract 2x from both sides. -1=5

As we know, adding 2x to both sides produces an equation with exactly the same solution set as the original one. But -1=5 is a false statement. No matter what the value of x is, it is never true. In other words, there is no value of x that would satisfy it. We conclude: The equation 2x-1=2x+5 does not have a solution, and we must write 'no solution' as the answer.

• Solve the following equation: 2(x-3) = 2x-6.

2(x-3) = 2x - 6	Remove parentheses.
2x - 6 = 2x - 6	Subtract $2x$ from both sides.
-6 = -6	

The statement -6 = -6 is true. It is true regardless of the value of x. It is true if x = 1, x = 2, or x = 1000. It is true for any value of x. All real numbers make it true. Therefore:

The solution of 2(x-3) = 2x-6 is all real numbers. We must write 'All Real Numbers' as the answer.

In general, when in the process of solving an equation the variable is cancelled (disappears from the equation), then the equation either has no solution or its solution is all real numbers:

If an equation reduces to a false numerical statement, the original equation does not have a solution.

If an equation reduces to a true numerical statement, the original equation has the solution consisting of all real numbers.

Examples and Problems with Solutions

<u>Example 9.1</u> Determine whether the following mathematical sentences represent an equation or an algebraic expression. Any time you find an equation, circle its left hand side.

a)
$$4x-2$$

Solution:
 $(x^2 + 3y^2 = 4 - x)$

Only b) and c) are equations. The left hand sides: 4x-2=7, $x^2+3y^2=4-x$

Example 9.2 Determine which of the following numbers are solutions of the equation: $x^3 = 6 + x$:

a) x = 2 b) x = -2

Solution:

a) Evaluate the left-hand side and the right hand side of the equation when x = 2:

 $x^3 = 2^3 = 8$, 6 + x = 6 + 2 = 8. Since both sides of the equations are equal, 2 is a solution. b) Evaluate the left-hand side and the right hand side of the equation when x = -2:

 $x^3 = (-2)^3 = -8, 6 + x = 6 + (-2) = 6 - 2 = 4$. Since the left-hand side is not equal to the right-hand side, -2 is not a solution.

Example 9.3 Determine if x = 4, y = 3, z = -5 is a solution of the equation: 2 = x + y + z. Solution:

Replace x with 4, y with 3, and z with -5 in the equation, evaluate, and determine if the left hand side is equal to the right hand side: x + y + z = 4 + 3 + (-5) = 4 + 3 - 5 = 2. Since both sides are equal, x = 4, y = 3, z = -5 is a solution.

Example 9.4 Solve the following equation: 3 = -2x - 5, and check your solution. Solution:

3 = -2x - 5	Add 5 to both sides (x was multiplied by -2 , and then 5
	was subtracted, so to 'undo' that, we first add 5)
8 = -2x	Divide each side by -2 (divide, since x is <i>multiplied</i> by
	-2, there is no subtraction!). Simplify.
-4 = x	

The solution of 3 = -2x - 5 is x = -4.

To check the answer we replace x with -4 in the original equation: 3 = -2(-4) - 5, evaluate both sides: 3 = 3, and conclude that, indeed, -4 is a solution.

Example 9.5 Solve the following equations: b) 4x - 5 = 15 + xa) 98 = -xc) 4x = 4(x-1)Solution: -x = 98To eliminate the minus sign, multiply (or equivalently a) divide) both sides by -1 (recall that $-x = -1 \cdot x$). -1(-x) = -1(98)Perform the indicated operations x = -98The solution of -x = 98 is x = -98. 4x - 5 = 15 + xb) 'Bring all x's on one side' by subtracting x from both sides (subtracting 4x from both sides would also be correct). 4x - x - 5 = 15Collect like terms. 3x - 5 = 15Add 5 to both sides 3x = 15 + 5Perform the operation of addition. 3x = 20Divide both sides by 3. $x = \frac{20}{2}$ The solution of 4x-5=15+x is $x=\frac{20}{3}$. 4x = 4(x-1)Remove parentheses using the Distributive Law c) 4x = 4x - 4Subtract 4x from both sides. 0 = -4

Since the statement 0 = -4 is false, the equation 4x = 4(x-1) has no solutions.

Common mistakes and misconceptions

Mistake 8.1

Algebraic expressions cannot be 'solved'. To solve means to find all values of variables for which the left-hand side of the equation is equal to the right-hand side. So, for example, it would be meaningless to use the phrase 'Solve 3x + 2'.

Mistake 8.2

While solving an equation, DO NOT display the work as a sequence of equations: 3x + 2 = 6 = 3x = 4... does not have assigned meaning. Each equation must always have the left-hand side and the right-side. Any time you perform an operation rewrite the whole equation.

Mistake 8.3

When solving 6x + 9 = 5, first subtract 9, NOT divide by 6.

Mistake 8.4

If in the equation 5x-5 = 4x you decide to subtract 4x from both sides, remember that on the right hand-side you get 0, and you must record that zero. (right-hand side DOES NOT 'disappear'): 5x-5-4x = 0

Mistake 8.5

DO NOT write 'no solution' when you get x = 0.0 is the solution.

Mistake 8.6

When asked to solve an equation and after all operations you get 3 = -5, you need to write the final answer: *there is no solution* (similarly, write 'all real numbers are solutions', when you get 3 = 3).

Exercises with Answers (For answers see Appendix A)

Ex.1 Fill in the blanks using the following words: 'equation', 'algebraic expression', 'solution' as appropriate:

One can solve a(n) ______ but not a(n) ______. If the left hand side of an equation is equal to the right hand side of the equation for x = 7, then 7 is called a ______. The ______(s)_ of an equation are all values of variables that make the equation true. The statement that contains two quantities separated by an equal sign is called a(n) ______. A(n) ______ always makes the ______ true.

Ex.2 Determine whether the following mathematical sentences represent an equation or an algebraic expression. In the case of an equation, circle the right-hand side of the equation.

a)
$$5x$$
 b) $5x = 2$ c) $x^2 = 36$ d) $\frac{x-1}{2} + x$ e) $x = -4 + 2x$
Ex.3 Tom found the solution of an equation to be x = 3, but the teacher gave as a correct solution 3 = x. Is Tom's answer right? Mary's answer to the same question is -x = -3. Did Mary correctly solve the equation? Tell why or why not for both Tom and Mary?

Ex.4 Does x = 7 make the statement 2(x+1) - x = 7 true or false? Does that mean that 7 is, or is not a solution of 2(x+1) - x = 7?

Ex.5 Determine if any of the following numbers $-2, 16, \frac{1}{2}, 2$ is a solution of the equation: $-x^4 = 16$. How about the equation $x^4 = 16$?

Ex.6 Is x = -1 a solution of a) (-x)(-x) = 2 b) -x - x = 2

- *Ex.*7 Is x = 2 a solution of a) $6^x = 36$ b) $-6^x = 36$ c) $(-6)^x = 36$
- *Ex.8* Does $a = \frac{1}{2}$, b = -2 make the following statements true or false? a) ab = 1b) ab = -1c) $\frac{a}{b} = -\frac{1}{4}$ d) $a + b = -\frac{3}{2}$

Ex.9 Determine if x = 1, y = 2 is a solution of any of the following equations:

a)
$$3x - 2y = 1$$

b) $-x - y = 3$
c) $\frac{-y}{x} = -2$

Ex.10 Is x = -1, y = 0 a solution of

a)
$$2xy = -2$$
 b) $2xy = 0$ c) $y - x = 1$ d) $\frac{x}{y} = 0$

Ex.11 Is
$$x = \frac{1}{2}$$
, $y = 2$ a solution of
a) $\frac{x}{y} = 4$ b) $\frac{y}{x} = 4$ c) $y - x = \frac{3}{2}$ d) $xy = y - 2x$

Ex.12 Check each of the following to determine whether or not it is a solution of the equation: x + 2y = z - x?

a) x = -2, y = -3, z = -10 b) x = -3, y = 5, z = 4

Ex.13 Check each of the following to determine whether or not it is a solution of the equation: xyz = 1?

a)
$$x = \frac{1}{3}, y = \frac{2}{11}, z = \frac{33}{2}$$

b) $x = 0.1, y = -0.01, z = -1000$

Ex.14 Find a number that is b) not a solution of -x = 5-x = 5

Ex.15 Guess three solutions of the following equation (remember that each solution will consist of two values: value of x and value of y): xy = 1

Ex.16 Can you guess a solution of x + y = 5 (remember that each solution will consist of two values: value of x and value of y)? Can you find a value for x and y that would not be a solution of x + y = 5?

Ex.17 Mr. X tried to solve the following equation : x = 2x by dividing each side by x. As a result, he obtained the equation: 1 = 2 and concluded that the equation x = 2x is a contradiction. The correct solution of this equation is x = 0. What did Mr. X do wrong?

Ex.18 Solve the following linear equations and check your solution:

a) x + 4 = 1 b) 3x = 18 c) -7 + x = -2 d) $-5 = \frac{x}{4}$

Ex.19 Solve the following equations and check your solution:

a) $-x - 13 = 5$	b) $15 + 2x = 7$	c) $-4x = x + 12$	d) $6x = 15x$
e) $3x = 5 - 4x$	f) $4(x-3) = 12$	g) $0 = 2(1+x)$	h) - (x+3) = 2x

Ex.20 Determine which of the following equations has no solution, exactly one solution, or solution consisting of all real numbers:

a) x-8 = x-9b) -x = 0c) x-x = 0d) 4(x+2) = 2+4xe) $3x+1 = 3(x+\frac{1}{3})$

Ex.21 Solve the following equations:

a) $6x = -12$	b) $\frac{a}{4} = -15$	c) $-x = -7$	d) $-x = 0$
e) $\frac{1}{6}a + 4 = 4$	f) $3x - 7 = 8$	g) $0.8y - 0.1 = 1.5$	h) $22 - 5y = y$
i) $-x-4x = -2x$ m) $3(2B+5) = 16$	j) $x-5=3x+7$ n) $-3x=12-$	k) $4x + 3 = 1 - x$ 7x o) -3	1) $2.3a - 5 = 1.8a$ (x - 2) = 6 - 3x
p) $x - 7 = -3(x + 5)$ s) $(x + 2) - 2x + 2(1)$	q) $3-7x = 2-$	-4x r) 3(2)	(m-4) = 6m + 6
(x+3) = 2x + 3(1)	$(-x)$ $(-x) = 2(4x - 1)^{-1}$	f(x) = 4 - x u) 4(y)	y = 2) = -(1 + y)

Homework/Class Work/Quiz

1. Determine whether the following examples represent an equation or an algebraic expression. In the case of an equation, circle the right-hand side of the equation:

a) 4x - 2 + 5y

b) $x^3 - 2x$

c) 4x - 2 = 5y - 9

2. Is x = -1 a solution of the following equations? Please, show how you arrived at your answer:

a) $-2x^2 = -2$

b) $-(2x)^2 = -4$

c) $(-2x)^2 = -4$

1. Determine whether the following examples represent an equation or an algebraic expression. In the case of an equation, circle the left-hand side of the equation:

a) 3x - 7y = 5

b) $x^2 - 2y^2 = 0$

c) -4x + 5 - y + z

d) x = y

2. Which of the following values of x is a solution of the following equation: $x^2 = -x$. Please, explain how you arrived at your answer:

a) -1

b) 1

c)
$$-\frac{1}{2}$$

d) 0

- 1. Find a number that is a solution of the following equation:
- a) $\frac{1}{x} = \frac{1}{2}$
- b) $\frac{1}{x} = -\frac{1}{2}$
- 2. Which of the following statements are true? Why?

a)
$$x = \frac{2}{3}$$
 is a solution of $\frac{3x}{2} = 1$

b)
$$x = 0$$
 is a solution of $\frac{x}{3} = 0$

c)
$$x = 0$$
 is a solution of $\frac{3}{x} = 0$

d) n = 8 is a solution of $2^n = 16$

e) x = -1 is a solution of $-1 = x^{15}$

1. Determine which of the following numbers are solutions of -x = x. Show how you arrived at your answer: a) x = 1

b) x = -1

c) x = 0

2. Is x = 0, y = 0.1 a solution of the following equations? Show how you arrived at your answer:

a) 10y = x

b)
$$\frac{y}{0.1} = x + 1$$

c) 3xy = 0.3

Homework/Class Work/Quiz

- 1. Which of the following statements are true? Why?
- a) x = 2, y = -3 is a solution of -4x + y = 7x + y

b) x = 5, y = -4 is a solution of $x^2 + y^2 = 9$

c)
$$x = \frac{1}{3}$$
, $y = -\frac{1}{3}$ is a solution of $x + y = -\frac{x}{y}$

d)
$$x = 2.2$$
, $y = -0.11$ is a solution of $\frac{x}{y} = 2 - 10x$

1. Find a number that is

a) a solution of $3^x = 9$

b) not a solution of $3^x = 9$

2. Is $x = \frac{-1}{2}$, y = 4, $z = \frac{3}{4}$ a solution of the following equations? Show how you arrived at your answer:

a) $xyz = -\frac{3}{2}$

b) 4z - 2x = y

c)
$$\frac{x}{y} = -\frac{3}{z}$$

- 1. Solve and check your answer:
- a) -2a = 5

b) x + 5 = -1

c) -2 = 7 + y

d) -0.5a = 5

e)
$$a + \frac{1}{2} = -\frac{1}{2}$$

8H

- 1. Solve the following equations. Check your answer.
- a) 4x = -8

b) 3+b=-2

c) -0.9 + x = 8

d)
$$-2 = -x$$

e)
$$\frac{2a}{3} = 4$$

1. Solve the following equations:

a)
$$-3 = \frac{x}{4}$$

b)
$$\frac{2}{3} = -3x$$

c)
$$-0.7x = 0.07$$

d)
$$y + 7 = -0.2$$

e)
$$-1 = \frac{x}{-2}$$

a)
$$-\frac{z}{4}=5$$

b) -3 = -6w

c) -7x = 0

d) 0 = 1 - x

e)
$$-x = 0$$

1. Solve the following equations and check your answer:

a) 2x - 2 = 4

b)
$$-\frac{x}{2} + 1 = 1$$

c) -2 = 0.1x + 5

d) -0.2x + 3 = -1

a)
$$1 = \frac{x}{5} - 2$$

b) -0.1x + 0.2 = -0.3

c) 2 = -3x + 7

d) 2 - 4x = -2

a) 2x - 5 = -x

b) 4n + 5 = -3n - 4

c) 6n - 1 = 4 + 4n

d) -2x-4 = 11+x

- 1. Solve the following equations and check your answer:
- a) 2(a-1) = 7

b) -(x+3) = 2x

c) 3x - 1 = 5(1 - x)

d) -2(a+1) = 3a-2

1. Determine which of the following equations has no solution, exactly one solution, or solution consisting of all real numbers:

a) 1 - 3x = -3x + 1

b) -x = x

c) 2x = 0

d) -(x-4) = -(4-x)

1. Determine which of the following equations has no solution, exactly one solution, or solution consisting of all real numbers:

a) 2(x-4) = -8 + 2x

b) 2(x-4) = -8

c) 2(x-4) = 2x

d) 2(3-5x) = -10x + 6

a) 6x = 3(2x - 4)

b) 1 - 4x = -2(1 + 2x)

c) x - (2 - x) = -2 + 2x

d) 3(1-x) = -3x + 2

a) -8 = 5 - A

b) -(5-x) = x

c) 3x - 4 = -5x + 2

d) 0.7y = -2 - 0.3y

e) 2(3-6x) = -3(4x-2)

Lesson 9

Topics:

Solving linear equations involving fractions; Literal expressions: solving equations for a given variable; Review for Test 3;

We will continue with solving of equations:

Solving linear equations involving fractions:

If you are to solve an equation involving fractions, you can eliminate the fractions by applying the following 'trick': **multiply both sides by a common multiple of all denominators of the equation**. This step *or 'trick'* is not a requirement; however, if you choose to omit this step, you would have to continue to work with fractions in order to solve the equation.

Solve the following equation $\frac{4x}{5} + 1 = \frac{2}{3}$.	
$\frac{4x}{5} + 1 = \frac{2}{3}$	In this equation, "15" is a common multiple of both
	denominators (which are: 5 and 3). We multiply both sides by 15:
$15 \times \left(\frac{4x}{5} + 1\right) = 15 \times \frac{2}{3}$	Remove parentheses.
$15 \times \frac{4x}{5} + 15 \times 1 = 15 \times \frac{2}{3}$	Cancel the denominators.
$3 \times 4x + 15 = 5 \times 2$	Simplify.
12x + 15 = 10	'Bring 15 to the other side' by subtracting 15 from both sides
12x = -5	Divide each side by 2.
$x = -\frac{5}{12}$	
The solution of the equation $\frac{4x}{5} + 1 = \frac{2}{3}$ is	$x = -\frac{5}{12} \cdot \blacksquare$

Solving equations for a given variable:

Often, equations express a relationship between more than one variable. For instance,

$$A = L \cdot h$$
, $P = 2L + 2W$, or $v = \frac{d}{t}$

Consider the first equation. In this equation A is expressed in terms of L and h. Suppose that we are asked to find h when the value of L and A are given, and then to repeat this calculation

for several different values of L and A. Rather then substituting the values of L and A each time in the original equation and then solving it for h, it seems to be easier to first express h in terms of L and A. Then we could more easily calculate the value of h. Expressing h in terms of the other variables is called solving an equation for h:

Solving an equation for
a given variableTo solve an equation for a given variable means to isolate
that variable on one side of the equation with all other
quantities on the other side.

The steps used in the process of solving for a given variable are exactly the same as those used in solving linear equations. Treat the specified variable as if it were the only variable in the equation and treat the other variables as if they were numbers. Isolate the specific variable by adding, subtracting, multiplying or dividing both sides by a suitable expression:

• Solve $A = L \cdot h$ for h. $A = L \cdot h$ Since h is multiplied by L, we divide both sides by L (we assume that $L \neq 0$, otherwise the operation cannot be performed). $\frac{A}{L} = \frac{L \cdot h}{L}$ Simplify. $\frac{A}{I} = h$ The equation $A = L \cdot h$ is solved for h: $h = \frac{A}{L}$. • Solve $d = \frac{d^2}{t}$ for t (assume, $t \neq 0$) $d = \frac{d^2}{t}$ Remove t from the denominator by multiplying both sides by t. $td = \frac{d^2}{t} \cdot t$ Simplify. $td = d^2$ Divide both sides by d (assume that $d \neq 0$). $\frac{td}{d} = \frac{d^2}{d}$ Simplify. t = d

The equation $d = \frac{d^2}{t}$ is solved for t: t = d.

• Solve xa = c + ya for a.

xa = c + ya	By subtracting ya from both sides, group all terms with a
	on one side.
xa - ya = c	If a variable for which we solve an equation appears in
	several terms, factor it. In this case: factor a.

a(x-y) = c Divide each side by x - y (assume $x - y \neq 0$). $a = \frac{c}{x - y}$

The equation xa = c + ya is solved for $a: a = \frac{c}{x - y}$.

Examples and Problems with Solutions

Example 9.1 Solve the following equation: $\frac{3x-1}{2} = 1 - \frac{x}{3}$. Solution:

$\frac{3x-1}{2} = 1 - \frac{x}{2}$	To get rid of fractions, multiply both sides by 6 (common
2 5	multiple of all denominators on both sides, i.e. 2 and 3).
$6 \cdot \frac{3x-1}{2} = 6 \cdot \left(1 - \frac{x}{3}\right)$	Remove parentheses applying the Distributive Law
$6 \cdot \frac{3x-1}{2} = 6 - 6 \cdot \frac{x}{3}$	Simplify.
3(3x-1) = 6 - 2x	Remove parentheses once again.
9x - 3 = 6 - 2x	Add $2x$ to both sides to 'group' all x's on one side.
9x - 3 + 2x = 6 - 2x + 2x	Collect like terms on each side separately.
11x - 3 = 6	Add 3 to both sides.
11x - 3 + 3 = 6 + 3	Perform the indicated operations.
11x = 9	Divide each side by 11 to isolate x
$x = \frac{9}{2}$	
11	
The solution of $\frac{3x-1}{2} = 1 - \frac{x}{3}$ is $x =$	$=\frac{9}{11}$.

Example 9.2 Let a = 3x - 2 and b = -x + 2. Find the value of x so that the following is true:

a)
$$a = b$$
 b) $a = \frac{b}{2}$

Solution:

a) In the equation a = b, use substitution to replace a with 3x-2, and b with -x+2. Solve the obtained equation:

3x - 2 = -x + 2	'Bring all x's on one side' by adding x to both sides.
3x + x - 2 = 2	Collect like terms.
4x - 2 = 2	Add 2 to both sides. Simplify.
4x = 4	Divide each side by 4. Simplify.
x = 1	
a a 11 a	

If a = 3x - 2 and b = -x + 2, then a = b when x = 1.

b) In the equation $a = \frac{b}{2}$, substitute *a* with 3x - 2, and *b* with -x + 2. Solve the obtained equation:

$$3x-2 = \frac{-x+2}{2}$$

$$2(3x-2) = 2 \cdot \frac{-x+2}{2}$$

$$6x-4 = -x+2$$

$$7x-4 = 2$$

$$7x = 6$$

$$x = \frac{6}{7}$$
Multiply each side by 2.
Remove parentheses on the left-hand side of the equation.
Add x to both sides of the equation.
Add 4 to both sides. Simplify.
Divide each side by 7.
If $a = 3x-2$ and $b = -x+2$, then $a = \frac{b}{2}$ when $x = \frac{6}{7}$.

Example 9.3 Solve the equation $\frac{A-B^2}{B} = B$ for A, and then find the value of A, if B = 3. Solution:

$$\frac{A-B^2}{B} = B$$
Multiply each side by B.

$$B \cdot \frac{A-B^2}{B} = BB$$
Simplify.

$$A-B^2 = B^2$$
Add B^2 to both sides.

$$A = 2B^2$$

To find the value of A, if B = 3, replace B in $A = 2B^2$ with 3 and evaluate. $A = 2 \times 3^2 = 2 \times 9 = 18$.

Example 9.4 Solve each equation for the specified variable. Always assume that the denominator (divisor) is different from zero. Simplify your final answer

a)
$$\frac{a}{b} - b^2 = 3b^2$$
 for a b) $xy = 1 - y + x$ for x

Solution:

a) $\frac{a}{b} - b^2 = 3b^2$	Add b^2 to both sides.
$\frac{a}{b} = 3b^2 + b^2$	Collect like terms.
$\frac{a}{b} = 4b^2$	Multiply each side by b .
$b \cdot \frac{a}{b} = 4b^2b$	Simplify.
$a = 4b^3$	
b) $xy = 1 - y + x$	Bring all terms with <i>x</i> 's on one side.
xy - x = 1 - y	Factor x.
x(y-1) = 1 - y	Divide each side by $y-1$

$$x = \frac{1 - y}{y - 1}$$

Factor -1 from the numerator (or you could choose
to factor -1 from the denominator).
$$x = \frac{-1(y - 1)}{y - 1}$$

Cancel y-1.

Exercises with Answers (For answers see Appendix A)

When needed, assume that all denominators (divisors) are not equal to zero:

Ex.1. Solve the following equations:

a)
$$\frac{x-2}{4} = -5$$

b) $-1 = 8x - \frac{x}{2}$
c) $-3x - \frac{x}{5} - 1 = 0$
d) $\frac{x-1}{3} = 1 - \frac{x}{2}$
e) $\frac{1}{5}y - \frac{1}{5} = \frac{3}{10}y$
f) $\frac{x}{2} + \frac{2}{3} = \frac{3}{4}$

Ex.2. Solve the following equations:

a)
$$8x - 2 = 3x + 5(x + 2)$$

b) $-x + (2 - x) = 5x - 12x$
c) $1 - \frac{2}{3}a = \frac{5}{6}$
d) $\frac{3x}{2} - \frac{3}{4} = -1$
e) $4(3x - 1) = 12x - 2$
f) $4x - 2 = 3x - (1 - 5x)$
g) $2(x - 3) + 4 = x - 2$
h) $-\frac{3x}{2} = -1 + \frac{3x}{5}$
i) $3(4 - x) = -2(x - 6) - x$
j) $\frac{5}{8}x - \frac{7}{12} = x - \frac{3}{4}$
k) $3(4 - 7x) = 1 - 5(3 + x)$
l) $-(x + 2) + 2(x - 1) = 3x + 5$

Ex.3 Let A = 2x, B = -x. Find x, so the following are true (Hint: Substitute algebraic expressions for A and B. Then solve for x):

a)
$$A + B = 0$$
 b) $A - 3B = 1$ c) $\frac{A}{3} = \frac{B}{8}$

Ex.4 Let P = 3(x-1), Q = 4x+5. Find x such that the following is true: (Hint: Substitute algebraic expressions for P and Q. Then solve for x)

a)
$$P = Q$$
 b) $P = -Q$ c) $\frac{Q}{2} = P$

Ex.5 Let x = -3a + 2, y = 2a + 1, z = 2 - a. Find *a*, so the following are true: (Hint: Substitute algebraic expressions for *x*, *y*, and *z*. Then solve for *a*)

a)
$$x + y = z$$
 b) $2x = y + z$ c) $\frac{x}{8} + \frac{y}{2} = \frac{z}{4}$

Ex.6	Does the following sta	atement make sense: "Solve	$\frac{x+1}{2} - 5y$ for x? ". Why or why not?
<i>Ex</i> .7	Solve for <i>x</i> :	a) $\frac{x}{2} = 7$	b) $\frac{x}{a} = b$
Ex.8	Solve for <i>x</i> :	a) $x + 3 = 8$	b) $x + a = b$
Ex.9	Solve for <i>x</i> :	a) $3x - 7 = 11$	b) $ax - b = c$
Ex.10	Solve for <i>x</i> :	a) $5x - 2x - 6 = 0$	b) $ax - bx - c = 0$
Ex.11	Solve for <i>x</i> :	a) $4x = 2(x+1)$	b) $ax = b(x+c)$
<i>Ex.</i> 12.	Solve for the indicated	l variable. Simplify your answ	ver whenever possible $a \qquad b \qquad c_{a}$
	a) $-x = a$ for x	b) $-=ac$ for b	c) $\frac{1}{b^3} = \frac{1}{c}$ for a

d) $ax + b = 4b$ for a	e) $\frac{a^2}{u} = a$ for u	f) $abc^2 = (ac)^3$ for b
g) $\frac{2m}{n^2} = 4n$ for m	h) $2x + y = t + 3x$ for y	i) $x^3 + xy = 2x^3$ for y
j) $AX - A = 1 - X$ for A	k) $s(x-1) = s^2$ for x	1) $ax - by = 3ax + 4by$ for a
m) $3(v-t) = s$ for v	n) $ax + x(a+2) = 1$ for x	o) $\frac{m-2n}{x} = 2na - ma$ for x

Ex.13 Solve the following equation
$$\frac{x}{y} = d + e$$
 for a) d b) x c) y

Ex.14 Solve the following equation 3at + b = 2at + t for a) b b) a c) t

Ex.15. Solve $\frac{x}{a^3} = a^6$ for x, and then evaluate, if a = -1.

Ex.16. Solve $3mn = mn - 2m^2$ for *n*, and then evaluate, if m = -4.

Ex.17. If *b* represents the base of a triangle, *h* its height, and *A* is the area of the triangle, then the following is true: $A = \frac{bh}{2}$

a) Solve the above formula for h

b) Find the height of a triangle with base b = 2 inches and the area A = 4 square inches *Ex.18.* If *L* represents the length of a rectangle and *W* its width, then the perimeter *P* of the rectangle is given by the formula: P = 2(L+W)

a) Solve the above formula for W.

b) Find the value of W when P = 4, and L = 1.

c) Find the width of a rectangle with perimeter 10 inches and length equal to 3 inches.

a)
$$\frac{v+1}{4} = \frac{2v}{5}$$

b)
$$\frac{x-4}{7} + \frac{x}{2} = -2$$

c)
$$\frac{x}{2} - 1 = \frac{3x}{8}$$

a)
$$-\frac{x+2}{10} = \frac{2x-1}{5}$$

b)
$$\frac{A+5}{4} - 1 = \frac{2A}{3}$$

c)
$$\frac{x+1}{2} - \frac{5}{6} = \frac{2-x}{3}$$

a)
$$-3x + \frac{x}{2} - 1 = 4x - \frac{1}{3}$$

b)
$$\frac{x}{2} - \frac{2x}{3} = -\frac{x+1}{6}$$

c)
$$\frac{3x-4}{9} + 1 = \frac{1+2x}{6}$$

a)
$$-(x+4) = 5 - 2(1-3x)$$

b)
$$\frac{3x-1}{9} + x = \frac{3x+1}{3} + 4$$

c)
$$3(2x-10) = 5(x-6) + x$$

a)
$$\frac{-A}{3} + 2A = \frac{2A+5}{6} - \frac{1}{12}$$

b)
$$1.1 - 0.9(2x - 1) = 0.2(x - 10)$$

c)
$$-A - (3A + 2) = 4(1 - A) - 5$$

1. Let P = 2x + 3 and Q = -x. Find x so that the following is true:

a) P = 0

b) P = Q

c) P - Q = 3

d) P = 2Q

1. Let $P = \frac{1-x}{3}$, and $Q = \frac{2+x}{4}$. Find x such that the following is true: a) $Q = \frac{1}{2}$

b) 6P - 4Q = 0

c) P = Q

1. Let a = 3x - 1, b = -x, and c = 4x - 2. Find x such that the following is true:

a) a-b=c

b)
$$a + b = \frac{c}{2}$$

c)
$$\frac{a}{2} + \frac{b}{5} + \frac{c}{10} = 0$$

Homework/Class Work/Quiz

1. Solve the following equations for x:

a)
$$3x = 5$$
 b) $ax = m$

2. Solve the following equations for x:

a) x - 4 = 6 b) x - a = m

3. Solve the following equations for x:

a)
$$\frac{x}{2} = 7$$
 b) $\frac{x}{a} = b$
1. Solve the following equations for x:

a)
$$2 - x = 4$$
 b) $a - x = m$

2. Solve the following equations for x:

a)
$$2x - 7 = 3$$
 b) $ax - b = c$

3. Solve the following equations for x:

a)
$$\frac{x-2}{5} = 3$$
 b) $\frac{x-a}{b} = c$

1 Solve the following equations for x:

a) 3x = 21 - 4x b) ax = b - cx

2. Solve the following equations for x:

a) 2(3x+1) = x

b) a(bx+c) = x

- 1. Solve for F. Simplify your final answer, if possible.
- a) 2nF = 4k

b) $F + 3k^2 = k$

c) a - F = b

d) Fa - Fc = a

- 1. Solve the following equations for a. Simplify your final answer, if possible.
- a) ax + y = 2y

b)
$$\frac{ax}{y} = y^2$$

c)
$$\frac{c}{a} = 1 - c$$

d)
$$2ax = x + x^2$$

1. Solve the following equations for x. Simplify your final answer, if possible.

a) $a^3 x = a^4$

b)
$$\frac{x}{a} + a = c$$

c) $2(bx-b^2) = 0$

d) ax - bx = cx + 1

1. Solve A = P(1 + rt) for

a) *P*

b) *r*

2. Solve
$$D = \frac{C-S}{t}$$
 for
a) t

b) *C*

c) *S*

1. Solve for v. Simplify your answer.

a) $3sa = as - \frac{v}{a}$

b) av + b = bv + a

2. Solve 3(y-x) = -6x for y, and then find the value of y if x = -4.

3. Solve $a^2bx = a^3b^2$ for x, and then find the value of x if $a = \frac{2}{11}$, and $b = \frac{22}{3}$.

1. Solve $\frac{c^5}{x} = 2c^3$ for x, and then find the value of x if c = -2.

2. Solve
$$\frac{x^2 - x}{P} = x$$
 for *P*, and then find the value of *P* if $x = \frac{1}{2}$.

3. Solve
$$\frac{x-1}{P} = x$$
 for x, and then find the value of x if $P = \frac{1}{2}$.

1. Let F be the temperature in Fahrenheit. The same temperature expressed in Celsius C is given by the formula:

$$C = \frac{5(F-32)}{9}$$

a) Convert 41 degrees Fahrenheit to Celsius temperature.

b) Convert -4 degrees Fahrenheit to Celsius temperature.

c) Solve
$$C = \frac{5(F-32)}{9}$$
 for *F*.

d) Convert 10 degrees Celsius to Fahrenheit temperature.

e) Convert -15 degrees Celsius to Fahrenheit temperature.

Lesson 10

Topics:

Test 3; Linear Inequalities; Graphing sets of the type x < a, $x \le a$ on a number line.

We will be now discussing inequalities:

Meaning and symbolical notation for 'less (greater) than', 'less (greater) than or equal to':

Recall that

' <i>x</i> <2'	means that a number x is less than 2
' <i>x</i> >2'	means that a number x is greater than 2.

Notice, that x < 2 has exactly the same meaning as 2 > x, and x > 2 the same as 2 < x (just as 1 < 2 has exactly the same meaning as 2 > 1).

The following symbols are also in use:

' <i>x</i> ≤2'	means that a number x is less than or equal to 2
' <i>x</i> ≥2'	means that a number x is greater than or equal to 2.

Again, $x \le 2$ is equivalent to $2 \ge x$, $x \ge 2$ is equivalent to $2 \le x$.

The only difference between ' $x \le 2$ ' and 'x < 2' is that 2 satisfies the first inequality but does not satisfy the second one (2 is not less than 2). Similarly, 2 satisfies ' $x \ge 2$ ', but does not satisfy 'x > 2'.

Graphing of inequalities:

To Graph an Inequality To graph an inequality means to plot all numbers satisfying the inequality on a number line.

Graphing gives a visual representation of a given set of numbers. For example:

The graph of x < 2 consists of all numbers less than 2. These numbers are represented by points that are to the left of 2 on a number line. The number 2 does not belong to the set. We shade **all points to the left of 2 and place 'an open circle' at 2, to indicate that 2 is excluded.**

			'		•	× .	•	•	~
-4	-3	-2	-1	0	1	2	3	4	

The graph of $x \ge 2$ consists of all numbers greater than or equal to 2, represented by points to the right of 2 on a number line, including 2. The number 2 does belong to the set. We will **shade all points to the right of 2 and place 'a closed circle' at 2, to indicate that 2 is included.**



A solution, the solution set, and what it means to solve an inequality:

The concept of solution(s) of an inequality is identical to the concept used in equations:

Solution Set of an Inequality The values of the variables for which the inequality is true are called solutions. The solution set of the inequality is the set of all solutions of the inequality.

For example, x = 3 is a solution of x + 2 < 6 (when we replace x with 3, we obtain the true statement: 3 + 2 < 6). However, it is not the solution set, because there are other solutions of the inequality, for example x = 1 (1 + 2 < 6). As expected:

To Solve an InequalityTo solve an inequality is to find the set of solutions of the
inequality or prove that it does not have a solution.

Operations that can be performed on any inequality without changing its solution:

The following operations can be done to any inequality without changing its solution:

Any quantity can be added to,	or subtracted from both sides of an inequality.
For example:	If $x < y$ then $x + 2 < y + 2$
	If $x < y$ then $x - 2 < y - 2$

Both sides of an inequality can be multiplied, or divided by any **nonzero** quantity, but **if the quantity is negative the direction of the inequality symbol has to be reversed.**

For example:	If $x < y$ then	2x < 2y	but $-2x > -2y$
	If $x < y$ then	$\frac{x}{2} < \frac{y}{2}$	but $\frac{x}{-2} > \frac{y}{-2}$

We must always perform the same operation on both sides of an inequality, just as we did in our work with equations.

Solving linear inequalities in one variable:

There are lots of types of inequalities, but in this course we will focus on linear inequalities only. To solve a linear inequality, we apply the same strategy as in the case of a linear equation. That

is, we add, subtract, multiply or divide both sides of an inequality by suitable quantities with the goal of isolating the variable on one side of an inequality. However, we must remember about the following:

Any time we multiply or divide both sides of an inequality by a *negative* number, the direction of the inequality sign must be reversed.

Consider the following example (you can find more examples in section 'Examples and Problems with Solutions'):

• Solve the inequality: $-4x + 9 \ge 1$.

$-4x+9 \ge 1$	Subtract 9 from both sides.
$-4x+9-9 \ge 1-9$	Perform the indicated operations (on each
	side separately).
$-4x \ge -8$	Divide both sides by -4 and reverse the
	direction of the inequality sign.
$\frac{-4x}{-4} \le \frac{-8}{-4}$	Cancel fractions.
$x \le 2$	

The solution of the inequality $-4x + 9 \ge 1$ is $x \le 2$

This means that all numbers that are less than or equal to 2 make this inequality true. That is, if in the original inequality x is replaced by any number less than or equal to 2, the resulting numerical statement will always be true. It also means that any other number, i.e. any number that is greater than 2, when substituted for x, would give us a false numerical statement. Observe that there are infinitely many solutions and that is why, instead of listing them, we must use the appropriate newly introduced notation.

Linear inequalities with no solution or solution consisting of all real numbers:

It might happen that during the process of solving an inequality, the variable gets cancelled and the inequality reduces to a numerical statement (you can remember this occurring when solving some equations):

• Solve the following inequality: $x + 3 \le x + 5$.	
$x+3 \le x+5$	Subtract <i>x</i> from each side and obtain
$x + 3 - x \le x + 5 - x$	Simplify
$3 \leq 5$	

The statement $3 \le 5$ is always true, regardless of the value of x. Thus, the solution set of $x+3 \le x+5$ consists of all real numbers.

• Solve the following inequality: $x + 5 \le x + 3$.

 $5 \leq 3$

This time, after subtracting x from each side we obtain:

This is a false statement, false for all values of x. There is no value of x that would make this statement true. Thus $x + 3 \le x + 5$ has no solutions.

Examples and Problems with Solutions

Example 10.1 Describe the following set of numbers using inequality signs:

a) All numbers x that are positive

b) All numbers x that are non-negative

c) All numbers x that are at most equal to 9

d) All numbers x that are at least equal to 9

e) All numbers *x* that are not less than 9

Solution:

b) x > 0 (zero should not be included)

c) $x \ge 0$ (zero should be included)

d) $x \le 9$ (at most means that amount or less)

e) $x \ge 9$ (at least means that amount or higher)

f) $x \ge 9$ (not less than includes the given number and higher)

Example 10.2 Determine which of the following numbers satisfies the condition: $x \le -2$:

-4, -2.1, -2, -0.9, 0, 1

Solution:

For each number you should check if it satisfies the inequality by replacing x with this number. For example, the number -4 satisfies the inequality: $-4 \le -2$. (It might help to plot the 6 points on a number line and note which ones are either equal to -2 or to the left of -2.). The final answer is:

-4, -2.1, -2

Example 10.3 If $x \le -1$, what inequality is true for

a)
$$x-2$$
 b) $\frac{x}{-2}$

Solution:

a) One should determine what operation was performed on x to get x-2. Since 2 was subtracted from x, the same operation must be performed on the other side of inequality: $x-2 \le -1-2$, which is equivalent to $x-2 \le -3$.

b) To obtain $\frac{x}{-2}$, x was divided by -2. One has to remember about reversing an inequality sign whenever both sides of an inequality are divided by a negative number: $x \le -1$ becomes $\frac{x}{-2} \ge \frac{-1}{-2}$. After simplification, we have $\frac{x}{-2} \ge \frac{1}{2}$.

Example 10.4 Knowing $6 \le -3x$, which of the following inequalities must also be true?

a)
$$-2 \le x$$
 b) $0 \le -3x - 6$

Solution:

a) Since x appears without -3 on the right-hand side of the inequality $-2 \le x$, we conclude that the original inequality must have been divided by -3 on both sides. But any time you divide an inequality by a negative number, the inequality sign must be reversed. The resulting inequality would have been $-2 \ge x$ (not $-2 \le x$). Thus $-2 \le x$ does not have to be true.

b) Since 6 on the left-hand side of the inequality $6 \le -3x$ is 'replaced' with zero, we conclude that 6 must have been subtracted from both sides of the inequality. The resulting inequality would be $6-6 \le -3x-6$, which after simplification gives us $0 \le -3x-6$. Thus, if $6 \le -3x$ is true, $0 \le -3x-6$ must also be true.

Example 10.5 Solve the following inequalities and graph their solution:

`a) $-x \le 4$	b) $3x - 7 \le -2$
Solution:	
a) $-x \le 4$	Multiply both sides by -1 , remember about reversing the inequality sign.
$(-1)(-x) \ge (-1)(4)$	Perform the indicated operations.
$x \ge -4$	

The graph of the solution:





a) -3(2x+5) < -6x b) $\frac{4-x}{2} - \frac{x+2}{4} > 5$

Solution:

a) - 3(2x+5) < -6x	Remove parentheses.
-6x - 15 < -6x	Add $6x$ to both sides.

$$-6x - 15 + 6x < -6x + 6x$$
 Simplify.
-15 < 0

Since the obtained inequality -15 < 0 is always true, we conclude that all real numbers are solutions of the inequality -3(2x+5) < -6x.

b) $\frac{4-x}{2} - \frac{x+2}{4} > 5$	To get rid of fractions, multiply both sides by 4 (a common
2 1	multiple of both denominators).
$4 \cdot \left(\frac{4-x}{2} - \frac{x+2}{4}\right) > 5 \cdot 4$	Apply the Distributive Law to remove parentheses.
$4 \cdot \frac{4-x}{2} - 4 \cdot \frac{x+2}{4} > 5 \cdot 4$	Simplify (place the parentheses correctly)
2(4-x) - (x+2) > 20	Again apply the Distributive Law to remove parentheses.
8 - 2x - x - 2 > 20	Collect like terms on each side of the inequality separately.
-3x + 6 > 20	Subtract 6 from both sides.
-3x+6-6 > 20-6	Simplify.
-3x > 14	Divide each side by -3 and reverse the inequality sign.
$\frac{-3x}{-3} < \frac{14}{-3}$	Simplify.
$x < -\frac{14}{3}$	
solution of $\frac{4-x}{2} - \frac{x+2}{4} > 5$ is	$x < -\frac{14}{3}.$

Common mistakes and misconceptions

Mistake 12.1

The

DO NOT forget to reverse the inequality sign any time you multiply or divide both sides of an inequality by a negative number.

Mistake 12.2

When solving an inequality DO NOT change the inequality sign to an equation sign.

Mistake 12.3

DO NOT forget to write the final answer. If you are getting -3 > 5 that means there is no solution. You must write that there is 'no solution'. Likewise, if you get something like 5>3, which is true regardless of the value(s) of the variable(s), that means all numbers are solutions. You must write "all real numbers".

Exercises with Answers (For answers see Appendix A)

Ex.1 List two numbers satisfying x < 5. List two numbers satisfying a < 5. Is the second question different from the first one? How about asking: List two numbers satisfying 5 > x?

Ex.2 Name three numbers that satisfy the condition: $x \le -1$

Ex.3 Find a number that satisfies $x \ge \frac{2}{3}$ but does not satisfy $x > \frac{2}{3}$.

- *Ex.4* Circle all numbers that satisfy the following inequality x > -3-3, -2, -1, 0, 1, 2, 3, 4, 5
- *Ex.5* Determine which of the following numbers do not satisfy the inequality: x > -3-3, -2, -1, 0, 1, 2, 3, 4, 5
- *Ex.6* Determine which of the following numbers satisfies the inequality: $x \le -0.6$: $\frac{-1}{2}$, -.666, -6, -0.6 -.5999, 0
- *Ex.7* Describe the following sets of numbers using inequality signs:
 a) All negative numbers x.
 b) All non-positive numbers x.
 c) All numbers x that are at least equal to 6.
 d) All numbers x that are at most equal to 6.
 - e) All numbers x that are not more than 6
- *Ex.8* Draw a separate number line for each of the following and graph each set:

a) x < -4 b) $x \ge 1\frac{2}{3}$ c) -4 < x

- *Ex.9* Graph the following number sets on a number line:
 - a) All numbers that are at least -2
 - b) All numbers no more than 4
 - c) All non-negative numbers
 - d) All numbers that are at most -1
- *Ex.10* Using inequality symbols, describe the set that is graphed below: a)



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and also satisfies $x \le \frac{2}{3}$.

Ex.12 Plot the points in part (a) and (b) on separate number lines. Then write an inequality for each that is satisfied by all points from the set.

a) 3, 4, 6 b) -2, -1, 3

Ex.13 Plot the points in part (a) and (b) on separate number lines. Then write an inequality for each that is *not satisfied* by any of these points.

a) 0, 1, 4 b) $-\frac{1}{2}$, $1\frac{1}{2}$ 2

Ex.14 Find an inequality that is satisfied by -1 but not by 3 (if it helps, you might plot the points).

Ex.15 Find an inequality that is satisfied by $\frac{3}{4}$ but not by 5 (if it helps, you might plot the points).

Ex.16 The number -5 is a solution of which of the following inequalities. Determine your answer without solving the inequality. Show your work.

a) x+2 < 4 b) $-x+8 \ge 13$ c) -x+3 < 1 d) 3x > -15

- *Ex.17* The solution of a given inequality is: $x \ge 0$.
 - a) Is x = 2 *a* solution of this inequality?
 - b) Is x = 2 the solution of this inequality? Why?
 - c) How many solutions does this inequality have?
 - d) List three solutions of the inequality.
 - e) List three numbers that are not solutions of this inequality.
 - f) If we write the solution of the inequality as $0 \le x$, would that also be correct? Why?
 - g) If we write the solution of the inequality as 0 < x, would that also be correct? Why?
 - h) Would it be right to say that the solution consists of all positive numbers?
 - i) Would it be right to say that the solution consists of all non-negative numbers?
- *Ex.18* Determine which of the following operations requires the change of inequality sign.
 - a) Multiplying both sides of an inequality by -2.
 - b) Multiplying both sides of an inequality by 2.
 - c) Adding 2 to both sides of an inequality.

d) Subtracting $\frac{1}{2}$ from both sides of an inequality. e) Dividing both sides of an inequality by -2.

Ex.19 Name the operation that must be performed on both sides of an inequality to isolate z on one side. Determine if the operation requires the change of inequality sign (indicate it in writing), and then perform the operation, reversing the inequality sign, if needed (for example, to isolate z in the inequality z-1<3, 1 must be added to both sides, the operation of adding 1 does not require the change of sign, the resulting inequality is z < 4).

a) z + 5 < 8 b) -2 + z < 1 c) -12 < 4z d) -z > -3 e) $-\frac{z}{3} > 1$

Ex.20 If n > 3, what inequality is true for

a) n+7 b) n-5 c) -2n d) $\frac{n}{-3}$ e) $\frac{n}{0.1}$

Ex.21 If $q \le \frac{2}{5}$, what inequality is true for:

a)
$$-6+q$$
 b) $-3q$ c) $\frac{2q}{5}$ d) $-q$ e) $-\frac{4q}{5}$

*Ex.*22 Knowing x < -10, which of the following inequalities must also be true?

a)
$$x + 10 < 0$$
 b) $-2x > -20$ c) $\frac{x}{2} > -5$ d) $-\frac{3}{10}x > 3$

Ex.23 Knowing $-2x \ge 0$, which of the following inequalities must also be true? a) $x \ge 2$ b) $x \le 0$ c) $x \le -2$ d) $2x \le 0$

Ex.24	Solve the following inequalit	ties and graph their solutions:	
	a) $-2x > 8$	b) $x-1 \le x$	c) $-0.3x \le 0.6$
	d) $3 > 2a - 11$	e) $-6 - x < 4$	f) $2a - 1 \le 3$

Ex.25 Solve the following inequalities. Each time you perform the operation on both sides of inequality, name the operation together with the operand (for example write: "adding 2 to both sides", "dividing both sides by 3", and so on).

a)
$$4 > -3 - a$$

b) $3x + 1 < 6$
c) $\frac{3x}{4} < -15$
d) $-5 \ge \frac{-x}{4}$
e) $\frac{x}{4} + 1 \ge -1$
f) $\frac{x+1}{4} \ge -1$

Ex.26 Solve the following inequalities:

a) -3x > 18b) $\frac{-x}{2} > 1$ c) $2 - 3x \le 14$ d) $x \le x$ e) x < xf) -3 - a > 4ag) $4x - 1 \le 7x - 5$ h) -x + 1 > 6x - 2i) $\frac{3}{2}x > -1$ j) x - 8 > x - 9k) 2 - a > 4a - 5a - 2

l)
$$4x - 1 \le 2(x - 5)$$
 m) $-3(x + 2) \le -3x + 2$ n) $3x + 1 < 3(x + \frac{1}{3})$
o) $3(2x + 5) > 6x + 16$ p) $3(2x - 1) < 2(x - 2)$ q) $\frac{-x - 2}{4} \le -5$
r) $\frac{-y}{2} - \frac{y + 3}{3} \le -1$ s) $\frac{2a - 3}{5} - 1 > -5a + \frac{2}{3}$

1. List three numbers satisfying $x \ge 3$.

Would your answer also be correct if the directions were:

- a) List three numbers satisfying $y \ge 3$.
- b) List three numbers satisfying $z \ge 3$.
- c) List three numbers satisfying $3 \le x$.
- 2. Describe the following set of numbers using inequality symbols:
- a) All numbers x that are at least equal to 3.
- b) All numbers x that are greater than or equal to 5.
- c) All numbers x that are at most 6.
- d) All numbers *x* that are not more than 8.
- e) All numbers x that are negative.

2. List three numbers x that do not satisfy the following condition: $\frac{-2}{3} \le x$

3. Describe the following set of numbers using inequality symbols:

a) All numbers x that are less than or equal to -2.

b) All numbers x that are positive.

- c) All numbers x that are at least $\frac{1}{2}$.
- d) All numbers x that are not less than 3
- 3. Circle all numbers that satisfy the following inequality: $m \ge -\frac{2}{7}$.

$$-2$$
 -1 $-\frac{3}{7}$ $-\frac{2}{7}$ $-\frac{1}{7}$ $-\frac{1}{14}$ 0

4. Circle all numbers that satisfy the following inequality: $m > -\frac{2}{7}$.

$$-2$$
 -1 $-\frac{3}{7}$ $-\frac{2}{7}$ $-\frac{1}{7}$ $-\frac{1}{14}$ 0

- 1. Which of the following statements has the same meaning as x < -2:
- a) -2 > x
- b) $-2 \le x$
- c) x > -2
- d) $x \leq -2$ and $x \neq 2$
- e) All numbers x that is at most -2.
- f) All numbers x that is no more than -2.
- g) All numbers x that is at least -2.
- 2. Which of the following numbers satisfy the inequality: $x < \frac{1}{2}$? Circle all of them.
- $0.5 \qquad \frac{4}{5} \qquad 0.499 \qquad 0.511 \qquad \frac{2}{3} \qquad \frac{1}{2}$
- 3. Name all numbers that satisfy the inequality: $x \le -5$, but do not satisfy x < -5.



1. Using inequality symbols, describe the set that is graphed below: • 0 2 -4 -3 -2 -1 1 3 4 2. Using inequality symbols, describe the set that is graphed below: -2 0 1 2 3 -4 -3 -1 4 3. Graph the following inequalities on a number line. a) $x < -\frac{3}{2}$ ≻ Ó 1 b) $a \ge -2$ ≻ 0 1 4. Graph the following number sets on the number line. a) All numbers that are less than -3. ≻ Ó 1 b) All negative numbers. ≻

0

1

1. Describe the following sets of numbers using inequality signs, and then graph the sets on the number line:

a) All numbers at least 2.



-4, 7, 3, -1

and then for each condition below find two examples of inequality satisfying the condition:

a) An inequality that is satisfied by all above points.

b) An inequality that is not satisfied by any of above points.

c) An inequality that is satisfied by -4 and -1, but not by 3 and 7.

d) An inequality that is not satisfied by -4 and -1, but it is satisfied by 3 and 7.

e) An inequality that is satisfied by -4, -1, and 3 but not by 7.

- 1. The solution of a given inequality is: $x \le 2$
- a) Is x = 1 a solution of this inequality?
- b) Is x = 2 a solution of this inequality?
- c) Is x = 2 the solution of this inequality? Why?
- d) List four numbers that are not a solution of this inequality.

2. Without solving the inequality, check if the number 4 is a solution of $-x + 5 \le 8$. Show how you got your answer. Is the number -3 a solution?

3. Fill in blanks with either 'reverse' or 'do not reverse' words, so the resulting statement is true:

When both sides of an inequality are multiplied by 10, we ______ the inequality sign.

When we add to both sides of an inequality -2, we ______ the inequality sign.

When both sides of an inequality are divided by -3, we ______ the inequality sign.

When we subtract from both sides of an inequality -1, we ______ the inequality sign.

When both sides of an inequality are divided by $\frac{2}{7}$, we ______ the inequality sign.

1. The solution of an inequality is: $x \ge 2$ Here are the answers given by students:

Student A:x > 2Student B: $2 \le x$ Student C: $-x \le -2$ Student D: $-2 \le x$ Student E:x > 3

List all students who got the right answer.

2. Without solving the inequalities, determine if the number -3 is a solution of the following inequalities. Show your work.

a) x - 2 < -3

b) $-x+1 \ge 4$

3. The solution of an inequality is x > 100, list three numbers that satisfy this inequality, and three that do not satisfy it.

1. Name the operation that must be performed on both sides of an inequality to isolate x on one side. Determine if the operation requires reversing the direction of the inequality sign (indicate it in writing), and then perform the operation, actually reversing the inequality sign, if needed. For example, to isolate x in the inequality x-1 < 3, 1 must be added to both sides, the operation of adding 1 does not require reversing the inequality sign, and the resulting inequality is x < 4).

a) $x + 3 \le 4$

b) $-7x \ge 0$

c)
$$\frac{x}{2} > -\frac{2}{3}$$

d) 9 > -x

2. If n < 20, what inequality is true for:

a) *n*+5

b) 0.4*n*

c)
$$\frac{n}{-4}$$

1. Name the operation that must be performed on both sides of an inequality to isolate x on one side. Determine if the operation requires reversing the direction of the inequality sign (indicate it in writing), and then perform the operation, actually reversing the inequality sign, if needed. For example, to isolate x in the inequality x-1<3, 1 must be added to both sides, the operation of adding 1 does not require reversing the inequality sign, and the resulting inequality is x<4).

a) -4 > 3x

b) 9 < -2 + x

c)
$$-\frac{1}{3}x \ge 2$$

2. Knowing that n < -1, which of the following inequalities must also be true?

a) n + 1 < 0

b) 4n < -4

c) -5n < 5

1. Solve the following inequalities. Each time you perform the operation on both sides of the inequality, name the operation together with the operand (for example write: "adding 2 to both sides", "dividing both sides by 3", and so on).

a) $x+2 \ge -1$

b) 4*x* > 2

c) -2x > 5

d) $-3 \le 5 + x$

e)
$$\frac{x}{-3} \ge -1$$

1. Solve the following inequalities:

a)
$$-\frac{x}{3}-1 < 6$$

b) $-2 \ge a+5$

c) $0.6 \le -0.2x - 1$

$$d) - x + \frac{2}{3} \ge 0$$

1. Solve each of the following inequalities and graph each solution (The points on number lines are equally spaced; choose any point you wish for 0 and 1):

- | | | | | | | |

2. Solve the following inequalities:

+

a) -2x+5 < 4

b)
$$\frac{x}{3} + 5 > 1$$

-

1. Solve the following inequalities:

a) 1 - 4x < 0

b) $1 \le -0.3x - 2$

2. Solve the following inequalities and graph each solution: (The points on number lines are equally spaced; choose any point you wish for 0 and 1)

a) $x - 7 \ge -10$

b) -3x < 5

a) $-2 + x \ge -3 + 2x$

b) 2x - 4 > -1 + 5x

c) 2x - 7x > 6 - 5x

d) -(x+2) < 4x+5

a) -2(a-1) < a+4

b) 0.2 + 5x < 0.1(2 + 50)

c)
$$-3(y+1) \ge \frac{1}{2}$$

$$d) \ \frac{-x}{3} \le 1 - x$$
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1. Solve the following inequalities:

a) -2(x+2) < -2x

b) -(x+2)+3(x-2)>1

c) 5x - 4x - 8x < 7x

d) 2x - (1 - 4x) < 6x - 1

1. Solve
a)
$$\frac{2x-3}{8} - x > 2x - \frac{2}{3}$$

b)
$$1 - \frac{z}{2} \le z - \frac{5}{6}$$

c) 3-2(3a-5) < -6a

a)
$$\frac{2x}{5} - \frac{1}{2} < 4$$

b)
$$\frac{-x+2x-5x}{2} \ge 0$$

c)
$$\frac{-3a}{2} + 1 \ge 2(a-3)$$

Lesson 11

Topics:

Definition of a linear equation; Factorization of the difference of squares; Recognizing and matching patterns; Writing expressions in a prescribed way.

The definition of a linear equation:

Linear equation in one variable A linear equation in one variable x is an equation that can be written in the form:

ax+b=0,

where b is any real number, and a any real number different from zero.

The equation 2x + 3 = 0 is already in the form ax + b = 0:

$$2x + 3 = 0$$
$$ax + b = 0$$

In this representation a = 2, b = 3, and thus 2x + 3 = 0 is an example of a linear equation. To show that the equation 5x + 1 = x is also an example of a linear equation, one must be able to *rewrite it in the prescribed form* given in the definition above, namely ax + b = 0. To this end, we subtract x from both sides of the equation: 5x + 1 - x = x - x, collect like terms, and obtain the equation: 4x + 1 = 0 that matches the form: ax + b = 0:

$$4x + 1 = 0$$
$$ax + b = 0$$

We can now see that a = 4 and b = 1, and we have matched to the pattern given in the definition. Therefore, we can conclude that 5x + 1 = x is a linear equation in one variable.

Equations: $x^2 - 3 = 0$ and $4x^3 = 5$ are not linear equations. No algebraic operation would 'remove' x^2 or x^3 from the equation, and hence each of the equations can never be written to match the desired form ax + b = 0.

Factoring with the use of the difference of squares formula:

Consider the expression (a-b)(a+b). When we remove parentheses and simplify, we get:

 $(a-b)(a+b) = a^{2} + ab - ba - b^{2} = a^{2} - b^{2}$

It is called the Difference of Squares formula:

The Difference of Squares Formula

$$a^2 - b^2 = (a - b)(a + b)$$

One of the uses of the above formula is for factorization (writing as a product) of a certain type of expression. Consider the expression $x^2 - 9$. If we could *rewrite it to match the prescribed* form, namely $a^2 - b^2$, then we could use the Difference of Squares formula to factor it (i.e., write the expression as a product). Since $9 = 3^2$, we can rewrite $x^2 - 9 \operatorname{as} x^2 - 3^2$. The expression $x^2 - 3^2$ now matches the left side of the difference of squares formula:

$$\frac{x^2 - 3^2}{a^2 - b^2}$$

Note that a = x and b = 3. Once we match to the left side of the formula, we can rewrite the right side of the formula, substituting x in place of a and 3 in place of b. It is helpful to write the formula directly above the expression that needs to be factored. That makes it easier for us to see how x must replace a and 3 must replace b on the right hand side.

$$a^{2} - b^{2} = (a - b)(a + b)$$

 $x^{2} - 3^{2} = (x - 3)(x + 3)$

Thus, the expression $x^2 - 9$ has been factored: $x^2 - 9 = (x - 3)(x + 3)$.

Rewriting expressions in a prescribed form:

It is quite often that we must **rewrite a given expression in a prescribed form**. The examples above are just two of many instances when we are required to do so.

This lesson is devoted to developing your ability to rewrite expressions according to a prescribed rule or pattern, as well as enabling you to match variables in the formula with appropriate parts of the expression. The examples in "Examples and Problems with Solutions" provide additional information which will be helpful in solving the exercises of this section.

Examples and Problems with Solutions

Example 11.1 The expression $(x-2)^2 + (y-1)^2 = 3^2$ is written in the form:

$$(x-p)^{2} + (y-q)^{2} = r^{2}$$

What are the values of p, q, and r? Solution:

Compare: $(x - p)^{2} + (v - a)^{2} = r^{2}$ with

$$(x-2)^{2} + (y-1)^{2} = 3^{2}$$

to find that p = 2, q = 1, and r = 3.

Example 11.2 Write the expression

a) x - y as a sum of two expressions, that is in the form A + B.

b) x + y as a difference of two expressions, that is in the form A - B. Solution:

a) The difference of any two expressions always can be written as a sum: x - y = x + (-y)

b) The sum of any two expressions always can be written as a difference: x + y = x - (-y)

To see that (a) and (b) are true, recall the rules for signed numbers: x + (-y) = x - y, x - (-y) = x + y. Now, interchange the left-hand side of each of the equations with its right-hand one (it can always be done!), and we obtain exactly what we claimed in part (a) and (b) of the solutions.

Example 11.3 Write the following expression in the form A^2 , where A is any algebraic expression. Each time identify A.

a)
$$81x^2$$
 b) x^8 c) $\frac{0.09x^2}{x^6}$

a) Express 81 as a square of a number: $9^2 = 81$. We write: $81x^2 = 9^2x^2 = (9x)^2$ and thus A = 9x.

b) Recall that $(a^m)^n = a^{mn}$, and so $x^8 = (x^4)^2$. Notice that the power of x inside the parentheses multiplied by 2 must be equal to 8. Since $4 \times 2 = 8$, x needs to be raised to the fourth power. In the expression $(x^4)^2$ we identify A as: $A = x^4$.

c) Write all factors that are not already written in this form, as a square of some expression: $0.09 = 0.3^2$, $y^6 = (y^3)^2$ (the power of y inside the parentheses multiplied by 2 must be equal to 6, thus we have y^3). As a result, we have $\frac{0.09x^2}{y^6} = \frac{0.3^2x^2}{(y^3)^2} = \left(\frac{0.3x}{y^3}\right)^2$. The expression $\left(\frac{0.3x}{y^3}\right)^2$ is written in the form A^2 , with $A = \frac{0.3x}{y^3}$.

Example 11.4 Write the following expressions in the form Ax + By, where A, B are any numbers. Identify A and B in your representation:

a)
$$\frac{-x+8y}{4}$$
 b) $3x-(2y+x)$ c) $x-(y+x)$

Solution:

a) Recall that $\frac{-x+8y}{4} = \frac{-x}{4} + \frac{8y}{4} = -\frac{1}{4}x + 2y$. Compare the obtained expression with Ax + By to recognize that $A = -\frac{1}{4}$, B = 2.

b) We simplify the expression (hoping to obtain the desired form of the expression): 3x - (2y + x) = 3x - 2y - x = 2x - 2y = 2x + (-2)y

Notice how, in the last step, we expressed subtraction as the addition. Comparing 2x + (-2)y to Ax + By gives us A = 2, B = -2.

c) We simplify the expression: x - (y + x) = x - y - x = -y = 0x + (-1)y. Notice that after the simplification the variable x has 'disappeared' from the expression. Since zero multiplied by any quantity is equal to zero, in such situation, we can write 0x to match the prescribed form. This 'trick' is often used, and thus worth remembering. Example 11.5 Write the following equation in the form y = mx + b, where *m* and *b* are any numbers. In the expression you obtained, identify *m* and *b*.

a) y = 2x - 4Solution: b) y - x = 0c) 2y = -4x + 3

a) To match the form y = mx + b, subtraction of 4 must be expressed as an addition. We rewrite y = 2x - 4 as y = 2x + (-4); m = 2, b = -4.

b) To match the form y = mx + b, we must isolate y on one side of the equation (which means solving the equation for y). To this end, we add x on both sides and get: y = x, or equivalently, y = 1x + 0; We see that m = 1, b = 0.

c) We must solve the equation for y, and then rewrite the right-hand side in such way that it matches mx + b:

2y = -4x + 3	Divide each side by 2.
$y = \frac{-4x+3}{2}$	Write the fraction as a sum of two fractions.
$y = \frac{-4x}{2} + \frac{3}{2}$	Simplify.
$y = -2x + \frac{3}{2}$	
$a = -2, b = \frac{3}{2}.$	

As a result, m = -2, $b = \frac{5}{2}$.

Example 11.6 Determine if the following equations are linear in one variable. If so, express them in the form ax + b = 0, $a \neq 0$. Determine the values of a and b in your representation:

a) $\frac{x}{4} - 1 = 0$ b) 7x + 2 = 3x c) $2x(x+3) = 2x^2$ d) $3x^2 - 1 = 0$

Solution:

a) It s a linear equation since it can be rewritten it in the form: $\frac{1}{4}x + (-1) = 0$; $a = \frac{1}{4}$

 $\left(\frac{1}{4} \neq 0\right)$, b = -1. Notice, that this representation is not unique. If we multiply both sides of the

equation $\frac{x}{4} - 1 = 0$ by 4, we get: x - 4 = 0. This can be rewritten as 1x + (-4) = 0, and then a = 1, b = -4. Both answers would be correct. There are other representations possible.

b) Subtract 3x from both sides: 7x + 2 - 3x = 0. Collect like terms: 4x + 2 = 0. This matches the form ax + b = 0, with a = 4 ($4 \neq 0$), b = 2. Thus 7x + 2 = 3x is a linear equation in one variable.

c) Remove parentheses: $2x^2 + 6x = 2x^2$, subtract $2x^2$ from both sides:

 $2x^2 + 6x - 2x^2 = 0$, and simplify to obtain: 6x = 0. We rewrite 6x = 0 as 6x + 0 = 0 (adding 0 does not change the value of the expression, but allows us to match the form). Thus a = 6 $(6 \neq 0)$, b = 0 in our representation, and $2x(x+3) = 2x^2$ is a linear equation in one variable.

d) Since no operation can remove x^2 from the equation, it is not a linear equation.

Example 11.7 Using the formula $a^2 - b^2 = (a - b)(a + b)$, factor the following expressions:

a)
$$x^2 - \frac{1}{9}$$
 b) $m^2 - (1 - 3m)^2$

Solution:

a) Write the expression $x^2 - \frac{1}{9}$ in the form: $a^2 - b^2$. To this end, notice that $\frac{1}{9} = \frac{1}{3^2} = \left(\frac{1}{3}\right)^2$, and thus $x^2 - \frac{1}{9} = x^2 - \left(\frac{1}{3}\right)^2$. We recognize that a = x, $b = \frac{1}{3}$, and substitute those values in the right-hand side of the equation: $(a-b)(a+b) = \left(x - \frac{1}{3}\right)\left(x + \frac{1}{3}\right)$. The expression $x^2 - \frac{1}{9}$ is factored: $x^2 - \frac{1}{9} = \left(x - \frac{1}{3}\right)\left(x + \frac{1}{3}\right)$.

b) The expression is already written in the form $a^2 - b^2$, with a = m and b = 1 - 3m. We replace a and b in (a-b)(a+b) with m and 1-3m, respectively, and simplify: (a-b)(a+b) = [m-(1-3m)][m+(1-3m)] = (m-1+3m)(m+1-3m) = (4m-1)(1-2m). This gives us the desired factorization: $m^2 - (1-3m)^2 = (4m-1)(1-2m)$.

Exercises with answers

Ex.1 The expression $\left(y - \frac{3}{4}\right)^2$ is written in the form: $(y - a)^2$. What is the value of *a*? *Ex.2* The following expressions are written in the form: ax + by. What are the values of *a* and *b* in these representations? a) 3x + 4yb) $-4x + \frac{2}{3}y$

Ex.3 The expression -3+x is written a) in the form: p+x. Determine the value of p in this representation. b) in the form: -p+x. Determine the value of p in this representation

Ex.4 The expression x^{-2+5} is written in the form: x^{a+b} . What are the values of *a* and *b*?

Ex.5 The expression $(-7)^2 x$ is written in the form: $a^2 x$. What is the value of a?

Ex.6 The expression 4x - 2y is written in the form: A - 2B. What algebraic expression represents A and B.

*Ex.*7 The following expressions are written in the form: XY^2 For each of them determine what algebraic expression represents X and Y.

a) $3ab^2$ b) $3(ab)^2$

Ex.8 The following expressions are written in the form: $\frac{c}{b}^9$. For each of expressions identify (without rewriting the expression) what algebraic expression represents *c* and *b*. a)

$$\frac{(x-1)^9}{2}$$
 b) $\frac{x^9}{y^9}$

Ex.9 For each of the following expressions (1-6) indicate if they match A, B, C, D, E or F. Each time identify a and b.

(1)
$$x^3 - 1^3$$
(A) $(a - b)^3$ (2) $(x - 1)^2$ (B) $a^2 - b^2$ (3) $(8 + x)(64 - 8x + x^2)$ (C) $(a - b)(a + b)$ (4) $(3x - 5y)^3$ (D) $(a + b)(a^2 - ab + b^2)$ (5) $(3x)^2 - (5y)^2$ (E) $a^3 - b^3$ (6) $(3 - y)(3 + y)$ (F) $(a - b)^2$

Ex.10 Among the expressions below identify those that are written in the form A - B and those that are in the form A + B, where A and B are any expressions except 0. Those that are in the form A - B rewrite as A + B, those that are in the form A + B rewrite as A - B (In other words: rewrite sums as differences and differences as sums):

a)
$$5-(-n)$$
 b) $5+n$ c) $5+(-n)$ d) $5-n$

Ex.11 Rewrite the following expressions in the form $ax^3 + b$, where a and b are any numbers. Identify a and b in your representation:

a)
$$-x^3 + 3$$
 b) $2x^3 - 3$ c) $1 - \frac{x^3}{2}$ d) $\frac{4x^3 + 3}{2}$ e) $\frac{3 - 2x^3}{2}$

Ex.12 Write the following expressions in the form a^2 , where *a* is any algebraic expression or a number. In each case state what *a* is equal to.

a) 36	b) 400	c) 0.16	d) $\frac{9}{49}$	e) $25y^2$
f) $\frac{b^2}{100}$	g) $0.49c^2$	h) X^4	i) 4 <i>x</i> ⁶	j) $81x^2y^8$

Ex.13 Write the following expressions in the form a^3 , where *a* is any algebraic expression or a number. In each case state what *a* is equal to.

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a) -1	b) 27	c) 0.027	d) $\frac{8}{125}$	e) $-z^{3}$
f) $64x^3$	g) $\frac{-x^3}{8}$	h) y ⁶	i) 1000 <i>x</i> ⁹	j) $\frac{x^{15}}{8y^3}$

Ex.14 Write the expressions
$$x^{24}$$
 in the following forms:
a) a^4 b) a^6 c) a^{12}

Ex.15 Write the following expressions in the form A^7 : Identify A in your representation.

a)
$$-x^7$$
 b) x^{14} c) $x^{14}y^{21}$ d) $\frac{y^{21}}{z^{70}}$

Ex.16 Write the following expressions in the form a^m , where *a* is any algebraic expression and *m* is a positive integer different from 1. Identify *a* and *m* in your representation:

a)
$$x^2 x^3$$
 b) $s^7 (tv)^7$ c) $b(b^3)^2$ d) $\frac{B^3}{C^3}$ e) $\left(\frac{x+y}{z}\right)^2 \frac{x+y}{z}$ f) $64x^2$

Ex.17 Write in the form Ax + By + Cz, where A, B, C are any numbers. Identify A, B, and C.

a)
$$-x + \frac{3}{2}z - \frac{y}{4}$$
 b) $-3(2x + y) + z$ c) $\frac{3x - 2y + z}{4}$
d) $x - y$ e) $\frac{x + 3y}{4} - x + 2z$ f) $\frac{z}{3} - \frac{2}{5}z$

Ex.18 Write the following equations in the form $ax^2 + by^2 = 0$, where *a* and *b* are any numbers. Identify *a* and *b* in your representation:

a)
$$x^{2} - 2y^{2} = 0$$

b) $3x^{2} = y^{2}$
c) $\frac{x^{2}}{2} - (y^{2} - 6) = 6$
d) $x^{2} = 0$
e) $\frac{3x^{2} + y^{2}}{4} = 0$
f) $-x^{2} = \frac{8x^{2} - 5y^{2}}{2}$

Ex.19 Write the following equations to match the form y = mx + b, where *m* and *b* are any numbers. In each case determine the value of *m* and *b*.

a)
$$y = 3x - 2$$

b) $y = x$
c) $y = \frac{x}{5}$
d) $y + 3x - 2 = 0$
e) $3y = 6x - 1$
f) $y - x = -x - 5$
g) $3 - y = 3$
h) $\frac{-y + 4x + 2}{2} = 4$

Ex.20 Recall the formula for the square of the sum: $(a+b)^2 = a^2 + 2ab + b^2$. The following expressions are written in the form $(a+b)^2$. For each such expression, identify *a* and *b*, and then substitute their values in $a^2 + 2ab + b^2$. Simplify.

a)
$$(3+b)^2$$
 b) $(2x+3y)^2$ c) $\left(y^2 + \frac{2}{5}\right)^2$

Ex.21 Determine if the following equations are linear equations in one variable. If so, express it in the form ax + b = 0, where b is any real number, a is any real number except zero, and x is unknown. Determine the values of a, and b in your representation:

a)
$$3x^2 - 9 = 0$$
 b) $\frac{x}{3} - 1 = 0$ c) $-x = 0$ d) $\frac{x+8}{8} = 0.1$

e)
$$x(x+1) = x^2 - 2$$
 f) $-3x + 7 = x$ g) $-(x+2) = 0.5$ h) $\frac{x}{5} = 2x - \frac{1}{2}$

Ex.22 The equation 4x + 2 = 0 is a linear equation written in the form ax + b = 0. Record the values of *a* and *b* in this representation. Obtain an equivalent equation by multiplying both side of the equation by 3. What are the values of *a* and *b* in the new representation?

Ex.23 Using the formula for the difference of two squares: $a^2 - b^2 = (a - b)(a + b)$, factor the following expressions. Simplify your answer, if possible:

a)
$$x^{2} - 1$$
 b) $4 - 9x^{2}$ c) $y^{2} - 100a^{2}$ d) $x^{4} - \frac{1}{9}$ e) $x^{2}y^{2} - 0.25$
f) $\frac{a^{2}}{b^{2}} - 36$ g) $m^{2} - (2m+1)^{2}$ h) $(x+1)^{2} - (3x+5)^{2}$ i) $(2a-3)^{2} - 9a^{2}$

Ex.24 The following formula is true: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$. Factor each of the following expressions using the above formula. To this end:

- rewrite each expression to match the left-hand side of the equation
- identify the value of *a* and *b* in your representation
- replace a and b in $(a-b)(a^2+ab+b^2)$ with their representation
- simplify, if possible

a)
$$x^3 - 64$$
 b) $1 - 8y^3$ c) $8x^3 - 27y^3$ d) $x^6 - \frac{1}{27}$

Ex.25 Calculate $10.7^2 - 9.3^2$ using $a^2 - b^2 = (a - b)(a + b)$ (Show how you matched to the given identity in order to arrive at your answer).

11A

1. The expression $3x + \frac{2}{3}$ is written in the form: mx + b. What are the values of *m* and *b*?

2. The expression
$$-3x^2 + \frac{y^2}{2}$$
 is written in the form: $ax^2 + by^2$ What are the values of *a* and *b*?

- 3. The expression $\frac{-4-x}{3}$ is written in the form:
- a) $\frac{a-x}{b}$. Determine the values of *a* and *b* in this representation.
- b) $\frac{-a-x}{b}$. Determine the values of a and b in this representation.
- 4. The expression $\frac{2}{3}x$ is written in the form: a) Ax. What is the value of A?
- b) $\frac{a}{b}x$. What are the values of *a* and *b*?

- 1. The expression $\frac{1}{x^2}$ is written in the form:
- a) $\frac{1}{a}$. Identify the expression representing *a*.

b) $\frac{1}{a^2}$. Identify the expression representing *a*.

2. The following expressions are written in the form: 3A - B. For each of them determine what algebraic expression represents A and B. a) 3x - 4

b) $3(2xy) - \frac{x}{2}$

3. The following expressions are written in the form: $m + n^2$ For each of them determine what algebraic expression represents *m* and *n*.

a) $3 + a + b^2$

b) $3 + (a+b)^2$

c) $(3+a+b)^3 + (a+b)^2$

1. The following expressions are written in the form: a^5b . Without rewriting, identify the expressions representing *a* and *b*?

a)
$$\left(\frac{4}{9}\right)^5 (-2)$$

b) $(x^7)^5 x^5$

2. For each of the following expressions (1-5) indicate if they match A, B, C, D or E. Each time identify a and b.

- (1) $(3x)^3 y^3$ (A) $a^2 b^2$
- (2) $(x-y)(x^2 + xy + y^2)$ (B) (a-b)(a+b)
- (3) $(2x)^2 3^2$ (C) $a^3 b^3$
- (4) (2x-3)(2x+3) (D) $(a-b)^2$
- (5) $(4x-y)^2$ (E) $(a-b)(a^2+ab+b^2)$

1. Write the following expression as a difference of two expressions i.e. in the form A - B, where A and B are any expressions except 0: a) 2x + (-y)

b) *m*+*n*

c) $\frac{2-a}{2+a}$

2. Write the following expression as a sum of two expressions i.e. in the form A + B, where A and B are any expressions except 0: a) 2x - (-y)

b) *m*−*n*

c)
$$\frac{2-a}{2+a}$$

3. Write the following expressions: in the form $(x-p)^2$, where p is any algebraic expression. Identify p in your representation: a) $(x+3t)^2$



c)
$$\left(\frac{2x+3}{2}\right)^2$$

1. The following expressions are written in the form: $A^m - B^m$, where A and B are any algebraic expressions, and m any positive integer. Identify A, B, and m. a) $(3x)^2 - (y+1)^2$

b)
$$(3y^5 + y - 1)^5 - (4y)^5$$

2. Write the following numbers in the form a^2 :

c) 0.36 d)
$$\frac{9}{64}$$

g)
$$\frac{4900}{81}$$
 h) 0.16

3. Write the following numbers in the form a^3 :

c) 0.027 d)
$$\frac{8}{27,000,000}$$

1. Write the following expressions in the form a^2 , where *a* is any algebraic expression. In each case state what *a* is equal to.

a) $64x^2$

b) $4(3x+7)^2$

c)
$$\frac{x^2}{y^2}$$



e) x^{10}



g) $16x^4y^{12}$

1. Write the following expressions in the form a^3 , where *a* is any algebraic expression. In each case state what *a* is equal to.

a) $\frac{x^3}{27}$

b) $0.008y^{12}$

c) $-x^{3}$

2. Write the following expressions in the form a^4 , where *a* is any algebraic expression. In each case state what *a* is equal to.

a) 1000 y⁴

b)
$$\frac{m^4}{n^{12}}$$

3. Write the following expressions in the form A^5 , where A is any algebraic expression. Identify A in your representation:

a) x^{10}

b) $-x^{5}$

c) $-x^{10}$

1. Write the expressions in the form $x^6 y^{12}$ in the following form:

a) A^2

b) *A*³

c) A^{6}

2. Write in the form ax^m , where *a* is any number, and *m* is a non-negative integer. Identify *a* and *m* in your representation (notice, that what you are being asked is to write an expression as a single exponential expression and then to identify the numerical coefficient):

a) *x*

b) $(-x)^{13}$

c) $3x^4(-2x)$

d) $(2x^3)^2$

1. Write in the form Ax + By + C, where A, B, C are any numbers. Identify A, B, and C. a) $\frac{x}{3} - y + 1$

b) 2x + 3y - 3x + 5

c)
$$\frac{x - y + 18}{9}$$

d) 2(x-3y) - 2x + 1

e)
$$\frac{3x}{0.01} + \frac{0.03y}{0.01} + \frac{1}{2} - \frac{3}{4}$$

Homework/Class Work/Quiz

1. Write the following expressions in the form $x^3 = a$, where *a* is any number. Identify *a* in your representation:

a) $x^3 + 2 = 0$

b)
$$2x^3 = \frac{2}{3}$$

c)
$$\frac{2x^3}{3} + 1 = 0$$

d)
$$\frac{2x^3+1}{3} = 0$$

e)
$$3x^3 - \frac{1}{2} = x^3 + \frac{1}{2}$$

1. Write the following expressions in the form ax + by = c, where *a*, *b*, *c* are any numbers. Determine the value of *a*, *b*, and *c* in your representation.

a) x + 2 = 2y

b) 3(x-2y+1)=0

c) x = -y

d)
$$2x - x = 4$$

e)
$$\frac{2x+y}{2} = x$$

Homework/Class Work/Quiz

1. Write the following expressions to match the form $y = ax^2 + b$. In each case determine the value of *a* and *b*.

a)
$$y = \frac{-2x^2}{5} + 3$$

b) $2y = x^2 - 2$

c)
$$y - x^2 + 0.4 = 0$$

d)
$$y = 3(x^2 - \frac{2}{3}) + 2$$

2. Recall the formula for the square of the sum: $(a-b)^2 = a^2 - 2ab + b^2$. The following expressions are written in the form $(a+b)^2$. For each such expression, identify *a* and *b*, and then substitute their values in $a^2 + 2ab + b^2$. Simplify.

a) $(x-1)^2$

b)
$$\left(x - \frac{y}{2}\right)^2$$

Homework/Class Work/Quiz

1. Determine if the following equation is an example of a linear equation in one variable. If it is, write it in the form ax + b = 0, $a \neq 0$, and determine the value of *a* and *b* in your representation. If not, write 'not a linear equation'. a) 3x - 2 = 5

b) $3x^2 + 4 = 0$

c) -0.4x = 0

d)
$$2(3x-4) = 7x-2$$

e)
$$\frac{x+5}{4} = 3$$

f)
$$2x^4 - x = \frac{8x^4 - 4}{4}$$

11N

1. Using the difference of squares formula $a^2 - b^2 = (a - b)(a + b)$, factor the following expressions: a) $x^2 - 3600$

b) $1 - \frac{y^2}{4}$

c) $100 - 4c^2$

d)
$$x^4 - \frac{1}{16}$$

e) $b^2 - (b-1)^2$

f)
$$x^6 y^8 - x^4 y^{12}$$

Homework/Class Work/Quiz

1. Determine if the following equation is an example of a linear equation in one variable. If it is, write it in the form ax + b = 0 and determine the value of *a* and *b* in your representation. If not, write 'not a linear equation'. a) -(x+1)-(x-1)=0

b)
$$\frac{2x}{3} = \frac{x}{2}$$

c)
$$x^3 + x - 1 = x^3$$

2. Using the difference of squares formula $a^2 - b^2 = (a - b)(a + b)$, factor the following expressions: a) $25x^2 - 9y^2$

b) $(2x-1)^2 - (5x)^2$

c) $49a^2 - (3a+1)^2$

11P

1. The following formula is true: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$. Factor each of the following expressions using the above formula. To this end:

- rewrite each expression to match the left-hand side of the equation
- identify the value of *a* and *b* in your representation
- replace a and b in $(a-b)(a^2+ab+b^2)$ with their representation
- simplify, if possible

a) $1 + x^3$

b) $x^3 + 27y^3$

c) $1000 + x^6$

1. The following formulas are true:

$$a^{2} - b^{2} = (a - b)(a + b)$$

$$a^{3} + b^{3} = (a + b)(a^{2} - ab + b^{2}).$$

$$a^{3} - b^{3} = (a - b)(a^{2} + ab + b^{2})$$

Factor each of the following expressions using one of the above formulas. To this end, you must first match each expression with one of the above formulas, then identify the value of a and b in your representation, and finally replace a and b in the right-hand side of the used formula, Please, simplify your answer..

a) $8x^3 + 27y^3$

b) $(2x+7)^2 - (2-x)^2$

c) $x^6 - 8y^9$

11R

1. The following formula is true (you are not asked to check it, although you certainly can; you have enough knowledge to do so):

 $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Use the above formula to calculate 11^4 .

Hint: Use the fact that 11 = 10 + 1 to write 11^4 to match the form $(a+b)^4$.

2. Calculate $52^2 - 48^2$ using $a^2 - b^2 = (a - b)(a + b)$ (Show how you matched to the given identity in order to arrive at your answer).

Lesson 12

Topics:

Application of linear equations; Review for Test 4.

Ratios and proportional quantities:

The following terminology is often used:

Ratio The quotient of two quantities $\frac{a}{b}$, $b \neq 0$ is called a ratio.

For example:

If in a class there are 12 girls and 17 boys, the ratio of girls to boys is $\frac{12 \text{ girs}}{17 \text{ boys}}$. If 1 euro is equal 1.33 dollar, the ratio of euro to dollar is $\frac{1 \text{ euro}}{1.33 \text{ dollar}}$.

Proportional quantities The two quantities are proportional if they have the constant ratio.

For example, if you drive with the same speed, then the distance you drive is proportional to the time it takes you to drive it. If you double the distance, the time required must also be doubled. If the distance is tripled, time must be tripled. By whatever factor we change the distance, we change time by exactly the same factor.

Application of equations to solving problems with proportional quantities:

Recognizing that certain quantities are proportional often can be employed to answer a variety of questions:

■ Making 5 pies requires 10 apples. How many pies can we make if we have 14 apples?

We will start with naming the unknown. Let

x : be the number of pies we can make if we have 14 apples.

Notice that the number of apples we need is proportional to the number of pies we make. It means that the ratio of the number of pies to the number of apples is constant. We could say "5 pies to 10 apples is like x pies is to 14 apples". To help ourselves visualize this relationship, we will create the diagram:

5 pies is 'paired' with 10 apples - we will record this in the first row.

x pies is 'paired' with 14 apples - this will be recorded in the second row.

5 pies 10 apples _____ 14 apples x pies

This diagram can be 'translated' into an equation by writing the first row in the diagram as a ratio on one side of the equation and the second row of the diagram as a ratio on the other side $\frac{5 \text{ pies}}{10 \text{ apples}} = \frac{x \text{ pies}}{14 \text{ apples}}$ of the equation:

It is important to make sure that we compare ratios of the same quantities. In our example we compare ratios of number of pies (these should be in the numerators) to number of apples required (these should be in the denominators). It is thus helpful to include dimensions when writing the initial equation.

Without units the above equation looks as follows: $\frac{5}{10} = \frac{x}{14}$. To solve it, we apply the methods known to us, i.e. by performing appropriate operations on both sides of the equation we will isolate x on one side:

$$\frac{5}{10} = \frac{x}{14}$$

Multiply both sides by 14 to isolate x.
$$14 \cdot \frac{5}{10} = \frac{x}{14} \cdot 14$$

Simplify.
$$7 = x$$

Thus the answer is: We can make 7 pies with 14 apples. \blacksquare

It is worth noticing that this problem could have been set up in different ways. If the ratio of number of apples needed and number of pies is always the same, then it is also true that: the ratio of number of pies and number of apples needed is always the same. In other words, number of pies could be in the numerator and the appropriate number of apples in the denominator instead and this leads to the equation:

$$\frac{5 \text{ pies}}{10 \text{ apples}} = \frac{x \text{ pies}}{14 \text{ apples}}$$

The ratio of number of pies is always the same as the ratio of number of apples needed. This results in the equation:

 $\frac{10 \text{ apples}}{14 \text{ apples}} = \frac{5 \text{ pies}}{x \text{ pies}}$ or $\frac{14 \text{ apples}}{10 \text{ apples}} = \frac{x \text{ pies}}{5 \text{ pies}}$

All of these equations are equivalent to the one we used to solve the problem and thus solving any of them gives (as it should) the same answer.

■ Convert 4 yards to feet, if 1 yard is equal to 3 feet.

Let x: be the number of feet equal to 3 yards.

The ratio of number of yards to number of feet stays the same: number of yards is proportional to the number of feet. We represent the relationship "1 yard is to 3 feet as 4 yards is to x feet". with the diagram.

1 yard is 'paired' with 3 feet, this relationship is recorded in the first row.

4 yards is 'paired' with x feet, this is recorded in the second row.

We set up the equation:

$$\frac{1 \text{ yards}}{3 \text{ feet}} = \frac{4 \text{ yards}}{x \text{ feet}}.$$

We make sure that units on both sides are the same ($\frac{\text{yards}}{\text{feet}}$ in this example).

We solve the equation:

$$\frac{1}{3} = \frac{4}{x}$$
Multiply both sides by x feet.

$$x \cdot \frac{1}{3} = \frac{4}{x} \cdot x$$
Simplify.

$$\frac{1}{3}x = 4$$
Multiply both sides by 3 to isolate x.

$$x = 12$$

Thus the answer is: 4 yards is equal to 12 feet. \blacksquare

■ What percent of 60 is 3?

Let *x*: be the percent of the number 60 that 3 is.

The key to this problem is to identify the number that is set to be 100%. We are being asked to find out what percent *of* 60 is the number 3, thus all comparison is done with respect to 60 and thus the number 60 is considered to be 100%.

As usual: The number 60 is 'paired' with 100% - the first row.

The number 3 is 'paired' with x % - the second row.

"60 is to 100% as 3 is to *x*%"

The resulting equation is: $\frac{60}{100\%} = \frac{3}{x\%}$.

We make sure that units are the same on both sides of the equation, and solve the equation:

$$\frac{60}{100} = \frac{3}{x}$$
Multiply both sides by x. Simplify.

$$\frac{60}{100}x = 3$$
Multiply both sides by $\frac{100}{60}$.

$$\frac{100}{60} \cdot \frac{60}{100}x = 3 \cdot \frac{100}{60}$$
Simplify.

$$x = 5$$

Thus the answer is: The number 3 is 5% of 60. \blacksquare

■ 7 is 5% of what number?

Let x - be a number such that 5% of it is 7.

Notice, that this time the unknown number x is considered to be 100% (5% of what number). 7 is the number that pairs up with 5%. Thus we have:

 $\begin{array}{cccc} x & ---- & 100\% \\ 7 & ---- & 5\% \end{array}$

And the resulting equation is:

$$\frac{x}{100\%} = \frac{7}{5\%}$$

We solve the equation:

$$\frac{x}{100} = \frac{7}{5}$$
$$100 \cdot \frac{x}{100} = \frac{7}{5} \cdot 100$$
$$x = 140$$

Multiply both sides by 100.

Simplify.

Thus the answer is : 7 is 5% of 140. \blacksquare

Other applications of equations to solving word problems:

■ I thought of a number. I multiplied it by 2, and then subtracted 3 from it. As a result, I got 5. What is the number I thought of?

Let x be the number I thought of.

I multiplied the number by 2, the resulting number was 2x. Then, I subtracted 3 from it. The expression representing the new number is 2x-3. Since the result was equal to 5, we can write the equation: 2x-3=5.

We solve the equation:

2x - 3 = 5	Add 3 to both sides.
2x = 8	Divide both sides by 2.

x = 4

The number I thought of is 4. \blacksquare

Examples and Problems with Solutions

Example 12.1 If a car goes 182 miles in 7 hours and it continues to go at the same speed, how long will it take to drive 546 miles? Solution:

x: number of hours it will take to drive 546 miles

In x hours the car goes 546 miles: the first row of the diagram. In 7 hours the car goes 182 miles: the second row . We have:

x hours _____ 546 miles 7 hours _____ 182 miles

The resulting equation is: $\frac{x \text{ hours}}{546 \text{ hours}} = \frac{7 \text{ hours}}{182 \text{ miles}}$ (the units on both sides are the same).

We solve the equation:

 $\frac{x}{546} = \frac{7}{182}$ Multiply both sides of the equation by 546. $x = \frac{7}{182} \times 546$ Perform the indicated operations. x = 21

Answer: It will take 21 hours to drive 546 miles at the same speed.

Example 12.2 Convert 18 pints to quarts, if 1 quart is equal to 2 pints. Solution:

x : number of quarts equal to 18 pints.

1 quart 'corresponds' to 2 pints: the first row. x quarts 'corresponds' to 18 pints: the second row of the diagram. We have:

 1 quart
 2 pints

 x quart
 18 pints

The resulting equation is: $\frac{1 \text{ quart}}{2 \text{ pints}} = \frac{x \text{ quart}}{18 \text{ pints}}$ (the units on both sides are the same: $\frac{\text{ quart}}{\text{ pints}}$). We solve the equation:

$$\frac{1}{2} = \frac{x}{18}$$

Multiply both sides by 18.
$$18 \cdot \frac{1}{2} = \frac{x}{18} \cdot 18$$

Simplify.
$$9 = x$$

Answer: 18 pints is equal to 9 quarts.

Example 12.3
$$\frac{1}{3}$$
 is what percent of 1? Solution:

x: the percent that $\frac{1}{3}$ is of 1.

The number that is considered to be 100% is 1 ("what percent of 1"), $\frac{1}{3}$ 'corresponds' to x %. Thus:



Example 12.4 A number x was multiplied by 10. The same number (number x) was added to 38 and then divided by 2. The two results were equal to each other. What was the number x? Solution:

x - the number we are looking for

'The number x was multiplied by 10' can be expressed as 10x.

'The number x was added to 38 and then divided by 2' can be expressed as $\frac{38+x}{2}$.

Since the two results were equal, we obtain the following equation:

$$10x = \frac{38+x}{2}.$$

We solve the equation:

$10x = \frac{38 + x}{2}$	Multiply both sides by 2.
$2 \cdot (10x) = \frac{38+x}{2} \cdot 2$	Simplify.
20x = 38 + x	Subtract <i>x</i> from both sides.
19x = 38	Divide each side by 19
x = 2	

Answer: The number was equal to 2.

Common mistakes and misconceptions

Mistake 11.1

When solving word problems DO NOT forget to define the variable before you attempt to write an equation. It is impossible to determine the correctness of the equation, if the variable is not defined.

Mistake 11.2

When solving proportions, be careful to be consistent with units on both sides of the equation. For example: If a car travels 80 miles in 2 hours, how far will the car go in 5 hours if it travels at the same speed. It would be correct to write:

 $\frac{80 \text{ miles}}{2 \text{ hours}} = \frac{x \text{ miles}}{5 \text{ hours}}$ However, it would be incorrect to write: $\frac{80 \text{ miles}}{2 \text{ hours}} = \frac{5 \text{ hours}}{x \text{ miles}}$ If you set up $\frac{\text{miles}}{\text{hours}}$ on the left, you must do the same on the right.

Exercises with Answers (For answers see Appendix A)

Ex.1 The administration wants to keep the ratio of teachers to students at 1 to 20. how many students can be served with 33 teachers?

Ex.2 A child can run 5 blocks in 2 minutes. How long does it take the child to run 8 blocks, at the same speed?

Ex.3 The label on a medicine to mix it with water in the ratio 1 to 9. How much water should be mixed with $1\frac{2}{3}$ tablespoons of the medicine?

Ex.4 Three T-shirts cost 15 dollars. What is the cost for 5 T- shirts, assuming that they are at the same price, and there is no discount for a larger quantity?
Ex.5 Jack and Jill went up the hill to pick apples and pears. Jack picked 10 apples and 15 pears. Jill picked 20 apples and an unknown number of pears. The number of apples and number of pears picked by Jack is proportional to the number of apples and pears picked by Jill. How many pears did Jill pick?

Ex.6 A sample of 96 light bulbs consisted of 4 defective ones. Assume that the proportion of the defective bulbs in a batch of 6,000 light bulbs is the same as the proportion of defective bulbs in the sample. Determine the total number of defective bulbs in the 6,000 light bulb batch.

Ex.7 A bartender mixes champagne and orange juice in the ratio of 2 to 3. How much juice does he mix with $7\frac{1}{2}$ ounces of champagne?

Ex.8 If 12 pencils cost \$1 (assume that the price of a pencil does not change with the number of pencils you buy):

a) how much would 21 pencils cost?

b) how many pencils could one buy for 25 cents

Ex.9 Convert 3 cups to fluid ounces, if 1 cup equals 8 fluid ounces.

Ex.10 If 3 euros are worth \$3.90, then how much would 27 euros be worth in dollars?

Ex.11 If one pound is equal to 16 ounces, how many pounds is $\frac{1}{2}$ ounce?

Ex.12 Convert 3 decimeters to meters, if 1 decimeter equals $\frac{1}{10}$ meter.

Ex.13 Convert 6.5 millimeters to centimeters, if 1 centimeter equals 10 millimeters.

Ex.14 If one ton is equal 2000 pounds, how many tons is 250 pounds?

Ex.15 Answer the following question: 2 is 150% of what number?

Ex.16 What is 5% of 12?

Ex.17 What percent of 7 is 0.21?

Ex.18 Answer the following question: 4 is what percent of 0.4?

Ex.19 Answer the following question: 0.1 is 10% of what number?

Ex.20 What is $\frac{2}{3}$ % of 15? *Ex.21* Answer the following question: $\frac{1}{3}$ is 5% of what number? *Ex.22* What percent of $\frac{4}{5}$ is 3? *Ex.23* What is $\frac{3}{7}$ % of $\frac{14}{9}$?

Ex. 24 A number was divided by 3 and the result was 7. What was the number?

Ex. 25 I though of a number: x. I subtracted 7 from it. I divided it by 3. As a result, I got 5. What is the number I thought of?

Ex. 26 After subtracting 7 from a number, the result was doubled. As a result the number 4 was obtained. What was the original number?

Ex.27 If you double a certain number and then add 1 to it, you will get the same result as if you triple the number and then add 2. What is the number?

*Ex.*28 If you add 4 to a number and then divide the result by 6, you will get the same result as if you first subtract 1 from the number, and then multiply everything by 2. What is the number?

Ex. 29 A number x was divided by 3. The same number (number x) was decreased by 4 and then divided by 4. The two results were equal. What was the number x?

Ex. 30 A number is increased by $\frac{2}{3}$. The result is multiplied by 6 and the number $\frac{1}{4}$ is obtained. What is the original number?

1. Tom drives his car 800 miles in 3 days. At this rate, how far will he drive in 30 days?

2. Jane types 360 words in 10 minutes. How long will it take her to type 720 words (assume that Jane types at a constant rate)?

3. In a certain high school, the ratio of boys to girls in a freshman class is the same as in a junior class. There are 25 girls and 75 boys in the freshman class and 13 girls in the junior class. How many boys there are in the junior class?

1. A child can run at a rate of $2\frac{1}{2}$ blocks per minute. At the same rate, how long does it take the child to run 7 blocks?

2. Under typical conditions, $1\frac{1}{2}$ feet of snow will melt to 2 inches of water. To how many inches of water will $3\frac{4}{5}$ feet of snow melt?

1. One foot is equal to 12 inches. Convert 72 inches to feet.

2. Convert 5 gallons to quarts, if 1 gallon equals 4 quarts.

3. Convert 12.4 centimeters to meters, if 1 centimeter equals $\frac{1}{100}$ meter.

1. What percent of 4 is 18?

2. Answer the following: 5 is 2% of what number?

3. What is 3% of 21?

1. What percent of 6 is 0.3?

2. Answer the following: 4 is 0.5% of what number?

3. What is
$$1\frac{2}{3}\%$$
 of 6?

1. Answer the following question: 150% of what number is $\frac{3}{5}$?

2. What is
$$\frac{1}{7}$$
% of 70?

3. What percent of
$$\frac{1}{2}$$
 is $\frac{2}{5}$?

1. A number was multiplied by 3, and then 4 was added. The result was 6. What was the number?

2. If you divide a number by 3 you will get the same result as if you subtract 5 from the number. What is the number?

1. I added 6 to a number. I subtracted the result from 12. As a result of these operations 3 was obtained. What was the original number?

2. If you multiply a number by $\frac{1}{2}$ and add $\frac{2}{5}$ to it, you will get the same result as if you subtract $\frac{3}{10}$ from the number. What is the number you began with?

1. Five pounds of potatoes cost \$7 (assume that the price is proportional to the amount you buy):

a) How much would 8 pounds of potatoes cost?

b) How many pounds of potatoes could one buy for \$17.50 ?

2. Change 236.5 centimeters to meters, if 100 centimeters is equal to 1 meter.

3. What percent of 12 is
$$\frac{1}{2}$$
?

1. A number was increased by 3, then divided by 2. The result was 6. What was the number?

2. How many yards is equivalent to $2\frac{1}{2}$ feet, if 3 feet equals 1 yard?

3. Answer the following question: 0.3 is 0.01% of what number?

1. A 6-lb turkey breast contains 24 servings of meat. How many pounds of turkey are needed for 32 servings?

2. Answer the following question:
$$\frac{4}{5}$$
 is what percent of $1\frac{2}{9}$?

3. Convert $\frac{1}{4}$ inch to feet, if 1 foot is equal to 12 inches.

1. In a metal alloy, the ratio of zinc to copper is 3 to 13. If there are 26 pounds of copper, how many pounds of zinc are there?

2. The number $\frac{2}{5}$ is what percent of 4?

3. I thought of a number. I multiplied it by -2, and then I added 5. As a result I got twice the original number. What was the number I began with?

1. A number is multiplied by 6. The result is subtracted from 7 and then everything is multiplied by 3. As a result of these operations 5 is obtained. What is the original number?

2. A number is divided by 6, then 1 is subtracted, and then everything is multiplied by 3. The result is equal to the original number multiplied by 2. What is the original number?

1. If you multiply a number by 3 and then subtract 6 from it, you will get the same result as if you first subtract 2 from the number, and then multiply everything by 3. What is the number?

2. I thought of a number. I subtracted 2 from it and divided by 3. I added 4. As a result I got the same number I started with. What was my number?

ANSWERS TO EXERCISES APPENDIX A:

Lesson 1

3x+2, y^2 , $\frac{a+bc}{2}$, $(-2a+1)^3$ are examples of <u>algebraic expressions</u>. $\Psi x, y, a, b, c$ are 1 examples of variables but also examples of algebraic expressions. Variables represent unknown <u>numbers</u>. If we know the value of x, we can evaluate 3x + 2, and as a result we get a <u>number</u>. 2 a) "a squared" or "a raised to the second power." b) "a cubed" or "a raised to the third power."c) ʻʻa raised to the twelfth power." d) "2 to the m" or " 2 raised to the m-th power." e) "minus y" or "the opposite of y" f) "cd" or "c times d" g) "a minus b" h) "two-fifth times x" or "two-fifth x" a) $7 \times n$ b) $-5 \times k \times m$ c) "there is no multiplication performed" 3 e) $\frac{3 \times x}{2}$ d) $-x \times (-y)$ f) $2 \times x - y \times z + w \times (-t)$ a) subtraction b)multiplication c) exponentiation d) division e) multiplication 4 a) $\frac{1}{2}y$ b) $\frac{2}{3}y$ c) y+5 d) v-y e) y^2 f) y+3 g) y-x5 h) xy a) a + (-b) b) a - (-b) c) a(-b) d) -C f) -(-C) g) $-\frac{-a}{b}$ 6. h) v(-t)(-p) i) $\frac{c}{-B}$ j) $(-x)^m$ k) $\left(\frac{x}{v}\right)^m$ a) x + 10 b) $\frac{2}{3}x$ c) $\frac{x}{3}$ d) $\frac{100}{r}$ e) 30x f) x - 37 8 9 $\frac{1}{2}(bh)$ or $\frac{1}{2}bh$ 10 $A = 1 \cdot A = 1 \cdot A;$ $a^{2}b = 1 \cdot a^{2}b = 1 \cdot a^{2}b$ $-M = -1 \cdot M = -1 \cdot M$ $-2cde = -1 \cdot 2cde$ 11 a) $-\frac{x^3}{y}$ b) $-\left(-\frac{x^3}{y}\right)$ c) $-\left(-\frac{x^3}{-y}\right)$ 12 a) yx^8 b) $\left(\frac{m}{n}\right)^\circ$ parentheses cannot be removed c) $(-a)^4$ parentheses cannot be removed 13 d) $-a^4$ e) x + (-b) parentheses cannot be removed f) $a \div (-b)$ parentheses cannot be removed g) y(-x) parentheses can't be removed h) -xy i) 3bc j) 3b(-c) parentheses can't be removed a) 3+5=8 b) 3-2=1 c) $\frac{3}{2}=1$ d) $4 \times 3 = 12$ e) $3^2 = 9$ f) $\frac{6}{3} = 2$ 14 $\frac{1}{x}$ can not be evaluated with x = 0, because the denominator of a fraction can not be 0. 15

If
$$x = 0$$
, $\frac{1}{x-5}$ can be evaluated: $\frac{1}{x-5} = \frac{1}{0-5} = -\frac{1}{5}$, but if $x = 5$ then $\frac{1}{x-5} = \frac{1}{5-5} = -\frac{1}{0}$ is
not possible. Another example could be: $\frac{3}{y+4}$ cannot be evaluated with $y = -4$. (answers vary)
16. a) $3 \cdot 0 = 0$ b) $0 - 2 = -2$ c) cannot be evaluated d) $\frac{0}{7} = 0$ c) $\frac{2}{0-3} = -\frac{2}{3}$
17. a) $3^2 = 9$ b) $2^3 = 8$ c) $2^2 = 4$
18. a) $-A = -2$ b) $-A = 2$
19. a) $6 - 8 = -2$ b) $-10 - 6 = -16$ c) $-4 + 6 = 2$ d) $6 - 6 = 0$ e) $-2 + 6 - 6 = -2$
20. a) $2 + (-2) = 0$ b) $2 - (-2) = 4$ c) $-2 - (-2) = 0$ d) $-5 - (-2) + 4 = 1$
e) $6 + (-2) - 10 - (-2) = -4$
21. a) $3(10) = 30$ b) $-5(10) = -50$ c) $\frac{-200}{10} = -20$ d) $-\frac{10}{2} = -5$
e) $-5 \pm 10 = -0.5$ or $-\frac{1}{2}$ f) $10^4 = 10,000$
22. a) $-1000(-12) = 12,000$ b) $\frac{-12}{6} = -2$ c) $-5(-12) = 60$
d) $\frac{6}{-12 \pm 12}$ cannot be evaluated e) $-24 \pm (-12) = 2$ f) $(-12)^2 = 144$
23. a) $\frac{7}{3}$ or $2\frac{1}{3}$ b) $\frac{13}{15}$ c) $-\frac{8}{21}$ d) $-\frac{13}{12}$ c) $\frac{8}{3}$ or $2\frac{2}{3}$ f) $-\frac{11}{3}$ or $-3\frac{2}{3}$
24 a) $\frac{9}{10}$ b) $\frac{16}{5}$ c) $\frac{8}{5}$ or $\frac{13}{5}$ d) $-\frac{13}{20}$ e) $-\frac{29}{10}$ or $-2\frac{9}{10}$
25 a) $\frac{4}{7}$ b) -2 c) $-\frac{4}{3}$ d) $\frac{5}{8}$ e) $\frac{35}{2}$ or $17\frac{1}{2}$ f) $-\frac{1}{7}$
26 a) $\frac{9}{16}$ b) -1 c) $\frac{9}{20}$ d) $\frac{3}{2}$ e) $\frac{1}{4}$ f) 0
27 a) 3.41 b) 34.81 c) 0.05 d) -8 e) 0.06 f) -5
28 a) -3.9 b) -2.1 c) $2 - 4$ d) 360 e) -0.0006 f) -0.216
29 a) $\frac{19}{15}$ or $1\frac{4}{15}$ b) $-\frac{5}{14}$ c) -1.28
30 a) $-\frac{1}{15}$ b) $\frac{13}{14}$ c) 0.88
31 a) $\frac{4}{9}$ b) $\frac{18}{5}$ or $3\frac{3}{5}$ c) -0.002
32 a) $\frac{9}{121}$ b) $\frac{40}{9}$ c) -20
33 a) $-10,000,000$ b) 16 c) $-\frac{1}{8}$ d) 0.00001 or $\frac{1}{100,000}$
34 K can be any negative number. For example, K = -2. Answers vary.

34 *K* can be any negative number. For example, K = -2. Answers vary. 35 For example, let a = 4, b = -6. In this case: -(a + b) = 2. Answers vary.

1	a) $3x + y$ b) $4(a+b)$ c) $\frac{-(4x+2)}{6}$ d) $z(y-3) \text{ or } (y-3)z$ e) $9x^3$
	f) $(9x)^3$ g) $\frac{a-b}{c}$ h) $\frac{3}{y} + 3x + 1$ i) $(-x)^3 + (-y)^7$
2	a) $2(x-7)$ b) $\frac{2}{3}(x-1)$ c) $\frac{1}{4}x-5$ d) $9(x-8)$ e) $\left(\frac{x}{2}\right)^3$
	f) $4(-x)$ g) $x^3 + 6$ h) $y(x-4)$ i) $x(x+5)$ j) $(-x)^{121}$ k) $-x^2$
3	$\frac{9}{5}C + 32$
4	2(L+W)
5	mc^2
6	a) $a + (b^5)$ exponentiation b) $(a+b)^5$ addition
	c) $-(x^8)$ exponentiation $d(-x)^8$ opposite of x
	e) $a - b$ subtraction f) $a \div b > c$ division
	g) $4 - \begin{pmatrix} 7y \\ 7y \end{pmatrix}$ multiplication h) $3 + a \div b$ division
7	 a) Multiply x by 4 and then subtract y. b) Add a and 3 and then divide the result by x. c) Add x and 3 and then multiply the result by y. d) Divide s by t and then add 2 to the result. e) Square x and then multiply by 3. f) Multiply x by 3 and then square the result. g) Add a and c and then raise the result to the 4th power. h) Raise c to the 4th power and then add the result to a.
8	a) We cannot remove parentheses. b) $c-3-a$ c) $3a+2+x$ d) We cannot remove parentheses.
	e) We cannot remove parentheses. f) $\frac{-c+d}{a}$ g) We cannot remove parentheses. h) yx^8
	i) We cannot remove parentheses. j) We cannot remove parentheses.
9	a) 2 b) 2 c) 2 d) Yes, because we performed the same operations. e) $\frac{1}{2}$
10	a) $-2 \times 3 - 5 = -11$ b) $-4 \times 3 + 3^2 = -3$ c) $\frac{3}{3-3}$ cannot be performed
	d) $(-3)^2 = 9$ e) $-3^2 = -9$ f) $\frac{3-3}{4+3} = 0$
11	a) $-2^4 = -16$ b) $(-2)^4 = 16$ c) $(-4)^2 = 16$ d) $-4^2 = -16$ e) $4^4 = 256$
12	a) $1 + (-1)(1) = 0$ b) $1 - (-1)(1) = 2$ c) $(1)(1)(1) = 1$
13	a) -2 b) $-2\frac{1}{2} = -\frac{5}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{4}$ e) $-\frac{1}{4}$
14	a) 0.39 b) -5 c) -1 d) -273
15	a) 26 b) 40
16	b) c) and f) We would have 0 in the denominator in these cases.

17 a) -19 b) 60 c) -34 d) -35 e) -50
18 a)
$$8 \times \left(-\frac{1}{8}\right) -10 \times \frac{4}{5} = -9$$
 b) $10 \left(-\frac{1}{8}\right) \left(\frac{4}{5}\right) = -1$ c) $2 \left(\frac{4}{5} + \frac{1}{8}\right) = \frac{37}{20}$
d) $-8 \left(-\frac{1}{8}\right)^2 + \frac{4}{5} = \frac{27}{40}$ e) $\frac{4}{5} \div \left(\frac{1}{8} - \frac{1}{8}\right)$ cannot be performed for $\frac{1}{5} \div \frac{1}{8} - \frac{1}{8} = \frac{251}{40} = 6\frac{11}{40}$
19 a) $2 \times \left(\frac{1}{3}\right)^4 = \frac{2}{81}$ b) $\left(-\frac{2}{3}\right)^4 = \frac{16}{81}$ c) $-\left(-\frac{2}{3}\right)^4 = -\frac{16}{81}$
d) $\frac{\frac{1}{3} - \frac{2}{3}}{\frac{1}{3} + \frac{2}{3}} = -\frac{1}{3}$ e) $\frac{\frac{1}{3} \times \left(-\frac{2}{3}\right)}{\frac{1}{3} \div \left(-\left(-\frac{2}{3}\right)\right)} = -\frac{4}{9}$
20 a) -1 b) 64 c) $-\frac{1}{9}$ d) $\frac{1}{9}$ e) 1 f) -1
21 a) -2.2 b) -2.1
23 a) -2 b) -2
24 a) -5 b) 0 c) 1.3 d) $1\frac{1}{2} = \frac{3}{2}$ e) $2\frac{13}{15} = \frac{43}{15}$ f) $\frac{4}{35}$
25 a) 4 b) 7 c) 0.5 d) 7 e) $\frac{3}{14}$ f) $\frac{17}{3} = 5\frac{2}{3}$
26 a) 10 b) -50 c) -0.05
27 a) -1 b) -15 c) -24
28 a) 1 b) -6 c) 5 d) cannot be evaluated e) -5 f) 0

In the expression $4x^2 \times 2y$, $4x^2$ and 2y are called <u>factors</u>. In the expression $4x^2 + 2y$, $4x^2$ and 1 2y are called terms. 2 a) $2 \times a$; factors: 2, a b) $3 \times (a+b)$; factors: 3, (a+b)c) $-3 \times x \times \frac{2}{y}$; factors: $-3, x, \frac{2}{y}$ d) $4 \times (x+y) \times (b-c)$; factors: 4, (x+y), (b-c)a) 3, x b) ab, -cd c) $\frac{xy}{2}$, $2y^2$, -1 d) $-(2-b)^2$, $\frac{x}{y}$, -z3 b) Terms: $2x^2$, y^3 ; $2x^2 + y^3 = y^3 + 2x^2$ a) Terms: 2m, z; 2m + z = z + 2m4 c) Terms: c(d - f), y^2 ; $c(d - f) + y^2 = y^2 + c(d - f)$ All these expressions are equal (equivalent) because of the commutative property of addition. 5. a) m - n = -1, n - m = 1; they are not equivalent. 6 b) True a) Terms: $x_1 - 2$; x - 2 = -2 + xb) Terms: -3c, 2; -3c + 2 = 2 - 3c7 c) Terms: $-(x-y)^2$, s; $-(x-y)^2 + s = s - (x-y)^2$ d) Terms: $\frac{yx^2}{2}$, $-\frac{(cd+f)^2}{t}$; $\frac{yx^2}{2} - \frac{(cd+f)^2}{t} = -\frac{(cd+f)^2}{t} + \frac{yx^2}{2}$

8 a) Terms:
$$-x^2$$
, $x - x^3$; $-x^2 + x - x^3 = x - x^2 - x^3 = -x^3 + x - x^2$ (answers vary)
b) Terms: $-a^2$, $-2bc$, $\frac{3x}{2}$; $-a^2 - 2bc + \frac{3x}{2} = \frac{3x}{2} - 2bc = \frac{3x}{2} - 2bc - a^2$ (answers vary)
9 (1) - (C), (2) - (E), (3) - (A), (4) - (B), (5) - (D)
10 a) $mn = nm$ b) $-5 \times 7 = 7(-5)$ c) $-cd = d(-c)$ d) $-c(a + d) = (a + d)(-c)$
11 a) $vst = stv = svt = tsv$ b) $v(x - y)t = vt(x - y) = (x - y)vt = (x - y)tv$.
12 a) All these expressions are equal (equivalent) because of the commutative property of multiplication
b) $-3AB = -3 \times 1 \times 2 = -6$; $BA - 3 = 2 \times 1 - 3 = -1$ when $A = 1$ and $B = 2$, thus they are not equivalent,
13 a) $\frac{2 - 5}{7} = \frac{2}{7} - \frac{5}{7}$ b) $\frac{a + 6}{3} = \frac{a}{3} + \frac{6}{3}$ c) $\frac{a - 2}{a + b} = \frac{a}{a + b} - \frac{2}{a + b}$
d) $\frac{ab^2 + cd}{ab^2 - c} = \frac{ab^2}{ab^2 - c} + \frac{cd}{ab^2 - c}$
14 a) $\frac{m + n}{4}$ b) $\frac{7m - n^2}{4x}$ c) $\frac{5m - 2n^2}{4c - 2}$
15 a) $\frac{4 - 7 + 2}{5}$ b) $\frac{7m + n^2 - 3}{4x}$ c) $\frac{m - 3 - t}{s - 1}$
16 a) $\frac{2}{3}x$ b) $\frac{2}{3}x^2$ c) $\frac{-2}{3}x^2$ d) $\frac{-2}{3}(a + 2b)$ e) $\frac{1}{3}x$ f) $\frac{1}{3}(a + 2b)$
17 a) $\frac{3m}{n}$ b) $\frac{3(-m)}{n}$ c) $\frac{a(-b)}{4}$ d) $-\frac{a(-b)}{4}$ e) $\frac{(s - 4)t}{n}$ f) $\frac{4(-a)}{n-1}$
18 a) $-\frac{2a}{a} = \frac{-2a}{-b}$ b) $-\frac{2a + c}{-b} = \frac{-(2a + c)}{(c - 2d)} = \frac{2a + c}{-(c - 2d)}$
19 a) equivalent b) equivalent c) not equivalent d) equivalent e) equivalent f) equivalent
20 a) $\frac{2x + 4y}{3}$ b) $\frac{2x - y}{3}$ c) $\frac{2x - 7y}{3}$ d) $\frac{3x - y}{t}$ e) $\frac{-x - 4y}{3}$ f) $\frac{-2x + 3y}{t}$ f) $\frac{-2x + 3y}{t}$

23 a)
$$ab + 2c = ba + 2c = 2c + ab = 2c + ba$$

b)
$$\frac{2-x}{4} = \frac{-x+2}{4} = \frac{1}{4}(2-x) = \frac{1}{2} - \frac{x}{4}$$

c)
$$\frac{x}{y} = \frac{1}{y}x = x \cdot \frac{1}{y} = x \div y$$
 (answers vary)

All of the expressions are equal to
$$\frac{5x}{6}$$
 except $\frac{5}{6x}$

25
$$m-(n)$$
 and $-n+m$ are equivalent to $m-n$:

26 Determine which of the following expressions are equivalent to $x \div 2$:

$$\frac{x}{2}, \quad \frac{4x}{8}, \text{ and } \quad \frac{-x}{-2} \quad \text{are equivalent to } x \div 2$$

$$27 \qquad \frac{1}{6}(3a-b), \quad \frac{3a}{6}-\frac{b}{6}, \quad \frac{-b+3a}{6}, \text{ and } (3a-b)\frac{1}{6} \text{ are equivalent to } \frac{3a-b}{6}.$$

28
$$8a+3$$
, $3+a\cdot 8$, $2\cdot \frac{3+8a}{2}$ are equivalent to $3+8a$

29
$$\frac{m}{-1}$$
, $-\frac{m}{1}$, $m(-1)$ are equivalent to $-m$.

30
$$\frac{-a}{b}$$
, $-a \cdot \frac{1}{b}$, $-\frac{2a}{2b}$, $\frac{a}{-b}$ are equivalent to $-\frac{a}{b}$.

31
$$\frac{m-n}{3}$$
, $\frac{1}{3} \cdot m - \frac{1}{3}n$, $(m-n) \cdot \frac{1}{3}$ and $\frac{-n+m}{3}$ are equivalent to $\frac{m}{3} - \frac{n}{3}$.

32
$$-a+b-c+d$$
 and $d+b-c-a$ are equivalent to $-a-c+b+d$.
33 All except $(-b)(-a)$ are equivalent to $-a-b$.

34
$$32 + \frac{9C}{5}, \frac{9C}{5} + 4 \times 8, \frac{180C}{100} + 32, \frac{-9C}{-5} - (-32)$$
 are equivalent to $\frac{9C}{5} + 32$

35 a)
$$\frac{3x}{y} = x \cdot \frac{3}{y}$$
 b) $\frac{3x}{y} = 3 \cdot \frac{x}{y}$ c) $\frac{3x}{y} = \frac{1}{y} \cdot 3x$ d) $\frac{a+b-c}{y} = \frac{1}{y} \cdot (a+b-c)$

36 a)
$$a-2b+c = c-2b+a$$
 b) $x = \frac{x}{4} \cdot 4$ c) $\frac{xy}{4} = \frac{1}{4}xy$ d) $a \div 2 = \frac{a}{2}$
e) $\frac{x+y}{3} = \frac{x}{3} + \frac{y}{3}$ f) $z-c = -c+z$ g) $-xyz = yz(-x)$ h) $\frac{ab}{2} = a \cdot \frac{b}{2}$

37 a)
$$-4+a$$
 b) $-32a$ c) $2a$ d) $64x$ e) not possible f) not possible
g) $2x$ h) $2x$ i) 3^m j) x^2 k) $-xy^2$ l) not possible
m) $2x$ n) $2x$ o) $\frac{8}{5}-x$ p) $\frac{6y}{xz}$ q) $-1-x$ r) $\frac{bd}{3ac}$

38
$$x^2 + y^2 = 5$$
, $(x + y)^2 = 1$, when $x = -1$, $y = 2$. They are not equivalent.

39
$$m-n+p=-2$$
, $m-(n+p)=-4$, when $m=2$, $n=5$, and $p=1$. They are not equivalent.

40
$$(-x)^4 = 1, -x^4 = -1.$$

41 a)
$$x(y-z) = 0$$
, $xy - z = 2$ when $x = 2$, $y = 2$, and $z = 2$. Thus, they are not equivalent
b) $x(y-z) = 0$, $xy - z = 0$ when $x = 1$, $y = 1$, and $z = 1$. There is no contradiction. If there is one
set of values of variables for which two expressions are not equal (and we found such in (a)), we determine
that the two expressions are not equivalent, even if for some other values of variables the expressions are
equal.

42
$$(x+2y)(1+3a)$$
 and $x+2y(1+3a)$ are not equivalent. In $(x+2y)(1+3a)$ the entire expression of $(x+2y)$ is multiplied with $(1+3a)$ and in $x+2y(1+3a)$ only 2y is multiplied with $(1+3a)$.

43 a)
$$(-1)^m = -1$$
, $-1^m = -1$, when $m = 1$
c) $(-1)^m = -1$, $-1^m = -1$, when $m = 5$
d) $(-1)^m = -1$, $-1^m = -1$, when $m = 7$

e) No, we cannot. f) $(-1)^m = 1$, $-1^m = -1$, when m = 2. Yes, we can, they are not equivalent. It is enough to find one set of values of variables for which two expressions are not equal to determine that they are not equivalent.

- 44 The two algebraic expressions are not equivalent, (b)
- 45 No conclusion can be reached, (c)

1 We did not prove that these rules really apply to EVERY real number. We only saw how and why they work in some examples.

- 2 .coefficient, exponent or power, the base.
- 3 a) first b) zero

4 a) b b) ab c) de d) -a c) a f)
$$\left(\frac{2x}{y}\right)$$

5 a) $\left(\frac{a}{2}\right)^4$ b) $\left(\frac{2}{3}a\right)^3$ c) $\frac{c^3}{7}$ d) $(5a)^2$ c) $8a^2$ f) $(-x)^5$ g) $-x^{10}$
6 base exponent coefficient
a) x 4 3
b) x m -1
c) x 3 $\frac{2}{3}$
d) $a-bc$ 2 -1
e) $\frac{x}{y}$ m 1
f) $x+y$ 7 $\frac{1}{4}$
g) $\frac{3x+z}{w}$ 7 $\frac{3}{4}$
h) ab 5 $-\frac{1}{2}$
7 a) 6^5 b) z^4 c) $3^3 \cdot a^4$ d) $-x^5y^2$ e) $-a-a^4$ f) x^2y-xy^2
g) $(a+b)^3$ h) $(2t^3)^4$ i) not possible j) $\frac{k^3}{n} \cdot k = \frac{k^4}{n}$ k) $\frac{-z-z-z}{z^4}$
l) $\frac{(-z)^3}{z+z+z}$ m) $x^3 \cdot \frac{x}{2} = \frac{x^4}{2}$ n) $(3-x)^3$ o) $\left(\frac{2a}{b}\right)^3$ p) $\left(-\frac{3}{x}\right)^3$
q) $\frac{1}{(w+2v)^3}$ r) $\left(\frac{x-y}{m}\right)^2$ s) $(m+n)^2m+p$ t) $(m+p-n)^2(n+m-p)$
8 a) $(-4)(-4)(-4)(-4)(-4)$ b) $-4\cdot4\cdot4\cdot4\cdot4$ c) $(-m)(-m)(-m)$ d) $-m\cdotm\cdotm$
e) $(2a)(2a)(2a)$ f) $2\cdot a\cdot a\cdot a$ g) $(a+b)(a+b)$ h) $a+b\cdot b$
9 a) 1 b) 3 c) x d) a e) ab f) 1 g) ab+1 h) 1
10 a) x^3 b) $(-x^4$, necessary c) $-x^7$ d) $a+(2b)^3$, necessary
e) $(a+2b)^3$, necessary f) $a+2b^3$ g) $a(bc)^m$, necessary h) $\left(\frac{2x}{y}\right)^4$, necessary

The answers are different, since the order of operations is different. In part b, we must first complete operations within parentheses.

12 a)
$$(-m)(-2m^2) = 2m^3$$
 b) $\frac{3x^5}{18x^3} = \frac{1}{6}x^2$ or $\frac{x^2}{6}$ c) $(2a^5)^3 = 8a^{15}$

8

1 Based on the Commutative Law of Multiplication: (a + b)c = c(a + b), and from here we can apply the Distributive Law: c(a+b) = ca+cb. Based on the Commutative Law of Addition xb + yb + xc + yc = xb + xc + yb + yc, so the two 2 answers are equivalent. Some other ways (answers vary): (x + y)(b + c) = xb + xc + yc + yb = yc + yb + xb + xc = yc + xc + yb + xb = xc + xb + yc + yba) $(x^2 - y) \cdot 5 = 5x^2 - 5y$ b) -(-4x + 1) = 4x - 1 c) a(-c + 1) = -ac + a3 d) 2y(-a+2b+d) = -2ya+4yb+2yd e) $(x-1)(y^3+2) = xy^3 - y^3 + 2x - 2$ f) $-(x-x^2+2x^4) = -x+x^2-2x^4$ a) $3a + 3b = 3b + 3a = (a + b) \cdot 3 = a \cdot 3 + b \cdot 3 = b \cdot 3 + a \cdot 3$ 4 (answers vary) b) $2z - yz = -yz + 2z = z(2 - y) = z \cdot 2 - zy = 2z - zy$ (answers vary) (mn + mp)(n+p)m(pm+nm) mn+p5 $\langle mp + nm \rangle$ (-1)(x-y+z) (x-y+z)(-1) (x-y+z) (x-y+z)-16. -x+y-z -x-y+z -x+y+z -(x+z-y)a) 2L + 2W b) R - Rx c) $P + \Pr t$ d) $R^2 s - r^2 s$ e) $2ac + c^2$ f) $x^3 - 7zx$ 7 a) $\frac{5}{9}F - 10$ b) $18y^5 + 24y^4$ c) $x^6 - 2x^5$ d) -2c + 22d e) $a^3 + a^2b + a^2b^2$ 8 f) $2x^4y^2 + \frac{3}{7}xy - x^2y^5$ g) xz - yz + xw - yw h) $10a - 4b - a^4 + \frac{2}{5}a^3b$ i) ac + bc - gc - ad - bd + gd j) $x^{3}y + x^{2}y + xy + y - x^{3} - x^{2} - x - 1$ k) 1) 2ac - c - 2abc + bc - 4a + 2 + 4ab - 2b2xy - 4y + 3x - 6a) 2 9 b) 3 c) m a) 5(x + y)e) c(3-2c)b) 7(1-7a)c) 2(lw+lh+wh) d) -11(t-4)10 f) $x(4x^2-5x+5)$ g) $y(-8xy^5+1)$ a) $xy(2-a^2)$ b) xy(-x+y) c) xy(a+b-1)11 12 a) -1(-3-x) b) -1(a-b-1) c) $-1\left(-a+\frac{x+y-z}{2}\right)$ a) 5a(2-3a) b) $11t\left(-\frac{1}{2}t+4\right)$ c) $5x^{3}(3x^{2}+1)$ d) $-4y^{5}(2xy-1)$ 13 e) 2xy(1-y+2xy) f) $-3x^2y(1-3xy)$ g) $a^3b^3(b+5a^4-1)$ h) $17xy(x^4y^2 + 2x^2y + 3)$ i) $-7ab^2(5b + 2ab^2 - 3)$ j) $a^3b^3(b + 5a^4 - 1)$ k) $-4ac^{3}(4-2c^{4}+3c^{5}d)$ a) $\frac{2}{3}(x^2y-2z)$ b) $-\frac{1}{5}\left(x+\frac{1}{5}\right)$ 14 a) (a+b)(6-x)b) (a+b)[4-3(a+b)]15

16 a)
$$(x-2y)(-2z+z^2)$$
 b) $(c-d)[3a(c-d)-(a+b)^6]$ c) $(c+d)^2[1-4a(c+d)]$
d) $(b^3+c)^3[1-6(b^3+c)^3+8(b^3+c)]$ e) $(cd)^2(8-ad)$
17 $a\left((1-a+\frac{3}{2})\right)$

17
$$a\left(\left(1-a+\frac{3}{a}\right)\right)$$

18 a)
$$2(2-\frac{x}{2})$$
 b) $x\left(\frac{4}{x}-1\right)$ c) $2x\left(\frac{2}{x}-\frac{1}{2}\right)$ d) $4x\left(\frac{1}{x}-\frac{1}{4}\right)$

a) Numerator: One term, t. (can be viewed as two factors, 1 and t) Denominator: two terms: 2t (with factors 2 and t) and -ty (with factors -1, t and y). Therefore t is a common factor of ALL terms (numerator and denominator.) We can divide the numerator and the denominator by t.
b) Numerator: two terms, x (with factors 1 and x) and xy (with factors x and y) Denominator: two terms, 2x (with factors 2 and x) and -ax (with factors -1, a and x). ALL terms in the numerator AND in the denominator have a common factor, x. We can therefore divide both the numerator and denominator by x.
c) Numerator: one term, 3ab (with three factors 3, a and b). Denominator: two terms, ab (with factors a and b).

b) and -a (with factors -1 and a). ALL terms in the numerator AND in the denominator have a common factor, a. We can therefore divide both the numerator AND the denominator by a.

- 20 x is NOT a factor in the denominator, but it can be viewed as a factor in the numerator. We can NOT cancel x, because it is not a factor in the denominator.
- 21 x IS a factor in BOTH the numerator and the denominator, therefore we can cancel x. The result is $\frac{7}{xy}$
- 22 (a-b) IS a factor in BOTH the numerator and the denominator, therefore we can cancel it. The result: x 23 (a-b) can be viewed as a factor in the denominator, but NOT in the numerator. We can not cancel it.

24 a)
$$\frac{1}{3}$$
; $3xy$ b) $\frac{-1}{b^2}$; a^2 c) $\frac{1}{7}$; $(a+b)$ d) $\frac{c}{4}$; $2ab$
e) $\frac{3(a-b)}{5}$; $5x$ f) $\frac{a(b-c)}{2}$; a g) $\frac{xy}{4}$; $5y^3$ h) $-20x^2$; x
i) $\frac{1}{x+y}$; 2 j) $\frac{y+z}{3}$; x k) $4-5x$; x l) $x+y$; 2
m) $\frac{1}{1-3x}$; $3x$ n) $a^2b-4b^4a^3$; a o) not possible p) $c(b+e)$; b
25 a) -1 b) $\frac{1}{2-x}$ c) not possible d) not possible e) 4 f) $-x+3y+2z$
g) $\frac{1-y+x}{3}$ h) -1 i) $3(u-v)$ j) $3x^3-6xyz$

Lesson 6

1 Yes.
2 No. (All three are unlike)
3 b) and d)
4
$$6x^2y^2$$
 $-2x^2y$ $5xy$ $0.3yx^2$
5 $5(ab)^2$ $2a^2b$ $-\frac{3}{7}ab^2$ $\frac{1}{7}ba^2$
6 yx^2 xyx $2xy$ y^2x xxy

7
$$a^{2}b^{5}a^{3}$$
 $(-b^{2}a^{5}b)$ $(ab)^{3}b^{3}$ $(2(ab)^{3}a)$ $a^{3}b^{5}$
8. a) $2x$ b) not possible c) 0 d) not possible c) $2xt$ or $2ts$ i) $2ac^{2}$
g) not possible b) $6hmv$ i) $3xy^{3}z^{5}$ j) $9m^{3}n$ k) not possible i) $2a^{2}b^{3}$
9 a) $(3-4)x = -x$ b) $(\frac{1}{3}-\frac{2}{3})x = \frac{1}{21}x$ c) $(\frac{2}{11}-\frac{3}{22})x = \frac{1}{22}x$
d) $(0.3-0.5)x = -0.2x$ c) $(-\frac{7}{9}-\frac{2}{5})x = -\frac{53}{45}x$ f) $(\frac{1}{5}-\frac{2}{3}+\frac{3}{10})x = -\frac{1}{6}x$
10 a) $-x$ b) $-5a$ c) x d) $-x$ e) $-7ab$ f) not possible g) not possible h) $-1.2x^{2}y$
11 a) $-3x-8x-2x+8y = -13x+8y$ b) $-5a-7a-3b-2b = -12a-5b$
c) $-2ab-4ba+3ab+2-1 = -3ab+1$
12 a) j b) $2+\frac{1}{3}a$ c) $0.7z$ d) $-2-7m$ c) $-2x^{3}+x^{4}$ i) $3x-y$
g) $-x-1$ h) $y-\frac{1}{2}x$ i) $\frac{2}{3}cd-d+c$ j) $-4b^{2}a^{3}-5a^{2}b^{3}$ k) $-\frac{11}{38}ab+5\frac{7}{8}$
13 $x+y$; a) -4 b) $-\frac{9}{5}$ c) -2.7
14 x ; a) 4 b) -10 c) $\frac{2}{3}$.
15 a) $18a-2$ b) $2t+10$ c) $4y-6$ d) $1.3a-0.2$ e) $-\frac{8}{5}x+1$ f) $-29q-48$
g) $-2x+1$ h) $27a+2$ i) $-a-\frac{41}{5}b$ j) $-10x-9y$ k) $2a-5d$ i) $-a^{2}-3$
m) $3xy-3$ n) $-5bc+abc-2a$ o) $-\frac{1}{3}b$ p) $6x^{4}+3x^{2}+2$ q) $12x^{2}+6$
16 a) $(3x-1)+(-4x+2)=-x+1$ b) $(-4a^{3}+2a)-(4a^{3}-2)=-8a^{3}+2a+2$
c) $(-a+2+3b)-(-2b+a)=-2a+5b+2$ d) $3yx-(-2xy)=5xy$
e) $\left(a-\frac{1}{3}\left(\frac{2}{5}-2a\right)=-2a^{2}+\frac{16}{15}a-\frac{2}{15}$ f) $(2x^{2}-y)(3y-x^{2})=-2x^{4}+7x^{2}y-3y^{2}$
g) $-mnk+4nmk+(-3mn)=3mnk-3mn$ h) $(3x+2)^{2}=9x^{2}+12x+4$
i) $(2a-b)^{2}=4a^{2}-4ab+b^{2}$ j) $(5x+2)+2(-4x+1)=-3x+4$
17 a) $x^{8}+4x^{4}+4$ b) $9x^{2}-6x+1$ c) $a^{2}+2a-8$ d) $3b^{2}+\frac{1}{2}bc-\frac{1}{2}c^{2}$ c) $-c+\frac{29}{3}x$
f) $2x^{3}-5x^{2}+x+2$ g) $a^{5}-2a^{3}b+a^{2}b-2b^{2}$ h) $\frac{4}{9}-\frac{8}{3}x+4x^{2}$ i) $3b-2c$
18 $-3a-3b$; a) 9 b) -\frac{3}{2}ar -1\frac{1}{2} c) 54

		1				
1	a) 20	b) $\frac{1}{27}$	c) - 32			
2	a) 6	b) $-36 c$) 4				
3	a) – 4	b) – 4	c) 4			
4	a) –14	b) $-\frac{2}{7}$	c) 2	d) 4		
5	a) $-\frac{2}{7}$	b) $\frac{2}{7}$	c) $\frac{2}{7}$	d) $\frac{4}{49}$		
6	a) - 7	b) 7	c) -7	d) – 7	e) -7	
7	a) 1	b) -1	c) -1			
8	a) – 3	b) 3	c) $\frac{1}{3}$	d) 6	e) 27	
9	a) $-\frac{1}{3}$	b) $\frac{1}{3}$	c) $-\frac{1}{9}$	d) $-\frac{1}{3}$	e) - 3	
10	a) $-\frac{1}{2}$	b) -1	c) $4\frac{1}{6} = \frac{25}{6}$			
11	a) – 3	b) -0.4	c) -0.12			
12	a) -12	b) -2	c) -2			
13	a) $\frac{2}{3}$	b) $-\frac{2}{3}$	c) $\frac{2}{9}$			
14	a) $\frac{1}{2}$	b) 0.05 or $\frac{1}{20}$	c) 10			
15	a) $\frac{6x}{2} = 3x$					
16	a) $\frac{-3x^2}{2}$ or -	$\frac{3}{2}x^2$ b) $-\frac{7}{2}$	x c) $\frac{27}{2}$	x^{3} d) $-x$		
17	a) x^{2}	b) $25x^2$	c) x^{2}	d) $25x^2$	e) $x^2 + 2x + 1$	f) x^{6}
18	a) $1 - 2x + x^2$	b) x^2	c) $\frac{x^2}{4} + x^2 + x$	$x^2 = \frac{9}{4}x^2$	d) 0	
19	a) $-2x$ b) $-x$	c) $-3x$	c			
20	a) $2s^3 - 2s$	b) $2s^7$	c) $10s^3$ d) s^5			
21	<i>x</i>	0, 10	0 12	4		
22	a) $-10s^4$	b) $8s^{10}$	$-8s^{12}$	c) $-s^{4}$		
23	a) $26s^2$	b) 4 <i>m</i> ²	c) $4m^2$			
24	a) $(2b)^4 = 16b$	b^4 b) $16b^4$ c) o	equal d) 0	e) $b^4 - 4b^4 +$	$6b^4 - 4b^4 + b^4 = 0 \text{f})$	equal
25	4x					
26	a) – 3 <i>m</i>	b) $\frac{m}{2}$ c)	6 ^{<i>m</i>} d) <i>m</i>	³ e) <i>m</i>		
27	a) y b) -3	$3y^2$ c) y	d) - 5 y			
28	(6x-2)(4x+	2)				
29	$\frac{3(x+3z)}{x+3z} = 3;$	after evaluation	remains 3			

30	a) 4	b) 4	c) 4	d) 4	e) – 4
31	a) 1	b) 1	c) 0		

1 One can solve an equation but not an algebraic expression . If the left hand side of an equation is equal to the right hand side of the equation for x = 7, then 7 is called a <u>solution</u>. The <u>solutions</u> of an equation are all values of variables that make the equation true. The statement that contains two quantities separated by an equal sign is called an <u>equation</u>. A <u>solution</u> always makes the <u>equation</u> true.

2. Equations: b)
$$5x = 2$$
 c) $x^2 = 36$ e) $x = -4 + 2x$

- Both Tom's and Mary's answers are correct, because x = 3 is equivalent to both 3 3 = x and -x = -3.
- False. 7 is not a solution of 2(x+1) x = 7. 4

5 None of the numbers is a solution of
$$-x^4 = 16$$
. The number 2 and -2 are solutions of $x^4 = 16$.
6. a) No b) Yes

7	a) Yes	b) No	c) Yes	
8	a) No	b) Yes	c) Yes	d) Yes
9	a) No	b) No	c) Yes	
10	a) No	b) Yes	c) Yes	d) No
11	a) No	b) Yes	c) Yes	d) Yes
12	a) Yes	b) Yes		
13	a) Yes	b) Yes		
14	a) – 5	b) 5 (answer	rs vary.)	
15	E.a	1 		

15 For example
$$x = \frac{1}{3}$$
 and $y = 3$ (answers vary)

- For example x = 2 and y = 3 is a solution, and x = 2 and y = -3 is not a solution. (answers 16 vary)
- We can only divide by a variable if we assume it is not 0. It is better if we always try to avoid dividing by a 17 variable or by an algebraic expression. A better way to solve this equation is: x = 2x2x-2x

$$\begin{array}{l} x - 2x = 2 \\ -x = 0 \end{array}$$

$$x = 0$$

a) x = -3 b) x = 6 c) x = 5 d) x = -2018

19 a)
$$x = -18$$
 b) $x = -4$ c) $x = -\frac{12}{5}$ d) $x = 0$ e) $x = \frac{5}{7}$
f) $x = 6$ g) $x = -1$ h) $x = -1$

20 a) no solution
d) no solution
21 a)
$$x = -2$$
 b) $a = -60$ c) $x = 7$ d) $x = 0$ e) $a = 0$ f) $x = 5$
g) $y = 2$ h) $y = \frac{22}{6} = \frac{11}{3}$ i) $x = 0$ j) $x = -6$ k) $x = -\frac{2}{5}$
l) $a = 10$ m) $B = \frac{1}{6}$ n) $x = 3$ o) all real numbers p) $x = -2$
q) $x = \frac{1}{3}$ r) no solution s) no solution t) no solution u) $x = \frac{7}{5}$

1	a) $x = -18$	b) $x = -\frac{2}{15}$ c	$) x = -\frac{5}{16}$	d) $x = \frac{8}{5}$	e) $y = -2$	f) $x = \frac{1}{6}$
2	a) no solution	b) $x = -$	$\frac{2}{5}$	c) $a = \frac{1}{4}$	d) $x = -\frac{1}{6}$	e) no solution
	f) $x = -\frac{1}{4}$	g) $x = 0$	h) <i>x</i> =	$\frac{10}{21}$	i) all real numbe	ers
	$j) \ x = \frac{4}{9}$	k) $x = \frac{13}{8}$	3	1) $x = -\frac{9}{2}$		
3	a) $x = 0$	b) $x = \frac{1}{5}$ c) $x = 0$			
4	a) $x = -8$	b) $x = -\frac{2}{7}$ c	$x = \frac{11}{2}$			
5	a) no solution	b) $a = \frac{1}{7}$ c	$a = -\frac{2}{7}$			
6	No. $\frac{x+1}{2} - 5y$	is not an equation, s	so we can not s	olve it.		
7	a) $x = 14$	b) <i>x</i> = <i>ab</i>				
8	a) $x = 5$	b) $x = b - a$				
9	a) $x = 6$	b) $x = \frac{c+b}{a}$				
10	a) $x = 2$	b) $x = \frac{c}{a-b}$				
11	a) <i>x</i> = 1	b) $x = \frac{bc}{a-b}$				
12	a) $x = -a$	b) $b = a^2$	$c^{2}c$	c) $a = \frac{b^4}{c}$	d) <i>a</i> =	$\frac{3b}{x}$
	e) <i>u</i> = <i>a</i>	f) $b = a^2$	С	g) $m = 2n^3$	h) <i>y</i> =	t + x
	i) $y = x^2$	j) $A = \frac{1}{x}$	$\frac{-x}{-1} = \frac{-(x-1)}{x-1}$	$\frac{1}{2} = -1$	k) <i>x</i> =	<i>s</i> + 1
	1) $a = -\frac{5by}{2x}$	m) $v = \frac{s}{3}$	$t + t$ or $v = \frac{s}{s}$	$\frac{+3t}{3}$	n) $x = \frac{1}{2a+2}$	
	o) $x = \frac{m-2x}{a(2n-x)}$	$\frac{n}{m} = \frac{-(2n-m)}{a(2n-m)}$	$=-\frac{1}{a}$			
13	a) $d = \frac{x}{y} - e$	b) $x = y($	(d+e)	c) $y = \frac{x}{d+e}$		
14	a) $b = t - at$	b) $a = \frac{t}{-1}$	$\frac{-b}{t}$ or $a = 1$ -	$-\frac{b}{t}$ c) $t = \frac{b}{t}$	$\frac{b}{1-a}$ or $t = \frac{-b}{a-b}$	<u>b</u> -1
15	$x = a^9 = -1$					
16	n = -m = 4					

17 a)
$$h = \frac{2A}{b}$$
 b) $h = 4$ inches

18 a)
$$W = \frac{P - 2L}{2}$$
 b) $W = 1$ c) $W = 2$ inches

	For example $x = 3$ or $x = -2$ (answers vary). The same numbers could be solutions for the other two inequalities
	For example $x = -1$ or $x = -3$ or $x = -10$ (answers vary.)
	$x = \frac{2}{3}$
	-3, $(-2$, $(-1$, $(0, 1, 2, 3, 4, 5)$
	(-3,) $-2, $ $-1, $ $0, $ $1, $ $2, $ $3, $ $4, $ 5
	666, -6, -0.6
	a) $x < 0$ b) $x \le 0$ c) $x \ge 6$ d) $x \le 6$ e) $x \le 6$
	a) $x < -4$
	b) $x \ge 1\frac{2}{3}$, -3 -2 -1 0 1 $1\frac{2}{3}$ 2 3 4 5 c) $-4 < x$ -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6
	a) All numbers that are at least -2
	-4 -3 -2 -1 0 1 2 3 4 5
	b) All numbers no more than 4
	-4 -3 -2 -1 0 1 2 3 4 5 c) all non-negative numbers
	-4 -3 -2 -1 0 1 2 3 4 5 d) All numbers that are at most -1
	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
0	a) $x < -1$ b) $x > -4$ c) $x \ge -1$ d) $x \le -1$

- 4) ——				+			
-3 -2	2	-1	0	.2	1	2	2	3	4	ł	5	
a) $x \ge 3$ or	<i>x</i> < 7	answe	ers vary))								
-4 -3	-2	-1	0	1	2	• 3	⊷•– 4	5	6	7	8	
b) $x \ge -2$ o	$r x \leq$	3 (ans	wers var	ry)								
-4 -3	-2	-1	0	1	2	⊷• 3	4	+ + 5	6	7	8	•
a) $x > 4$ or	<i>x</i> < 0) (answ	ers vary)								
-4 -3	-2	-1	0	1	2	3	4	5	6	7	8	
b) $x > 2$ or	<i>x</i> < -	$-\frac{1}{2}$ (at	nswers v	vary)								
	 		• <u> </u>		•-•	· · ·		· · ·		· · · -	· · ·	•
-4 -3	-2	-1	÷ 0	1 1	÷2	3	4	5	0	7	8	
$-4 -3$ $x \le 0 (answe$	-2 ers vary.	-1 .)	$\frac{1}{2}$ 0	1 1	i 2	3	4	5	0	7	δ	
$ \begin{array}{rrrr} -4 & -3 \\ x \leq 0 & (answed) \\ x \leq 4 & (answed) \end{array} $	-2 ers vary ers vary	-1	1 <u>2</u> 0	1 1	2 2	3	4	5	0	7	δ	
-4 -3 $x \le 0$ (answer $x \le 4$ (answer a) -5 + 2 < 4	-2 ers vary ers vary and	-1	$\frac{1}{2}$ 0 + 8 ≥ 13	1 1 3 ; -	2 2 - 5 is n	Э ot a solı	4 ution of	f(c) and	o (d)	7	8	
$-4 -3$ $x \le 0 (answer)$ $x \le 4 (answer)$ $a) -5 + 2 < 4$ $-3 < 4$	-2 ers vary ers vary and	-1	$\frac{1}{2}$ 0 + 8 ≥ 13 ≥ 13	1 1 3 ; -	+ <u>2</u> 2 - 5 is n	Э ot a solu	4 ution of	f(c) and	(d)	7	ð	
-4 -3 $x \le 0$ (answer $x \le 4$ (answer a) -5 + 2 < 4 -3 < 4 a) yes b) 1 10 100	-2 ers vary ers vary and b) no	$-1 - \frac{1}{2}$ b) 5 - 13 2 (there a	$\frac{1}{2}$ 0 + 8 \ge 13 are infin	1 1 3 ; -	$\frac{1}{2}$ 2 - 5 is n any sol	3 ot a solu utions)	4 ution of c) in	5 finitely	(d) many	7	ð	
-4 $-3x \le 0 (answerx \le 4 (answera) -5 + 2 < 4-3 < 4a) yesd) 1, 10, 100$	-2 ers vary ers vary and b) no e) -	$-1 - \frac{1}{2}$ b) 5 - 13 2 c (there a 1, -3	$\frac{1}{2}$ 0 + 8 \ge 13 are infin 3, -5	1 1 3 ; - itely m (answ	$\frac{1}{2}$ 2 - 5 is n any sol wers van	3 ot a solu utions) ry.)	4 ution of c) in f) ye	5 f(c) and finitely s g) 1	(d) many no h)	no	i) yes	
-4 -3 $x \le 0 (answer x \le 4 (answer a) -5 + 2 < 4-3 < 4a) yesd) 1, 10, 100a) and e)a) subtract 5: x$	-2 ers vary ers vary and b) no e) -	$-1 - \frac{1}{3}$ b) 5 - 13 2 c (there a 1, -3	$\frac{1}{2} 0$ $+ 8 \ge 13$ are infin $3, -5$ $5 \le 3$	1 1 3 ; - itely m (answ	$\frac{1}{2}$ 2 - 5 is n any sol wers van	3 ot a solu utions) ry.)	4 ution of c) in f) ye	5 finitely g g r	(d) many no h)	no :	i) yes	
-4 -3 $x \le 0$ (answer $x \le 4$ (answer a) -5 + 2 < 4 -3 < 4 a) yes d) 1, 10, 100 a) and e) a) subtract 5; re- $a) divide by 4;$	-2 ers vary ers vary and b) no e) -	-1	$\frac{1}{2} = 0$ $+ 8 \ge 13$ are infin 3, -5 z < 3	1 1 3 ; - itely m (answ	$\frac{1}{2}$ 2 - 5 is n any sol wers van	3 ot a solu utions) ry.) b) add	4 ution of c) in f) ye 2; no s	f(c) and finitely $f(c) = \frac{1}{2} \int_{0}^{2} \frac$	(d) many no h) nge; <i>z</i> < 3	no :: < 3	ð i) yes	
$-4 -3$ $x \le 0 (answer)$ $x \le 4 (answer)$ $a) -5 + 2 < 4$ $-3 < 4$ $a) yes$ $d) 1, 10, 100$ $a) and e)$ $a) subtract 5; rec) divide by 4;$ $e) multiply by$	-2 ers vary ers vary and b) no e) - no sign c no sign	-1	$\frac{1}{2} 0$ $+ 8 \ge 13$ ≥ 13 are infin 3, -5 z < 3 $z > -3$ $z < -3$	1 1 3 ; - itely m (ansv 3 c	$\frac{1}{2}$ 2 - 5 is n any sol vers val	3 ot a solu utions) ry.) b) add e by -1;	4 c) in f) ye 2; no s sign ch	5 finitely s g) r ign chan	(d) many no h) nge; $z < 3$	no : < 3	ð i) yes	
$-4 -3$ $x \le 0 (answere x \le 4)$ $x \le 4 (answere x \le 4)$ $x \le 4 (answere x \le 4)$ $x \le 4 (answere x \le 4)$ $(a) -5 + 2 < 4$ $-3 < 4$ $(a) yes$ $(b) 1, 10, 100$ $(a) and e)$ $(b) 3 (a) and e)$ $(c) divide by 4;$ $(c$	-2 ers vary ers vary and b) no e) - no sign c no sign -3; sign	$-1 - \frac{1}{2}$ b) $5 - \frac{1}{2}$ c) (there a $1, -3$ change; change; changes	$\frac{1}{2}$ 0 + 8 ≥ 13 are infin 3, -5 z < 3 z > -3 z < -2 z < -2	1 1 $3 ; -$ $itely m$ $(answ)$ $3 c$ -3 $-2n s$	$\frac{1}{2}$ 2 - 5 is n any sol wers van d) divid	3 ot a solu utions) ry.) b) add e by -1;	4 ution of c) in f) ye 2; no s sign ch d) n	f(c) and finitely s g) r ign chan nanges; < -1	(d) many no h) nge; $z < z < 3$	no < 3 < 3 < 2 < 3	ð i) yes	
$-4 -3$ $x \le 0 (answer)$ $x \le 4 (answer)$ $x = 4 (answer)$ $x =$	-2 ers vary ers vary and b) no e) - no sign c no sign -3; sign b) <i>n</i> 28	$-1 - \frac{1}{3}$ b) $5 - \frac{1}{3} \ge 0$ (there a $1, -3$) change; change; a > -2	$\frac{1}{2}$ 0 + 8 ≥ 13 ≥ 13 are infin 3, -5 z < 3 z > -3 c; z < - c)	1 1 $3 ; -$ itely m (answ) $3 c$ -3 $-2n < 3$	$\frac{1}{2}$ 2 - 5 is n any sol vers var d) divid < -6	3 ot a solu utions) ry.) b) add e by -1;	4 c) in f) ye 2; no s sign ch d) <i>n</i>	f(c) and finitely s g) r ign chan nanges; < -1	(d) many no h) nge; $z < z < 3$ e)	no < 3 n > 3 2a	i) yes	
$-4 -3$ $x \le 0 (answere x \le 4 (answ$	-2 ers vary ers vary and b) no e) - no sign c no sign -3; sign b) n $-\frac{28}{5}$	$-1 - \frac{1}{2}$ b) $5 - \frac{1}{3} = \frac{1}{2}$ c) (there a $1, -3$ change; change; changes; a > -2 or -1	$\frac{1}{2} 0$ $+ 8 \ge 13$ $\Rightarrow 13$ are infin 3, -5 $z < 3$ $z > -3$ $z < -z$ $c)$ $6 + q \le 3$	$1 1$ $3 ; -$ itely m (answ) $3 c$ $-3 - 2n <$ $5 - 5 \frac{3}{5}$	$\frac{1}{2}$ 2 - 5 is n any sol vers val d) divid < -6	3 ot a solutions) ry.) b) add e by -1; b) -3	ution of c) in f) ye 2; no s sign ch d) n $q \ge -$	f(c) and finitely s g) r ign chananges; < -1 $\frac{6}{5}$	(d) many no h) nge; z < z < 3 e) c)	no <3 n > 3 $\frac{2q}{5} \le $	i) yes $0 \frac{4}{25}$	
$-4 -3$ $x \le 0 \text{ (answard)}$ $x \le 4 \text{ (answard)}$ $x = 4 (an$	$\begin{array}{r} -2 \\ \text{ers vary} \\ \text{ers vary} \\ \text{and} \\ \text{b) no} \\ \text{e)} - \\ \text{no sign c} \\ \text{no sign c} \\ \text{no sign c} \\ \text{no sign c} \\ \text{b) } n \\ -\frac{28}{5} \end{array}$	$-1 - \frac{1}{3}$ b) 5 - 13 2 o (there a 1, -3 change; change; changes a > -2 or -	$\frac{1}{2} 0$ $+ 8 \ge 13$ are infin 3, -5 $z < 3$ $z > -2$ $z < -2$ $c)$ $6 + q \le 4$	$1 1$ $3 ; -$ itely m (answ) $3 c$ -3 $-2n c$ $5 -5\frac{3}{5}$	$\frac{1}{2}$ 2 - 5 is n any sol wers van d) divid < -6	3 ot a solu utions) ry.) b) add e by -1; b) -3	ution of c) in f) ye 2; no s sign ch d) n $q \ge -$	f(c) and finitely s g) r ign chan hanges; < -1 $\frac{6}{5}$	(d) many no h) nge; z < z < 3 e) c)	no < 3 $n > 3$ $\frac{2q}{5} \le$	i) yes $0 \frac{4}{25}$	
$-4 -3$ $x \le 0 (answere x \le 4)$ $x \le 4 (answere x \le 4)$ $x \le 4 (answere x \le 4)$ $x \le 4 (answere x \le 4)$ $(answere x \ge 4)$ $(answe$	-2 ers vary ers vary and b) no e) - no sign c no sign -3; sign b) n $-\frac{28}{5}$	$-1 - \frac{1}{3}$ b) 5 - 13 $\frac{1}{2}$ c) (there a 1, -3) change; change; changes a > -2 or -1	$\frac{1}{2} = 0$ $+8 \ge 13$ ≥ 13 are infin 3, -5 $z < 3$ $z > -3$ $z < -z$ $c)$ $6 + q \le -\frac{4}{5}q \ge 13$	1 1 $3 ; -$ $itely m$ $(answ)$ $3 c$ $-2n <$ $3 - 2n <$ $3 - 2n$	$\frac{1}{2}$ 2 - 5 is n any sol vers val d) divid < -6	3 ot a solu utions) ry.) b) add e by -1; b) - 3	ution of c) in f) ye 2; no s sign ch d) n $q \ge -$	f(c) and finitely s g) r ign chananges; < -1 $\frac{6}{5}$	(d) many no h) nge; z < z < 3 e) c)	no <3 n > 3 $\frac{2q}{5} \le$	i) yes $0 \frac{4}{25}$	
$-4 -3$ $x \le 0 (answerthing)$ $x \le 4 (answerthing)$ $x \le 4 (answerthing)$ $x \le 4 (answerthing)$ $x \le 4 (answerthing)$ $a) -5 + 2 < 4$ $a) yes$ $d) 1, 10, 100$ $a) and e)$ $a) subtract 5; redirectors (answerthing)$ $a) - 6 + q \le 4$ $d) - q \ge -\frac{2}{5}$ $a) and d)$	-2 ers vary ers vary and b) no e) - no sign c no sign c no sign -3; sign b) n - $\frac{28}{5}$	$-1 - \frac{1}{2}$ b) $5 - \frac{1}{13} = \frac{1}{2}$ c) (there a $1, -3$ change; change; change; 2 - 2 or -1 e) -1	$\frac{1}{2} = 0$ $+ 8 \ge 13$ are infin 3, -5 $z < 3$ $z > -3$ $z < -2$ $c)$ $6 + q \le -\frac{4}{5}q \ge 0$	$1 1$ $3 ; -$ $itely m (answ)$ $3 c$ -3 $-2n <$ $5 - 5\frac{3}{5}$ $-\frac{8}{25}$	$\frac{1}{2}$ 2 - 5 is n any sol wers van d) divid < -6	3 ot a solu utions) ry.) b) add e by -1; b) - 3	ution of c) in f) ye 2; no s sign ch d) n $q \ge -$	f(c) and finitely s g) r ign chan nanges; < -1 $\frac{6}{5}$	(d) many no h) nge; z < z < 3 e) c)	no < 3 n > 3 $\frac{2q}{5} \le$	i) yes $0 \frac{4}{25}$	
$-4 -3$ $x \le 0 (answere x \le 4)$ $x \le 4 (answere x \le 4)$ $x \le 4 (answere x \le 4)$ $(a) -5 + 2 < 4$ $(a) -3 < 4$ $(a) - 3 < $	-2 ers vary ers vary and b) no e) - no sign c no sign -3; sign b) n $-\frac{28}{5}$	$-1 - \frac{1}{3}$ b) 5 - 13 $\frac{2}{3}$ o (there a 1, -3 change; change; changes a > -2 or - $\frac{1}{3}$	$\frac{1}{2} = 0$ $+8 \ge 13$ $\frac{1}{2} = 13$ are infin 3, -5 $z < 3$ $z > -3$ $z > -3$ $z < -2$ $c)$ $6 + q \le -\frac{4}{5}q \ge 13$	$1 1$ $3 ; -$ itely m (answ) $3 c$ $-2n <$ $3 - 5\frac{3}{5}$ $-\frac{8}{25}$	$\frac{1}{2}$ 2 - 5 is n any sol wers val d) divid < -6	3 ot a solu utions) ry.) b) add e by -1; b) - 3	tition of c) in f) ye 2; no s sign ch d) n $q \ge -$	f(c) and finitely s g) r ign chananges; < -1 $\frac{6}{5}$	(d) many no h) nge; z < z < 3 e) c)	no <3 n > 3 $\frac{2q}{5} \le$	i) yes $0 \frac{4}{25}$	
$-4 -3$ $x \le 0 (answere x \le 4 (answ$	-2 ers vary ers vary and b) no e) - no sign c no sign c no sign -3; sign b) n $\frac{28}{5}$	$-1 - \frac{1}{2}$ b) 5 - 13 2 c) (there a 1, -3 change; change; changes a > -2 or - e) -	$\frac{1}{2} = 0$ $+ 8 \ge 13$ are infinitions $z < 3$ $z > -3$ $z < -z$ $c)$ $6 + q \le -\frac{4}{5}q \ge 13$	$1 1$ $3 ; -$ $itely m (answ)$ $3 c$ $-3 - 2n <$ $5 - 5\frac{3}{5}$ $-\frac{8}{25}$	$\frac{1}{2}$ 2 - 5 is n any sol wers van d) divid < -6	3 ot a solu utions) ry.) b) add e by -1; b) - 3	ution of c) in f) ye 2; no s sign ch d) n $q \ge -$	f(c) and finitely s g) r ign chan nanges; < -1 $\frac{6}{5}$	(d) many no h) nge; z < z < 3 e) c)	no < 3 n > 3 $\frac{2q}{5} \le$	i) yes $0 \frac{4}{25}$	
$-4 -3$ $x \le 0 (answere x \le 4)$ $x \le 4 (answere x \le 4)$ $x \le 4 (answere x \le 4)$ $(a) -5 + 2 < 4$ $(a) - 3 <$	-2 ers vary ers vary 1 and b) no e) - no sign c no sign -3; sign b) n $-\frac{28}{5}$	$-1 - \frac{1}{2}$ b) 5 - 13 $\frac{1}{2}$ o (there a 1, -3 change; change; changes a > -2 or - $\frac{1}{2}$	$\frac{1}{2} = 0$ $+ 8 \ge 13$ ≥ 13 are infin 3, -5 $z < 3$ $z > -2$ $z < -2$ $c)$ $6 + q \le -\frac{4}{5}q \ge -\frac{4}{5}q = -\frac{4}{5}q \ge -\frac{4}{5}q = -\frac{4}{5}q =$	$1 1$ $3 ; -$ itely m (answ) $3 c$ -3 $-2n c$ $5 -5\frac{3}{5}$ $-\frac{8}{25}$	$\frac{1}{2}$ 2 - 5 is n any sol vers var d) divid < -6	3 ot a solu utions) ry.) b) add e by -1; b) − 3	ution of c) in f) ye 2; no s sign cl d) n $q \ge -$	f(c) and finitely s g) r ign chan hanges; < -1 $\frac{6}{5}$	(d) many no h) nge; z < z < 3 e) c)	no < 3 n > 3 $\frac{2q}{5} \le$	i) yes $0 \frac{4}{25}$	1
$ \begin{array}{rcl} -4 & -3 \\ x \leq 0 & (answer) \\ x \leq 4 & (answer) \\ a) -5 + 2 < 4 \\ -3 < 4 \\ a) yes \\ d) 1, 10, 100 \\ a) and e \\ a) subtract 5; r \\ c) divide by 4; \\ e) multiply by - \\ a) n > 10 \\ a) -6 + q \leq -2 \\ d) -q \geq -\frac{2}{5} \\ a) and d \\ b) and d \\ a) x < -4 \\ \hline -7 & -6 \\ \end{array} $	-2 ers vary ers vary -2 and b) no e) - no sign c no sign c no sign c -3; sign b) <i>n</i> -3; sign -285	$-1 - \frac{1}{3}$ b) 5 - 13 $\frac{1}{2}$ o (there a 1, -3 change; change; changes a > -2 or - e) -	$\frac{1}{2} 0$ $+ 8 \ge 13$ ≥ 13 are infin 3, -5 $z < 3$ $z > -3$ $z < -z$ $c)$ $6 + q \le -\frac{4}{5}q \ge -\frac{4}{5}q = -\frac{4}{5}q =$	$1 1$ $3 ; -$ $itely m (answ)$ $3 c$ $-2n <$ $3 - 2n <$ $3 - 5\frac{3}{5}$ $-\frac{8}{25}$ $-\frac{8}{25}$	$\frac{1}{2}$ 2 - 5 is n any sol vers val d) divid < -6	3 ot a solutions) ry.) b) add e by -1; b) -3	ution of c) in f) ye 2; no s sign ch d) n $q \ge -$	f(c) and finitely s g) r ign chananges; < -1 $\frac{6}{5}$	(d) many no h) nge; z < z < 3 e) c)	no < 3 n > 3 $\frac{2q}{5} \le$ 6	i) yes $0 \frac{4}{25}$, ε

	d) <i>a</i> < 7												~	_
	-6 -5 e) $x > -10$	5 -4	-3	-2	-1	0	1	2	3	4	5	6	7	
	-11 -10 -0 f) $a \le 2$	9 -8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
25	-5 -4 a) "add 3 to both	-3 n sides", "o	-: divide (2 each si	-1 de by -	-1";	a > -7	1 7 -	2		3		4	5
	b) "subtract 1 fro	om both si	des", "	divide	each s	ide by	3"; <i>x</i>	$<\frac{5}{3}$						
26	c) "multiply each d) "multiply each e) "subtract 1 fr f) "multiply each a) $x < -6$	h side by $\frac{2}{3}$ h side by $\frac{2}{3}$ com both s h side by $\frac{2}{3}$ b) $x < \frac{2}{3}$	4", "div 4", "div ides", ' 4", "su –2	vide ea vide ea 'multip btract (c) 2	ch side ch side oly each 1 from $x \ge -4$	by 3" by – h side both s	; x < - 1"; x by 4"; ides"; d) all	$-20 \ge 20$ $x \ge -3$ $x \ge -3$ $real nu$	8 5 ımbers		e) no	soluti	on	
	f) $a < -\frac{3}{5}$	g) $x \ge \frac{1}{2}$	$\frac{4}{3}$	h) 2	$x < \frac{3}{7}$		i) <i>x</i> >	$>-\frac{2}{3}$	j)	all rea	l numt	pers		
	k) all real numbe	ers	l) <i>x</i> ≤	$\frac{9}{2}$	m	n) all r	eal nun	nbers	n)) no sol	ution			
	o) no solution		p) <i>x</i> <	$< -\frac{1}{4}$	q) <i>x</i> ≥	18	r) <u>y</u>	$y \ge 0$	s) <i>a</i>	$>\frac{34}{81}$	 		
Less	<u>on 11</u>													
1	$a = \frac{3}{4}$													
2	a) $a = 3$ and	b = 4		b) <i>a</i>	<i>!</i> = −4	an	d b	$=\frac{2}{3}$						
3	a) $p = -3$	b) <i>p</i> =	3					5						
4	a = -2 and	<i>b</i> = 5												
5	a = -7	D												
6 7	A = 4x ana a) $X = 3a ax$	D = y nd Y =	: h	b)	X = C	3 ar	nd Y	=ab						
8	a) $c = x - 1$	and b=	= 2	0)	b) $c =$		ind I	$b = v^{9}$)					
9	(1)-E; $a = x a$	$nd \ b = 1$	-	(2)-F	; <i>a</i> = .	x ana	l b = 1	. (.	3)-D; a	a = 8 a	nd b	<i>= x</i>		
	(4)-A; $a = 3x$	and $b =$	-5 y	(5)-E	B; <i>a</i> =	3x at	nd b =	= 5 y	(6) - C;	<i>a</i> = 3	and	<i>b</i> = y	,	
10	a) form of $A - I$ c) form of $A + I$	B; 5 - (-B; 5 + (-B))	(n) = 5 (-n) = 5	5+n 5-n	b) f d) f	orm o orm o	f A + A f A - A	B;5- B;5	n = 5 n = 1	5 - (- <i>1</i> 5 + (-	ı) n)			
11	a) $a = -1$ and	<i>b</i> = 3	b) <i>a</i> =	= 2 an	d b =	-3		c) -	$-\frac{1}{2}x^3$	+1; <i>c</i>	<i>i</i> =	$\frac{1}{2}$ and	d b =	1
	d) $2x^3 + \frac{3}{2}$;	<i>a</i> = 2 <i>an</i>	d b =	$\frac{3}{2}$	e)	$(-x^3)$	$+\frac{3}{2};$	<i>a</i> =	-1 <i>an</i>	d b =	$\frac{3}{2}$			

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20 a)
$$a = 3; b = b; 3^{2} + 2 \times 3 \times b + b^{2} = 9 + 6b + b^{2}$$

b) $a = 2x; b = 3y; (2x)^{2} + 2 \times 2x \times 3y + (3y)^{2} = 4x^{2} + 12xy + 9y^{2}$
c) $a = y^{2}; b = \frac{2}{5}; (y^{2})^{2} + 2 \times y^{2} \times \frac{2}{5} + \left(\frac{2}{5}\right)^{2} = y^{4} + \frac{4}{5}y^{2} + \frac{4}{25}$
21 a) not linear b) $\frac{1}{3}x + (-1) = 0; a = \frac{1}{3}; b = -1$ c) $-x + 0 = 0; a = -1; b = 0$
d) $\frac{1}{8}x + 0.9 = 0; a = \frac{1}{8}; b = 0.9$ or $x + 7.2 = 0; a = 1; b = 7.2$ e) not linear
f) $-4x + 7 = 0; a = -4; b = 7$ or $4x + (-7) = 0; a = 4; b = -7$
g) $-x + (-2.5) = 0; a = -1; b = -2.5$
h) $18x + (-5) = 0; a = 18; b = -5$ or $-18x + 5 = 0; a = -18; b = 5$
22 $a = 4; b = 2; 12x + 6 = 0$ where $a = 12; b = 6$
23 a) $(x - 1)(x + 1)$ b) $(2 - 3x)(2 + 3x)$ c) $(y - 10a)(y + 10a)$
d) $\left(x^{2} - \frac{1}{3}\right)\left(x^{2} + \frac{1}{3}\right)$ e) $(xy - 0.5)(xy + 0.5)$ f) $\left(\frac{a}{b} - 6\right)\left(\frac{a}{b} + 6\right)$
g) $[m - (2m + 1)][m + (2m + 1)] = (-m - 1)(3m + 1)$
h) $[(x + 1) - (3x + 5)][(x + 1) + (3x + 5)] = (-2x - 4)(4x + 6)$
i) $[(2a - 3) - 3a][(2a - 3) + 3a] = (-a - 3)(5a - 3)$
24 a) $x^{3} - 4^{3}; a = x; b = 4; (x - 4)(x^{2} + 4x + 16)$
b) $1^{3} - (2y)^{3}; a = 1; b = 2y; (1 - 2y)(1 + 2y + 4y^{2})$
c) $(2x)^{3} - (\frac{1}{3})^{3}; a = 2x; b = 3y; (2x - 3y)(4x^{2} + 6xy + 9y^{2})$
d) $(x^{2})^{3} - (\frac{1}{3})^{3}; a = x^{2}; b = \frac{1}{3}; (x^{2} - \frac{1}{3})(x^{4} + \frac{1}{3}x^{2} + \frac{1}{9})$
25 $10.7^{2} - 9.3^{2} = (10.7 - 9.3)(10.7 + 9.3) = 1.4 \times 20 = 28$

660 students 1 2 3.2 minutes 3 15 tablespoons 4 \$25 5 30 pears 250 defective bulbs 6 $11.25\left(11\frac{1}{4}\right)$ ounces 7 a) \$1.75 24 fluid ounces 8 b) 3 pencils 9 10 \$35.10 $\frac{1}{32}$ or 0.03125 pounds 11 $\frac{3}{10}$ or 0.3 meters 12

14 0.65 centimeters 0.125 tons $\frac{4}{3}$ 0.6 3% 1000 % $\frac{1}{10}$ or 0.1 $\frac{20}{3}$ 375% -1 $\frac{16}{11}$ -12 $-\frac{5}{8}$
APPENDIX B:

SAMPLE TESTS

Sample Tests: Lesson 1-3

Sample Test 1 (Lesson 1-3)

1. Write the following statements as algebraic expressions. Remember to place parentheses where needed (please, place them only when needed) :

a) two seventh of *xy*

- b) the product of x and y + 2
- c) *xy* raised to fourth power
- d) Subtract A from B, and then multiply by C
- e) take the opposite to x, and then raise it to the third power

2. Remove parentheses, if unnecessary. In cases when removing parentheses would change the order of operations (and thus parentheses cannot be removed), indicate so.

a) 2(x-3) b) (2x)-3 c) $(bc)^5$ d) a(-c)

3. Let m = -2. Rewrite the expression replacing the variable with its value (Remember to place parentheses) and evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a)
$$-m$$
 b) $5m$ c) $-m^4$ d) $\frac{1}{2+m}$

4. Let a = 0.1, b = -0.3. Rewrite the expression replacing the variable with its value (Remember to place parentheses) and evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a)
$$a - b$$
 b) 100 ab c) $\frac{b}{a}$

5. Let $x = \frac{3}{4}$, $y = \frac{1}{5}$. Rewrite the expression replacing the variable with its value (Remember to place

parentheses) and evaluate, if possible. If evaluation is not possible, explain why it is not possible.

a)
$$8x - 5y$$
 b) $5xy$ c) $-\frac{x}{y}$

6. Write an algebraic expression representing the opposite of (do not remove parentheses): $\frac{-A}{-B}$

7. Determine which of the following expressions are equal to -x:

- $x(-1) \qquad -1 \cdot x \qquad \frac{x}{-1} \qquad \frac{-x}{1} \qquad 1 2x$
- 8. Write the following expressions as a single fraction: $-\frac{2}{3}x \frac{1}{3}y$.
- 9. Fill in the blanks to make a true statement:

a)
$$-\frac{A}{y} = -A \cdot _$$
 b) $\frac{2a+b}{c} = \frac{2a}{c} + _$ c) $\frac{3a}{4} = a \cdot _$

10. Perform all numerical operations that are possible. If none is possible, clearly indicate so:

a)
$$-4+2x$$
 b) $(-4x)(-2)$ c) $-(-2x^2)$ d) $\frac{3x}{9}$ e) $-12 \cdot \frac{x}{3}$

Sample Test 2 (Lesson 1-3)

1. Rewrite the following expressions removing all unnecessary parentheses, if possible (if not possible, clearly indicate so).

c) 3b + (a + 2)

- a) 5 (ab) b) 3x (5 y)
- 2. Write the following statements as algebraic expressions:
 - a) x raised to the 4^{th} power, and then subtracted form y.
 - b) the sum of 5 and three-forth of a number.
 - c) the opposite of b+4
 - d) 6 less than the square of a number.
- 3. Let m = -3. Evaluate the following expressions if possible (if not possible, clearly indicate so):

a)
$$m^2$$
 b) $\frac{m+5}{2}$ c) $\frac{m+1}{m+3}$ d) $3-2m$

4. Evaluate uv^2 if possible (if not possible, clearly indicate so), when $u = \frac{2}{3}$ and $v = \frac{1}{3}$.

5. Evaluate $\frac{m}{p} - n$ if possible (if not possible, indicate so), when m = -0.1, n = 0.02, p = -1.

- 6. Let F be a variable representing the temperature in Fahrenheit. Use F to write the following statement as an algebraic expression: five-ninth times the difference of F and 32.
- 7. Perform all numerical operations if possible (if not possible, clearly indicate so):

a)
$$\frac{10b}{-5}$$

 $2x^{3}(2-5)-4$
b) $2+3c$
c) $x-(-3)^{2}$
d) $\frac{2x^{3}}{3}\left(-\frac{9y}{5}\right)e$

- 8. Write $\frac{-5a}{2b}$ in three different ways.
- 9. Determine which of the following expressions are equivalent to $\frac{2a}{b}$

$$2 \cdot \frac{a}{b}$$
, $2a \div b$, $\frac{1}{b} \cdot 2a$, $\frac{-2a}{-b}$, $(1+1a) \div b$, $\frac{(1+1)a}{b}$

10. Determine which of the following expressions are equivalent to mn - k:

$$k-mn$$
, $nm-k$, $mn+(-k)$, $m(n-k)$, $-k+mn$

11. Replace Ψ with expressions such that the resulting statement is true. Use parentheses when needed.

a)
$$-xyz = yz\Psi$$
 b) $A = 7 \cdot \frac{A}{\Psi}$ c) $\frac{a+b}{c} = \frac{\Psi}{c}(a+b)$

Sample Test 3 (Lesson 1-3)

1. Use the letter x to represent a number and write the following statements as algebraic expressions.

- (a) Half of a number is subtracted from 7.
- (b) Twice a number is added to 5, and then the result is multiplied by 3.
- (c) The number is tripled, and then squared.
- 2. Write the opposite. a) -2x + 3y b) $\frac{x y}{x + y}$
- 3. Remove parentheses if it would not change the presumed order of operations.
- (a) 3(x+y) (b) 5(x)y (c) $-4(x)^2$ (d) $(-4x)^2$ (e) (3x)+y4. If possible (if not possible, clearly indicate so), evaluate -3xy when

(a)
$$x = -6$$
 and $y = 2$ (b) $x = \frac{5}{6}$ and $y = \frac{3}{4}$ (c) $x = -0.1$ and $y = -0.2$

5. If possible (if not possible, clearly indicate so), evaluate $-\frac{x}{y}$ when

(a)
$$x = -6$$
 and $y = 2$ (b) $x = \frac{5}{6}$ and $y = \frac{3}{4}$ (c) $x = -0.1$ and $y = -0.2$

6. If possible (if not possible, clearly indicate so), evaluate $\frac{x}{1-x}$ when

(a)
$$x = 0$$
 (b) $x = 1$ (c) $x = -1$

7. State which expressions are equivalent to $\frac{x-y}{2}$

(a)
$$x - y \div 2$$
 (b) $(x - y) \div 2$ (c) $\frac{x}{2} - \frac{y}{2}$ (d) $\frac{-y + x}{2}$ (e) $\frac{1}{2}(x - y)$

8. Replace X with expressions such that the resulting statement is true. Use parentheses when needed.

a)
$$-x + 2y - 3z = -3z + 2y + X$$
 b) $3 \cdot \frac{a}{y} \cdot b = \frac{X}{y}$ c) $\frac{m+n}{v} = \frac{n}{v} + X$.

9. Perform all possible numerical operations and simplify completely.

(a)
$$-5 - (6 - 2)x$$
 (b) $-6(2x)$ (c) $(6 \div 2)x$ (d) $\frac{2}{3} - \frac{5}{6}x$ (e) $\frac{2}{3} - \frac{5}{6} - x$
10. Show that $(-x)^2$ is not equivalent to $-x^2$ by evaluating both expressions when $x = -\frac{2}{3}$.

Sample Tests: Lesson 4-6

Sample Test 1 (Lesson 4-6)

1. Write the following statements as an algebraic expression using parentheses where appropriate, and then remove parentheses:

- a) the product of A and B A + 3
- b) The opposite of -x + 2

2. Factor $2a^3$ from the following expression: $-8a^5b + 10a^3$

3. Factor $\frac{2}{7}$ from the following expression: $\frac{4}{7}a - \frac{2}{7}$

4. If possible, add (or subtract) the following expressions. If not possible, clearly say so:

a)
$$2hk - \frac{1}{2}kh$$
 b) $ab - ab^2$ c) $mn^2 - nmn$ d) $\frac{x}{3} - \frac{1}{6}x$

5. Collect like terms: -4yx - y + 3xy + 8y - 7y, and then evaluate if when x = -2, $y = \frac{1}{4}$.

6. Remove parentheses and then collect like terms

a)
$$-7b - \frac{1}{3}(3b - 6)$$
 b) $-4x - (x + 2)$ c) $(3x - b)^2$

7. Simplify, if possible (assume that all denominators are different from zero). If not possible, clearly say so.

a)
$$\frac{ax-b}{ax}$$
 b) $\frac{xy^3}{x}$ c) $\frac{2(a+b)}{b+a}$ d) $\frac{2c}{xy} \cdot xy^2$ e) $\frac{x-2}{2-x}$

8. Write as a single exponential expression. For each expression, identify the numerical coefficient.

a)
$$\frac{(-a)^5}{a^3}$$
 b) $x^7 (2x^2)^3$ c) $\frac{2ab^3}{(2ab)^3}$

9. Evaluate the following expressions:

a)
$$\frac{2^{45}}{2^{43}}$$
 b) $3a^{0}$

10. Find such X that the following is true:

a)
$$4^7 = 2^x$$
 b) $\left(\frac{1}{3}\right)^x = \left(\frac{1}{9}\right)^5$

Sample Test 2 (Lesson 4-6).

1. Write the following statements as algebraic expressions using parentheses if appropriate, and then remove parentheses. Simplify, if possible:

a) The product of 2a - 3 and $-3a^3 + 4a$

b) 2 times the opposite of -3y+9

c) The difference of *x* and 2, then squared.

- 2. Factor *b* from the expression: $9b^2 5b$
- 3. Factor $5x^3$ from the expression: $15x^7 + 5x^3$
- 4. Factor $\frac{2}{3}$ from the expression: $\frac{4}{3}y^2 + \frac{2}{9}y \frac{8}{3}$
- 5. Collect like terms:

a)
$$-3ab + 4ba - 5ab$$

b) $3x + 4x^2 - 5 + 3x^2 - 6$

6. If possible, add the following expressions. If not possible, clearly say so:

a)
$$-\frac{1}{3}ab^2 + \frac{1}{2}b^2a$$
 b) $-mn^2 + (mn)^2$ c) $-x^2y^2 + (4xy)^2$

- 7. Subtract 2a b from -3a + 2b and simplify.
- 8. Remove parentheses. Simplify:

a)
$$-7x + 2\left(3x - \frac{1}{4}\right)$$
 b) $-2b - (1-b)$

9. Simplify if possible. If not possible, clearly say so:

a)
$$\frac{3a-2b}{2b-3a}$$
 b) $\frac{3}{6b-3}$ c) $\frac{x-2y}{x+2y}$ d) $\frac{a^6}{a^2}$

10. Write as a single exponential expression.

a)
$$2x(-3x^2)(x^3)^4$$
 b) $\left(\frac{x^8}{x^4x}\right)^2$ c) $\frac{a^5}{2a} \cdot 4a^3$

Sample Test 3 (Lesson 4-6)

1. Circle all expressions equivalent to $x^2 y^6$

$$xy^6x$$
 $(xy^3)^2$ $x(y^3)^2$ $y^3x^2y^2$
2. Write as a one exponential expression:

a)
$$\frac{3x^4}{(2x)^2}$$
 b) $\frac{2}{a^3} \times a^9$ c) $3m(-3m)$

3. Evaluate the following expressions: $\frac{5^{27}}{5^{25}}$.

4. Replace Ψ with a number so the following is equal:

a)
$$9^7 = 3^{\Psi}$$
 b) $25^7 \times 25^8 = 25^{\Psi}$

5. Remove parentheses and identify numerical coefficient:

a)
$$(-2x^3)(-3x^2)x$$
 b) $(2a^2b^4)^3$ c) $(-5b) \cdot \frac{7b^3}{10b^2}$

6. Write each statement as an algebraic expression using parentheses, and then rewrite without parentheses and collect like terms.

a) opposite of -3x + 5yb) product of $x^2 + 5$ and 2x - 3c) sum of 5x - 4y and 2x + 7d) difference of $2x^2 - 7x + 3$ and $5x^2 - 8x - 6$ 7. Factor a) x from the expression $5x^3 - 4x^2 + 7x$. b) -3x from the expression $-15x^2 + 3x$ c) a + z from the expression $4(a + z) - x(a + z)^2$ d) -1 from the expression $\frac{c}{d} - 2$ 8. Simplify, if possible. If not possible then write "not possible".

a)
$$\frac{6xy}{9yx}$$
 b) $\frac{xy + xz}{4x}$ c) $\frac{5x - 3y}{-3y + 5x}$ d) $\frac{x + 4}{x + 6}$

9. Collect like terms, and then evaluate, when a = -3.

a)
$$-2a - 7a - (4 - 8a)$$
 b) $(3 - a)^2 + 6a$

Sample Tests: Lesson 7-9

Sample Test 1 (Lesson 7-9)

1. Evaluate the following expressions, if x = -1, and $y = \frac{3}{4}$ a) xy^2 b) $\frac{x}{y}$ 2. Evaluate a^2bc if a) a = -2, $bc = \frac{1}{4}$ b) $a^2c = -0.4$, b = 0.53. If l + d = 8, evaluate a) 2(l + d) b) 2l + 2d c) $-(l + d)^2$

4. Let x = 3 - m. Express the following expression in terms of m and simplify: $\frac{5+x}{3-x}$

5. Write the following expression 2x - y + z in terms of *a*, if z + 2x = 3m and y = -m. Simplify your answer.

6. Is $x = \frac{2}{3}$ a solution of the following equations? Explain how you arrived at your answer.

a)
$$2-3x = 0$$
 b) $\frac{x}{3} = 2$

7. Is x = -2, y = -3, z = 1 a solution of z + x - y = 2? Explain how you arrived at your answer. 8. Solve the following equations:

a)
$$b + \frac{1}{3} = 2$$

b) $5 - 2x = 1$
c) $2 - x = 2x - 4$
d) $-(x - 4) = 2 - x$
e) $\frac{6y - 4}{2} = 3y - 2$

9. If P = -x + 3 and Q = 2x + 4, find such x, that a) P = Q b) $\frac{P}{2} = Q + 1$

10. Solve the following equations for x. Any time you must perform the operation of division, assume that the devisor is different from zero. Simplify whenever possible:

a)
$$\frac{x}{b} = 2b$$
 b) $ax - b = 2b$ c) $ax + a^2x = a$

Sample Test 2 (Lesson 7-9)

- 1. Evaluate the following expressions, if abc = -2 a) $\frac{-abc}{4}$ b) $a^2b^2c^2$
- 2. Evaluate $x 2 + 3^{x-2}$ if a) x = 4 b) x 2 = 1
- 3. Write the following expressions in terms of a, if x + y = a and xy = a + 1. Simplify your answer.

a)
$$\frac{x}{3} + \frac{y}{3} - xy$$
 b) $\frac{x^2 y^2}{x + y}$

4. Determine whether the following mathematical sentences represent an equation or an algebraic expression. In the case of an equation, circle the right-hand side of the equation.

$$3(x-2) = 3x+4$$
, $5x + \frac{1}{x-2}$, $5x - 7 = 8$, $2+3=5$

5. Which of the following numbers are solutions of $-x^2 = 9$? a) -3 b) 3 c) 9 6. Is y = -1, $z = \frac{1}{2}$ a solution of the following equation: $y - z = -\frac{1}{2}$? Explain how you arrived at your answer.

7. Solve the following equations:

a)
$$-0.6y = -0.4$$

b) $-3x + 1 = 2$
c) $-\frac{2}{5}a = 4$
d) $2 - x = 2x - 4$
e) $2(2c - 1) - 4c = 2$
f) $\frac{x}{5} - \frac{x - 7}{3} = \frac{1}{3}$

8. Let m = 2y and n = 3y. Find y so that the following is true:

a) m + 1 = n b) m = n

9. Solve the following equations for d. Any time you must perform the operation of division, assume that the devisor is different from zero. Simplify your answer whenever possible.

a)
$$d-c = c^2 - 2c$$
 b) $\frac{b}{d} = -a$ c) $db^2 = b^2 - b$

Sample Test 3 (Lesson 7-9)

1. If
$$A + B = -\frac{3}{8}$$
, evaluate: a) $\frac{2}{A+B}$ b) $B + A$ c) $-A - B$

2. Evaluate the following expressions, if x - y = 0.2 and $\frac{1}{y} = -0.6$:

a)
$$\frac{1}{v} + x - y$$
 b) $\frac{x - y}{v}$

3. Evaluate 2a - (3b + c) if a) a = -3, b = -2, c = 2 b) a = 2, c + 3b = 0.64. Rewrite the expression $2a - b^2$ in terms of x. Write your answer without parentheses. Simplify. a) a = x + 2, b = x + 1 b) a = 3x, b = 2x

5. Write the following expression 3m(m-2n) in terms of t, if m = n = t. Simplify your answer.

6. Which of the following are solutions of xy = 24?

a)
$$x = -6$$
, $y = -4$ b) $x = -\frac{1}{2}$, $y = -48$ c) $x = 0$, $y = 24$

7. Solve the following equations:

a)
$$x-3 = 27$$

b) $2x = 7x$
c) $3(x-1) = 1+3x$
d) $-2x+1=5$
e) $\frac{3x}{2}-x=1$

8. Let A = 3x, B = x, and C = -x. Find x so that the following is true:

$$\frac{A}{2} - \frac{B}{3} = C$$

9. Solve for x. Any time you must perform the operation of division, assume that the devisor is different from zero. Simplify your answer, if possible

a)
$$-ax = b$$
 b) $\frac{x}{3d^2} = d$ c) $ax - a = cx$

Sample Tests: Lesson 10-12

Sample Test 1 (Lesson 10-12)

1. Describe the following set of numbers using inequality sign and then graph the set of on the number line: all numbers x at least equal to -1.

2. Which of the following numbers: -1, 0, 2.99, $\frac{7}{4}$, $\frac{10}{3}$, $\frac{9}{3}$ satisfy the inequality: $x \ge 3$?

3. Solve the following inequalities:

a) $-3x \le 2$ b) 3(x+1) > 4x c) $3-2x < \frac{1}{4}$

4. The following expression $\frac{x-3}{4}$ is written in the form $\frac{x-p}{2}$. Identify p in this representation. 5. Write the expression x-7 in the form: x+p, where x is unknown and p any number. Identify p in your representation

6. Write the following expressions in the form A^3 , where A is any algebraic expression. Identify A in your representation a) $\frac{y^3}{2}$ b) x^3y^6

7. Write the following expressions in the form $y = ax^3 + bx^2 + c$, where *a*, *b*, and *c* are any numbers. Identify *a*, *b*, and *c*? a) $y = -2x^2 + \frac{x^3}{4}$ b) $2y - 2x^3 = 4x^2 + 7$

8. Factor the following expressions: a) $x^2 - 100$ b) $\frac{1}{25} - y^4$

9. Convert $2\frac{1}{2}$ feet to yards, if 1 yard equals 3 feet.

- 10. 2.5 is 0.1% of what number?
- 11. Twice a number added to 25 is 9. What is the number?

Sample Test 2 (Lesson 10-12)

1. The number 4 satisfies which of the following statements? a) x < 2 b) $x \ge 4$ c) $x \le 4$

2. Using inequality symbols, describe the set that is graphed below.



3. Solve the following inequalities.

a) 5x-2 < 13b) 5 > -4 + xc) $-2x \le 8$ d) $\frac{-x-6}{5} \ge -3$ e) 4x-12 > 4(x-3)

4. Write the following expression as a difference of two expressions, that is in the form A - B, where A and B are any expressions except zero. a) a + 3b b) $\frac{m-n}{k}$

5. Write the following expressions in the form of a^n , where *a* and *n* is an algebraic expression and identify *a* and *n*: a) $2 \cdot 2^5$ b) $(2zy)^3(x)^3$

6. Write the following equations in the form ax + by = c, where a, b, c are any real numbers. Identify a, b, and c in your representation:

a) 2x - 3y = 5 b) 3y = 7(x - 1) (c) x = 0

7. Determine if the following equations are linear equations in one variable. If so, express it in the form ax + b = 0, where a is any real number except 0, b is any real number, and x is

unknown. a) $x^{2} + 1 = 0$ b) $\frac{x}{3} - 1 = 0$ c) $\frac{8x^{4} - x}{8} = x^{4}$ 8. Factor the following expressions: a) $1 - a^{2}$ b) $(2x - 1)^{2} - x^{2}$

9. What is $\frac{2}{3}$ % of 30?

10. A number was added to itself and then multiplied by 10. The result was 7. What was the original number?

11. Using the formula $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$, factor $1 - 27x^3$.

Sample Test 3 (Lesson 10-12)

1. Graph the following sets on the number line: a) x < -4b) all numbers that are at least -12. Solve the following inequalities:

a)
$$-x < 5$$
 b) $3x + 3 > -4$ c) $\frac{x-6}{5} \ge 0$
d) $-(2x+1) \ge 4 - 2x$ e) $\frac{x}{2}x - \frac{2}{3} < 1$
 $x > 1$, what inequality is true for a) $x - 2$ b) $\frac{x}{-2}$

3. If x > 1, what inequality is true for a) x - 2

4. The expression $3(a+2) + (-2)^2$ is written in the form: $3A + B^2$. What algebraic expression represents A and B?

5. Write the following expressions in the form x^3 , where x is any algebraic expression. Identify x in b) v^{18} your representation: a) $8v^{3}$

6. Write the following equations in the form $az^3 = b$, where a and b are any numbers. Identify a and

a) $z^3 - 3z^3 = -1$ b) $\frac{6z^3 + 3}{3} = 1$ *b* in your representation:

a) $0.25-9x^2$ b) $(3a+1)^2 - (2a)^2$ c) $1 - v^4$ 7. Factor the following expressions: 8. Determine if the following equations are linear equations in one variable. If so, express it in

the form ax + b = 0, where a is any real number except 0, b is any real number, and x is

a) 5x - 7 = 8 b) $\frac{x}{4} = 1$ c) 2(x-4) = 3xunknown.

9. Answer the following question: 4 is 0.2% of what number?

10. Tom bicycles 3 miles in 15 minutes. At this rate, how far can he go in 20 minutes?

11. Using the formula $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$, factor $x^3y^3 - 1000$.

1. Write the following statements as algebraic expressions. Remember to place parentheses where needed (please, place them only when needed) :

a) two sevenths of *xy*

b) the product of x and y + 2

c) xy raised to fourth power _____

d) Subtract A from B, and then multiply by C

e) take the opposite to x, and then raise it to the third power

2. Let F be a variable representing the temperature in Fahrenheit. Use F to write the following statement as an algebraic expression: five-ninths times the difference of F and 32.

3. Determine which of the following expressions are equal to -x: Circle your choices.

 $x(-1) \qquad -1 \cdot x \qquad \frac{x}{-1} \qquad \frac{-x}{1} \qquad 1 - 2x$

4. Determine which of the following expressions are equivalent to mn - k: Circle your choices.

k-mn, nm-k, mn+(-k), m(n-k), -k+mn

5. State which expressions are equivalent to $\frac{x-y}{2}$ Circle your choices.

(a) $x - y \div 2$ (b) $(x - y) \div 2$ (c) $\frac{x}{2} - \frac{y}{2}$ (d) $\frac{-y + x}{2}$ (e) $\frac{1}{2}(x - y)$

6. Let m = -3. Evaluate the following expressions if possible (if not possible, clearly indicate so): Show all steps in your process.

a) m^{2}

b)
$$\frac{m+5}{2}$$

c)
$$\frac{m+1}{m+3}$$

(a)
$$x = -6$$
 and $y = 2$

(b)
$$x = \frac{5}{6}$$
 and $y = \frac{3}{4}$

(c)
$$x = -0.1$$
 and $y = -0.2$

- 8. If possible (if not possible, clearly indicate so), evaluate $\frac{x}{1-x}$ when
 - (a) x = 0
 - (b) x = 1
 - (c) x = -1
- 9. Evaluate $\frac{m}{p} n$ if possible (if not possible, indicate so), when m = -0.1, n = 0.02, p = -1.

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10. Perform all possible numerical operations and simplify completely. If no work can be done, state that clearly by making the expression equal to itself with no changes.

(a) -5 - (6 - 2)x(b) -6(2x)(c) $(6 \div 2)x$ (d) $\frac{2}{3} - \frac{5}{6}x$

(e)
$$\frac{2}{3} - \frac{5}{6} - x$$

11. Replace X with expressions such that the resulting statement is true. Use parentheses when needed. Place your replacement for X on the space provided so the expressions are equivalent.

a)
$$-w + 2y - 3z = -3z + 2y + X$$
 X is _____

b)
$$3 \cdot \frac{a}{y} \cdot b = \frac{X}{y}$$

X is _____

c)
$$\frac{m+n}{v} = \frac{m}{v} + X$$
.

X is _____

Exam II

NAME:

Instructions: This is a closed-book exam. Neither calculators nor notes are allowed. Answers without supporting work will receive no credit. Read the instructions of each part carefully, as some parts carry partial credit and some others do not.

Part I. True or False. In true or false questions you get +1 for a correct answer, -1 for an incorrect answer, and 0 if the answer is left blank. Since you may get a negative score, it may be to your advantage to leave blank an item of which you are not completely sure. No partial credit.

1.
$$(2+3)^{10} = 2^{10} + 3^{10}$$
.

2.
$$(2 \cdot 3)^{10} = 2^{10} \cdot 3^{10}$$
.

3.
$$(2^5)^2 = 2^7$$
.

4. It is always true that
$$x(2x+3) = x(2x)(3)$$
.

5.
$$(1^1 + 2^2 + 3^3 + 4^4 - 5^5)^0 + 4$$
 equals 5.

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Part II. In each of the items below, select the choice, or choices, that is—or are—equivalent to the given algebraic expression. You get +1 if you bubble-in a correct answer, -1 if you bubble-in an incorrect answer, and 0 if you do not bubble-in the choice. No partial credit.

6.
$$10x\left(\frac{y}{5}+x\right) =$$

(A) $\frac{4y}{2}+10x^2$ (B) $2xy+10x$ (C) $10\left(\frac{y}{5}+x\right)x$ (D) $5x\cdot\left(x+\frac{y}{5}\right)\cdot 2$ (E) $2xy+10x^2$
7. $(a-2b)^2 =$
(A) $(a-2b)(a-2b)$ (B) a^2-4b^2 (C) a^2+4b^2 (D) $a^2-4ab+4b^2$ (E) $-(2b-a)(a-2b)$
8. $\frac{x-y}{y-x} =$
(A) 1 (B) -1 (C) -2 (D) $(x-y)^2$ (E) $\frac{y-x}{x-y}$
9. $(2x^2y^2)(3xy^4) =$
(A) $6x^3y^6$ (B) $6(xy^4)^2$ (C) $6(xy^2)^3$ (D) $\frac{12x^9y^{12}}{2x^3y^2}$ (E) $\frac{12x^6y^7}{2x^3y}$
10. $x(x^3-1)+2(x^3-1) =$
(A) $(x^3-1)x-2(-x^3+1)$
(B) $2(x^3-1)+x(-x^3+1)$
(C) $(x+2)(x^3-1)$
(D) x^4-x+2x^3-2
(E) $x^4-x+1(x^3-2)$

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Part III. Write each statement as an algebraic expression using parentheses, and then rewrite without parentheses and collect like terms. +3 points each. Some partial credit may be given for almost perfect answers. Show all work.

11. Twice the opposite of x - 2y + 4z.

12. The product of (x - 2y) and $(x^2 + 4y^2)$.

13. The product of $x^2 - x + 1$ and $x^2 + x - 1$.

Part IV. Factor the required expression from the given algebraic expression. +3 marks each. Some partial credit may be given for almost perfect answers. Show all work.

14. Factor $2a^3b^3$ from $4a^3b^4 - 10a^4b^3$.

15. Factor
$$\frac{3}{4}x$$
 from $\frac{9}{16}x^2 - \frac{3}{4}x$.

16. Factor -1 from -x + 2y - 3z.

Part V. Multiple-choice. Exactly one of the choices is the correct one, bubble it in. +2 mark for each correct item. 0 for incorrect items or items with more than one choice bubbled-in.

- 17. Collect like terms: x + 2y xy + 2x 2y + yx 2xy.
 - (A) $-16x^5y^5$ (B) 3x 2yx (C) 3x + 4y 4xy (D) 3x 4y + 4xy (E) $16x^5y^5$
- 18. Write as a single exponential expression: $(2x^2)(-3x^3)^3$.
 - (A) $-6x^5$ (B) $-18x^8$ (C) $-18x^{11}$ (D) $-54x^{11}$ (E) $54x^{11}$
- 19. If $4^1 \cdot 4^2 \cdot 4^3 = 2^n$ then n =

(A) 6	B 8	C 384	D 12	(E) 4096

- 20. Collect like terms: a(b-c+d) a(b+c-d).
 - (A) 0 (B) 2ab + 2ad 2ac (C) 2ac 2ad (D) 2ad 2ac (E) 2ad
- 21. Collect like terms: $\frac{x}{2}(4x-2) + x$.
 - (A) $2x^2 + 2x$ (B) $8x^2 3x$ (C) $2x^2 x$ (D) 2x 1 (E) $2x^2$

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Part VI. In the items below, use the algebraic expression in order to obtain the arithmetic result with very little computation. Some partial credit may be given for almost perfect answers. Show all work.

22. (2 points) Expand and collect like terms: $2x(x+1) - x^2 - x(x-1)$.

23. (1 point) Use problem 22 and a cleverly chosen value of x to evaluate

 $2 \cdot (11112) \cdot (11113) - 11112^2 - (11111) \cdot (11112)$

No credit will be given if the result of problem 22 is not used.

Exam II

Brief Answers and Solutions.

F Exponents do not distribute over sums.
 T Exponents distribute over products.
 F (2⁵)² = 2¹⁰.
 F x(2x+3) = 2x³ + 3x.
 T (1¹+2²+3³+4⁴-5⁵)⁰ = 1.

Problems 1—5, at most 5 marks. Running total: 5.

- 6. CDE
- 7. ADE
- 8. BE
- 9. ACE
- 10. ACD

Problems 6-10, at most 14 marks. Running total: 19.

11. 2(-1(x-2y+4z)) = 2(-x+2y-4z) = -2x+4y-8z.

12.
$$(x-2y)(x^2+4y^2) = x(x^2+4y^2) - 2y(x^2+4y^2) = x^3+4xy^2 - 2x^2y - 8y^3.$$

13. We have

$$\begin{array}{rcl} (x^2-x+1)(x^2+x-1) &=& x^2(x^2+x-1)-x(x^2+x-1)+1(x^2+x-1)\\ &=& x^4+x^3-x^2-x^3-x^2+x+x^2+x-1\\ &=& x^4-x^2+2x-1. \end{array}$$

Problems 9—13, at most 9 marks. Running total: 28.

- 14. $4a^{3}b^{4} 10a^{4}b^{3} = 2a^{3}b^{3}(2b 5a).$ 15. $\frac{9}{16}x^{2} - \frac{3}{4}x = \frac{3}{4}x\left(\frac{3}{4}x - 1\right).$
- 16. Factor -1 from -x + 2y 3z = -1(x 2y + 3z).

Problems 14—16, at most 9 marks. Running total: 37 marks.

Exam II

17. B.

18. D.

19. D.
$$4^1 \cdot 4^2 \cdot 4^3 = 4^{1+2+3} = 4^6 = (2^2)^6 = 2^{12}$$
.

20. D.
$$a(b-c+d) - a(b+c-d) = ab - ac + ad - ab - ac + ad = -2ac + 2ad$$
.

21. E.
$$\frac{x}{2}(4x-2) + x = \frac{x}{2} \cdot 4x - \frac{x}{2} \cdot 2 + x = 2x^2 - x + x = 2x^2$$
.

Problems 17—21, at most 10 marks. Running total: 47 marks.

22. We have

$$2x(x+1) - x^2 - x(x-1) = 2x^2 + 2x - x^2 - x^2 + x$$

= 3x.

23. Putting x = 11112 one obtains from the preceding part that

$$2 \cdot (11112) \cdot (11113) - 11112^2 - (11111) \cdot (11112) = 3 \cdot 11112 = 33336.$$

Problems 22—23, at most 3 marks. Running total: 50 marks.

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Instructions: This is a closed-book exam. Neither calculators nor notes are allowed. Answers without supporting work will receive no credit.

- 1. (2 points each) Evaluate $-3ab^2 + b$ in the following instances.
 - (a) a = 0 and b = -5

(b) a = -3 and b = 2

(c)
$$a = \frac{1}{3}$$
 and $b = 1$

(d) a = -0.1 and b = 0.2

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- 2. (2 points each) Write 5a 2 in terms of x in the following instances, and simplify.
 - (a) a = -x

(b)
$$a = \frac{x+2}{5}$$

(c) a = 5x - 2

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3. (3 points) Expand and collect like terms to demonstrate that $(2x - 3y)^2 - 4x^2 + 12xy$ is equal to $9y^2$.

4. (2 points each) Evaluate $(2x - 3y)^2 - 4x^2 + 12xy$ in the following instances. (Hint: It is more effective to use the result in problem 3 to aid your evaluation.)

(a) x = -6.66 and y = 1

(b) x = 2007 and y = -2

- 6. (2 points each) Determine whether the following equations have no solutions, infinitely many solutions, or exactly one solution. In the case when the equation has exactly one solution, find it, and verify that it is indeed a solution.
 - (a) 3x 12 = 9

(b) 2x + 9 = 2(x + 3) + 3

(c) x + 4 = x + 3

(d) 2 = 2x + 1

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7. Let A = 5x, B = 2x, and C = -x.

(a) (2 points) If
$$\frac{A}{5} - \frac{B}{2} = C$$
, find *x*.

(b) (1 point each) Find A, B, and C.



8. (2 points) Solve ax + b = cx + d for *x*.

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- 9. (2 points each) Determine whether the following equations have no solutions, infinitely many solutions, or exactly one solution. In the case when the equation has exactly one solution, find it.
 - (a) 9x + 8 = 2x + 5

(b) x + 8 = 2x

(c)
$$\frac{x}{4} - \frac{2}{3} = \frac{5}{6}$$

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(d) 0.45x + 10.4 = 7.5 - x

(e) 9(x-3) + 20 = 4x + 2(x+5)

(f)
$$\frac{5x-1}{3} = -\frac{x}{3} + 3$$

(g)
$$7x + 13 = 7(x + 2) - 3$$

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- 10. (1 point each) Attempt this problem for extra credit only after finishing all preceding problems.

(a) Factor
$$n + 1$$
 from $\frac{n(n+1)}{2} + (n+1)$ and simplify your answer.

(b) Observe that

$$1 = \frac{(1)(2)}{2},$$

$$1 + 2 = \frac{(2)(3)}{2},$$

$$1 + 2 + 3 = \frac{(3)(4)}{2},$$

$$1 + 2 + 3 + 4 = \frac{(4)(5)}{2},$$

$$1 + 2 + 3 + 4 + 5 = \frac{(5)(6)}{2}.$$

$$1 + 2 + \dots + 19 + 20 = \frac{(20)(21)}{2},$$

Would it be true that

$$1 + 2 + \dots + 19 + 20 = \frac{(20)(2)}{2}$$

How can you use the expression in 10a to deduce this?

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Brief Answers and Solutions

- 1. (a) $-3ab^{2} + b = -3(0)(1)^{2} + (-5) = -5$ (b) $-3ab^{2} + b = -3(-3)(2)^{2} + (2) = 38$ (c) $-3ab^{2} + b = -3\left(\frac{1}{3}\right)(1)^{2} + (1) = 0$ (d) $-3ab^{2} + b = -3(-0.1)(0.2)^{2} + (0.2) = 0.212$
- 2. (2 points each) Write 5a 2 in terms of x in the following instances, and simplify.
 - (a) 5a-2 = 5(-x) 2 = -5x 2(b) $5\left(\frac{x+2}{5}\right) - 2 = x + 2 - 2 = x$ (c) 5(5x-2) - 2 = 25x - 10 - 2 = 25x - 12
- 3. We have,

$$\begin{array}{rcl} (2x-3y)^2-4x^2+12xy&=&(2x-3y)(2x-3y)-4x^2+12xy\\ &=&2x(2x-3y)-3y(2x-3y)-4x^2+12xy\\ &=&4x^2-6xy-6xy+9y^2-4x^2+12xy\\ &=&9y^2, \end{array}$$

proving the assertion.

- 4. (a) Using the preceding problem, $(2x 3y)^2 4x^2 + 12xy = 9y^2 = 9(1)^2 = 9$ in this instance.
 - (b) Using the preceding problem, $(2x 3y)^2 4x^2 + 12xy = 9y^2 = 9(-2)^2 = 9(4) = 36$ in this instance.

5.
$$5x-4=2x-3 \implies 5x-2x=-3+4 \implies 3x=1 \implies x=\frac{1}{3}$$
. Hence choices (C) and (E).

- 6. (a) $3x 12 = 9 \implies 3x = 9 + 12 \implies 3x = 21 \implies x = \frac{21}{3} = 7$. Verification: $3(7) 12 \stackrel{?}{=} 9 \implies 9 \stackrel{\checkmark}{=} 9$.
 - (b) $2x+9=2(x+3)+3 \implies 2x+9=2x+6+3 \implies 0=0$, this is a tautology, that is, the assertion is always true, regardless of the value of *x*. Hence, the equation has infinitely many solutions.
 - (c) $x+4 = x+3 \implies 4 = 3$. This is a contradiction. The equation has no solutions.

(d)
$$2 = 2x + 1 \implies 2 - 1 = 2x \implies 1 = 2x \implies x = \frac{1}{2}$$
. Verification: $2 \stackrel{?}{=} 2\left(\frac{1}{2}\right) + 1 \implies 2 \stackrel{\checkmark}{=} 2$.

7. (a)
$$(2 \text{ points}) \frac{A}{5} - \frac{B}{2} = C \Longrightarrow \frac{5x}{5} - \frac{2x}{2} = -x \Longrightarrow x - x = x \Longrightarrow 0 = -x \Longrightarrow x = 0.$$

(b) $A = 5(0) = 0$ $B = 2(0) = 0$ $C = -0 = 0$

8. $ax + b = cx + d \implies ax - cx = d - b \implies x(a - c) = d - b \implies x = \frac{d - b}{a - c}$.

9. (a)
$$9x+8=2x+5 \implies 9x-2x=5-8 \implies 7x=-3 \implies x=-\frac{3}{7}$$
.
(b) $x+8=2x \implies x-2x=-8 \implies -x=-8 \implies x=8$.

(c)
$$\frac{x}{4} - \frac{2}{3} = \frac{5}{6} \implies \frac{x}{4} = \frac{5}{6} + \frac{2}{3} \implies \frac{x}{4} = \frac{5}{6} + \frac{4}{6} \implies \frac{x}{4} = \frac{9}{6} \implies 6x = 36 \implies x = \frac{36}{4} = 9.$$

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- (d) $0.45x + 10.4 = 7.5 x \implies x + 0.45x = 7.5 10.4 \implies 1.45x = -2.9 \implies x = -\frac{2.9}{1.45} = -2.$
- (e) $9(x-3) + 20 = 4x + 2(x+5) \implies 9x 27 + 20 = 4x + 2x + 10 \implies 9x 4x 2x = 10 + 27 20 \implies 3x = 17 \implies x = \frac{17}{3}.$

(f)
$$\frac{5x-1}{3} = -\frac{x}{3} + 3 \implies \frac{5x-1}{3} = -\frac{x}{3} + \frac{9}{3} \implies \frac{5x-1}{3} = \frac{-x+9}{3} \implies 5x-1 = -x+9 \implies 5x+x=9+1 \implies x = \frac{10}{6} = \frac{5}{3}.$$

(g) $7x + 13 = 7(x+2) - 3 \implies 7x + 13 = 7x + 14 - 3 \implies 13 = 11$, this is a contradiction. The equation does not have a solution.

10. (a)
$$\frac{n(n+1)}{2} + (n+1) = (n+1)\left(\frac{n}{2} + 1\right) = \frac{(n+1)(n+2)}{2}$$

(b) Using the above expression, if

$$1+2+\dots+n=\frac{n(n+1)}{2}$$

were true, we would expect, by adding n + 1 to both sides, that

$$1 + 2 + \dots + n + (n+1) = \frac{n(n+1)}{2} + (n+1) = \frac{(n+1)(n+2)}{2},$$

Since the formula is true for a few base cases and it appears to be true for general *n* from above, we would expect

$$1 + 2 + \dots + 19 + 20 = \frac{(20)(21)}{2} = 210.$$

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Instructions: This is a closed-book exam. Neither calculators nor notes are allowed. Answers without supporting work will receive no credit.

1. $(\pm \frac{1}{2} \text{ point})$ Circle all of the following numbers that satisfy the inequality $x \le 4$:

 $0, \quad 4, \quad 4.0001, \quad 3.9, \quad \frac{12}{4}, \quad \frac{13}{3}, \quad 5, \quad \frac{81}{10}.$

2. (3 points each) Solve the following inequalities and graph the solution sets:

(a) -x > 1



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(c)
$$\frac{y+4}{7} \ge 0$$









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(a) $(\frac{1}{2}$ point per parenthesis) Fill in the parentheses so that the equality be true, using the perfect square formula.

$$(2x+3)^2 = \left(\begin{array}{c} \\ \end{array} \right)^2 + 2 \left(\begin{array}{c} \\ \end{array} \right) \left(\begin{array}{c} \\ \end{array} \right) + \left(\begin{array}{c} \\ \end{array} \right)^2.$$

(b) (1 point per blank) Simplify your result above as much as possible by evaluating exponents and products and fill in the blanks below:

4. (1 point each) Rewrite each as the square of some quantity.

(a)
$$x^6 = ($$
)²

(b) $t^{10} = ($ $)^2$

(c)
$$9y^4 = ($$
 $)^2$

(d)
$$\frac{x^4}{4} = ($$
)²
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- 5. (2 points each) Use the difference of squares formula $A^2 B^2 = (A + B)(A B)$ to factor the following expressions:
 - (a) $9x^2 49$

(b) $p^2 - 4q^2$

- 6. Complete the following steps in order to factor $x^6 9y^4$:
 - (a) (2 points) Rewrite $x^6 9y^4$ in the form $A^2 B^2$, that is, fill in the parentheses below in order for the equality sign to be true:

 $x^6 - 9y^4 = \left(\begin{array}{c} \\ \end{array} \right)^2 - \left(\begin{array}{c} \\ \end{array} \right)^2.$

(b) (2 points) Use Difference of Squares Formula to factor $x^6 - 9y^4$, that is, fill in the parentheses below in order for the equality sign to be true:

$$x^6 - 9y^4 = \left(\qquad \right) \left(\qquad \right).$$

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7. (2 points each) Solve each proportion:

(a)
$$\frac{x}{3} = \frac{2}{7}$$

(b)
$$\frac{5}{4} = \frac{6}{x}$$

- 8. (2 points each) Percent problems.
 - (a) 6 is 0.5% of what number?

(b) What is 0.5% of 1200?

(c) 3 is what percent of 12?

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- 9. Complete the steps below in order to solve the following problem: Three pounds of apples cost \$4. How many pounds of apples can one buy at the same price for \$10?
 - (a) (1 point) Let *x* be the number of apples that one can buy at the same price for \$10. Set up a proportion (equation) involving *x* for the situation in this problem.

(b) (1 point) Solve the equation found in part 9a.

- 10. Complete the steps below in order to solve the following problem: A number was added to 4 and then this sum was multiplied by 3. The result was equal to the original number. What was the number?
 - (a) (1 point) If the original number was *x*, then the number added to 4 is _____.
 - (b) (1 point) In terms of *x*, the sum multiplied by 3 is _____.

(c) (1 point) Set up an equation for the situation in this problem involving the algebraic expressions found in parts 10a and 10b above.

(d) (1 point) Solve the equation found in part 10c above. The original number is thus

- 11. (Extra Credit) Use the result of problem 11a in order to solve problem 11b. You may avail of the formula $(A + B)^2 = A^2 + 2AB + B^2$.
 - (a) (1 point) Factor $x^2 + 4xy + 4y^2$

(b) (1 point) Evaluate $x^2 + 4xy + 4y^2$ when x = 0.72, y = 0.14.

10

10

10

10

10

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			Brief Answers and Solutions	
1.	(Run	ning total: 2) (0	$(4), (4), 4.0001, (3.9), (\frac{12}{4}), \frac{13}{3}, 5, \frac{81}{10}.$	
2.	(a)	(Running total: 5)	$-x > 1 \implies x < -1$	7 8 9
	(b)	(Running total: 8)	$3 - 8y \le 1 - 7y \implies -8y + 7y \le 1 - 3 \implies -y \le -2 \implies y \ge 2$	
	-	-10 -9 -8 -7 < 		7 8 9
	(c)	(Running total: 11)	$\frac{y+4}{7} \ge 0 \implies y+4 \ge 0 \implies y \ge -4$	
	-	-10 -9 -8 -7 <		7 8 9
	(d)	(Running total: 14)	$\frac{2}{3} + \frac{x}{4} < 1 \Longrightarrow \frac{x}{4} < 1 - \frac{2}{3} \Longrightarrow \frac{x}{4} < \frac{1}{3} \Longrightarrow x < \frac{4}{3}.$	
	-			7 8 9
	(e)	(Running total: 17)	$0.5(4x+2) > 1 \implies 2x+1 > 1 \implies 2x > 0 \implies x > 0$	
				7 8 9
3.	(a)	(Running total: 19)	$(2x+3)^2 = (2x)^2 + 2(2x)(3) + (3)^2$	1 1 1
	(b)	(Running total: 22)	$\underline{4}x^2 + \underline{12}x + \underline{9}.$	
4.	(a)	(Running total: 23)	$x^6 = (x^3)^2$	
	(b)	(Running total: 24)	$t^{10} = (t^3)^2$	
	(c) (d)	(Running total: 25) (Running total: 26)	$\frac{9y^{2}}{4} = \left(\frac{x^{2}}{2}\right)^{2}$	
5.	(a)	(Running total: 28)	$9x^2 - 25 = (3x - 5)(3x + 5)$	
	(b)	(Running total: 30)	$p^2 - 4q^2 = (p - 2q)(p + 2q)$	
6.	(a)	(Running total: 32)	$x^{6} - 9y^{4} = \left(x^{3}\right)^{2} - \left(3y^{2}\right)^{2}.$	
	(b)	(Running total: 34)	$x^6 - 9y^4 = \left(\mathbf{x^3} - 3\mathbf{y^2}\right)\left(\mathbf{x^3} + 3\mathbf{y^2}\right).$	
7.	(a)	(Running total: 36)	$\frac{x}{3} = \frac{2}{7} \implies 7x = 6 \implies x = \frac{6}{7}.$	
	(b)	(Running total: 38)	$\frac{5}{4} = \frac{6}{x} \implies 5x = 24 \implies x = \frac{24}{5}.$	

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8.	(a) (Running total: 40)	$6 = \frac{.5x}{100} \implies 600 = .5x \implies x = \frac{600}{.5} = 1200.$
	(b) (Running total: 42)	$x = \frac{.5}{100} \cdot 1200 = 6.$
	(c) (Running total: 44)	$3 = \frac{x}{100} \cdot 12 \implies 3 = \frac{12x}{100} \implies x = 3 \cdot \frac{100}{12} = 25.$
9.	(a) (Running total: 45)	$\frac{3}{4} = \frac{x}{10}$
	(b) (Running total: 46)	$\frac{3}{4} = \frac{x}{10} \implies x = 10 \cdot \frac{3}{4} = \frac{15}{2}$. Hence, one may buy 7.5 lbs.
10.	(a) (Running total: 47)	$\underline{x+4}$
	(b) (Running total: 48)	3(x+4)
	(c) (Running total: 49)	3(x+4) = x
	(d) (Running total: 50) number is <u>–6</u> .	$3(x+4) = x \implies 3x+12 = x \implies 3x-x = -12 \implies 2x = -12 \implies x = -6$. The
11.	(a) (Running total: 51)	$x^2 + 4xy + 4y^2 = (x + 2y)^2$

(b) (Running total: 52) $x^2 + 4xy + 4y^2 = (x + 2y)^2 = (0.72 + 2 \cdot 0.14)^2 = (0.72 + 0.28)^2 = 1^2 = 1.$

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Instructions: This is a closed-book exam. Neither calculators nor notes are allowed. Answers without supporting work will receive no credit.

- 1. Translate the following expressions into algebraic symbols, using parentheses, and then simplify, suppressing the parentheses and collecting any like terms:
 - (a) (2 points) The opposite of x 2.

(b) (2 points) The sum of x + 1 and x - 2.

(c) (2 points) The product of x + 1 and x - 2.

2

- 2. Evaluate the following expressions. Simplify the numerical result, or determine whether the expression is undefined.
 - (a) (2 points) $(x y)^2 + (x^2 y^2)$ when x = 3 and y = -2.

(b) (2 points)
$$\frac{4+a^2}{4-a^2}$$
 when $a = -2$.

(c) (2 points) $2x + x^2$ when x = -0.1.

(d) (2 points) r - 2s + 3t - 4u when r = x, s = 2x, t = -3x, and u = -4x.

3

- 3. Use the distribute law accordingly to expand and collect like terms. Simplify your answers.
 - (a) (2 points) -2(-2a+b-3c)+3(a-2b+c)

(b) (2 points)
$$\frac{4a}{5} - \frac{a}{2}$$

(c) (2 points) $2x(x-2) - 4(x^2 - x)$.

(d) (2 points)
$$\frac{12x^2 - 9x}{3x}$$

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- 4. Use the laws of exponents to simplify the following expressions.
 - (a) (2 points) $x^2 x^3 x^4$.

(b) (2 points)
$$\frac{x^6}{x^2} \cdot \frac{y^9}{y^3}$$
.

(c) (2 points) $(3x^2)^3$.

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5. (2 points) Factor -1 from b-2a+1.

6. (2 points) Factor
$$\frac{x^2}{4} - 1$$
.

7. (2 points) Factor ab from $a^2b - ab^2 + ab$.

8. (2 points) If $8^4 = 2^n$, find *n*.

9. (2 points) If
$$x = 2 \cdot \frac{x}{a}$$
, find *a*.

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10. (3 points) Solve the following e	equation for <i>x</i> an	d check your so	lution: $12-5x$	= 2.	

11. (3 points) Solve the following equation for *x* and check your solution: 2(3x + 4) = 4(2 - 3x).

12. (3 points) Solve the following equation for *x* and check your solution: $\frac{2x-1}{3} = -1$.

13. (2 points) Solve the following equation for *x*. You do not need to check your answer: 2x + a = b.

- 14. Solve each of the following word problems. You must identify an unknown (variable), set up an equation related to the problem for the unknown, and then solve the equation.
 - (a) (3 points) A pancake recipe calls for six eggs for every five pints of milk. How many pints of milk are needed if twenty eggs are used?

(b) (3 points) A number is doubled, and then one adds eight to the result. One obtains six as the final answer. What was the original number?

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- 15. Solve the following inequalities and graph the solution sets:
 - (a) (3 points) $3x \le x + 2$



(b) (3 points) x > 3x - 2



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16. (1 point per blank) If					
	2(x	(x-1) = 3(2x+2))		
were rewritten in the form Ax	+B = 0, then $A =$	an an	d <i>B</i> =		

17. (1 point per blank) Fill in the blanks to make the equality true:

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1 Arithmetic Problems

1 Problem Calculate: 1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 9 - 10. $(2 \cdot (-3) + 1)^2 - ((-5) \cdot 4 + 2 \cdot 9)^3$. **2 Problem** Calculate: $\frac{\frac{3}{5} \div \frac{5}{3}}{\frac{3}{5} + \frac{5}{2}}.$ **3 Problem** Calculate: $\left(\frac{2}{5}\right)^2 + 2 \cdot \left(\frac{2}{5}\right) \left(\frac{3}{10}\right) + \left(\frac{3}{10}\right)^2.$ 4 Problem Calculate: **5 Problem** Calculate: $(2+3+4)(2^2+3^2+4^2-2\cdot 3-3\cdot 4-4\cdot 2)$. **6 Problem** Calculate: $10(0.1)(-0.02) + 20(0.1)^2$. $\frac{100^3 + 100^3 + 100^3 + 100^3 + 100^3 + 100^3}{100^2 + 100^2 + 100^2}.$ **7 Problem** Calculate: $123456^2 - 123455^2$. **8 Problem** Calculate: $\frac{1}{2+\frac{3}{4+\frac{1}{5}}}$. **9 Problem** Calculate: **10 Problem** Calculate: $(123^2 \cdot 456^2 - 789)^0 + 3 \cdot 2^3$.

11 Problem What is the value of $((1 \times 2 \div (3 \times 4) - 5) \times 6 - 7) \div (8 \times 9)$?

12 Problem Write as a single fraction. If the result is an "improper fraction," leave it in that form.

$$\frac{1}{3} + \frac{2}{5} - \frac{3}{7}$$

13 Problem Write as a single fraction. If the result is an "improper fraction," leave it in that form.

$$\frac{1}{3} \cdot \frac{2}{5} \div \frac{3}{7}$$

14 Problem Convert into a fraction and express in lowest terms: 0.204

15 Problem Convert into a decimal: $\frac{9}{11}$.

16 Problem Calculate:

$$\frac{(10)^3 + (-5)^3 + (1)^3 - 3(10)(-5)(1)}{(10) + (-5) + (1)}$$

17 Problem Calculate: 1020.4016 ÷ 637.751.

2 Algebraic Notation Problems

18 Problem A group of p people charters a bus for a trip for D dollars. If they must pay an equal share, what is this share? If the night before the trip x people decide they can't attend the trip, what is the share of the people remaining?

19 Problem If you have *a* one-dollar bills, *b* five-dollar bills, *c* ten-dollar bills and *d* twenty-dollar bills, what amount of dollars do you have?

20 Problem If you have *a* one-dollar bills, *b* five-dollar bills, *c* ten-dollar bills and *d* twenty-dollar bills, how many bills do you have?

21 Problem If *x* is the unknown quantity, translate "the square of a quantity reduced by its triple" into symbols.

22 Problem You start the day with *A* dollars. Your uncle Bob gives you enough money to double your amount. Your aunt Rita gives you 10 dollars. You have to pay *B* dollars in fines, and spent 12 dollars fueling your camel with gas. How much money do you have now?

23 Problem If *a* and *b* are the unknown quantities, translate "the square of the sum two quantities" into symbols.

24 Problem If *a* and *b* are the unknown quantities, translate "the sum of the squares of two quantities" into symbols.

25 Problem If n = 1, 2, 3, ... indicates the position of the general term of the arithmetic progression 1, 7, 13, 19, ..., give a formula for its general term.

26 Problem You begin the day with *E* eggs. During the course of the day, you fry *O* omelettes, each requiring *A* eggs. How many eggs are left?

27 Problem Think of a number. Double it. Add 10. Half your result. Subtract your original number. After these five steps, your answer is 5 regardless of your original number! If *x* is the original number, explain by means of algebraic formulæ each step.

3 Evaluation Problems

28 Problem Evaluate the expression $(a + b)^2 + (a - b)^2$ when a = 3 and b = -2.

29 Problem Evaluate the expression $\frac{a}{b} + \frac{b}{a} - \frac{a+b}{a-b}$ when a = 3 and b = -2.

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30 Problem Evaluate the expression $a^{-b} + b^a$ when a = 3 and b = -2.

31 Problem Evaluate the expression $xy^2 + x^2y - 2xy$ when $x = -\frac{1}{2}$ and $y = \frac{2}{3}$.

32 Problem Evaluate the expression $(x^2 + 2xy + y^2)^0$ when x = 12345 and y = 54321.

33 Problem Evaluate the expression $\frac{x^2 + 1}{1 - x^2}$ when x = -1.

34 Problem Evaluate the expression $x + 2x^2 + 3x^3$ when x = -0.1.

35 Problem Evaluate a - 2b + 3c - 4d when a = x, b = 2x, c = 3x, and d = 4x.

36 Problem Evaluate $\frac{u}{v} + \frac{v}{u} + 2$ when $u = -\frac{x}{2}$ and $v = \frac{3x}{2}$.

37 Problem Evaluate $U^2 + (U+V)^2 - (U-V)^2 + V^2$ when U = 2x and V = -3x.

4 Algebraic Operations Problems

38 Problem Collect like terms: 2(a+2b+3c)-2(a-3b+c).

39 Problem Collect like terms: $\frac{a}{7} + \frac{a}{5}$.

40 Problem Expand and collect like terms: $(x - 1)^2$.

41 Problem Expand and collect like terms: $x(x-1) - (x-1)^2$.

42 Problem Expand and collect like terms: $(x + 2)(x^2 - 2x + 4) - (x - 2)(x^2 + 2x + 4)$.

43 Problem Expand and collect like terms: $3(2x-4) - \frac{24x-12}{4}$.

44 Problem Perform the division: $\frac{6x^3 - 12x^2}{-2x^2}$.

45 Problem Perform the division: $\frac{(6x^3)(-12x^2)}{-2x^2}.$

46 Problem If
$$\frac{(x^3)^3 x^7}{(x^3)(x^2)^2} = x^n$$
, find *n*.

47 Problem If $25^3 = 5^n$, find *n*.

5 Factoring Problems

48 Problem Factor -1 from 1-2x.

49 Problem Factor 2 from 1 - 2x.

50 Problem Factor
$$x^3y^2$$
 from $x^3y^3 + x^4y^2 - x^3y^2$.

51 Problem Factor $\frac{4x}{3y}$ from $\frac{12x^3}{y^5} + \frac{3y}{x}$.

52 Problem Factor $\frac{x^2}{4} - \frac{4}{x^2}$.

53 Problem Factor $a^2b^2 - x^2y^2$.

54 Problem Factor $a^3b^3 - x^3y^3$.

55 Problem Factor $\frac{x^3}{8} - \frac{8}{x^3}$.

56 Problem Factor $x^3 - x$ as the product of three non-constant polynomials.

57 Problem Factor $x^4 - 1$ as the product of three non-constant polynomials.

6 Equation Problems

58 Problem Determine which of the following equations are tautologies, conditional, or contradictions:

$$I: 4x + 2 = 2(2x + 1) \quad II: 4x + 2 = -2(2x + 1) \quad III: 4x + 2 = 2(2x + 1) + 1 \quad IV: 3x = 2x + 1 = 2(2x + 1) + 1 = 1 = 1$$

59 Problem Solve for x: 4x-3=9. **60 Problem** Solve for x: $\frac{3-x}{2} = \frac{1-2x}{3}$. **61 Problem** Solve for x: $\frac{3-x}{2} - 1 = \frac{1-2x}{3}$. **62 Problem** Solve for x: $\frac{ax-b}{a} = \frac{b-cx}{c}$. Page 487

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63 Problem The formula

$$F - 32 = \frac{9}{5}C$$

expresses the relationship between degrees Fahrenheit and Celsius. Solve for C. How many degrees Celsius are 77° F?

64 Problem	Solve for <i>x</i> :	4x - b = a.
65 Problem	Solve for <i>x</i> :	$\frac{bx}{a} + \frac{ax}{b} = 1.$
66 Problem	Solve for <i>x</i> :	$\frac{x}{3} + \frac{1}{2} = \frac{1}{6}.$
67 Problem	Solve for <i>x</i> :	$\frac{x-2}{3} = 5.$
68 Problem	Solve for <i>x</i> :	2(3x-4) - 4(2-3x) = 1.
69 Problem	Solve for <i>x</i> :	$x - \frac{x}{2} - \frac{x}{3} = 1.$
70 Problem	Solve for <i>x</i> :	$\frac{x-2}{2} = \frac{3-x}{3}.$
71 Problem	Solve for <i>x</i> :	$\frac{x}{a} - 1 = 2.$
72 Problem	Solve for <i>x</i> :	$\frac{ax}{b} = a.$
73 Problem	Solve for <i>x</i> :	ax+b=c.
74 Problem	Solve for <i>x</i> :	$\frac{x+a}{2} = 2x+1.$
75 Problem	Solve for <i>x</i> :	$\frac{x+1}{2} - \frac{x+2}{3} = \frac{x-1}{4}.$
76 Problem	Solve for <i>x</i> :	$\frac{a}{x} = b.$
77 Problem	Solve for <i>x</i> :	$\frac{ab}{cx}=d.$
78 Problem	Solve for <i>x</i> :	$\frac{3}{x-2} = 1.$
79 Problem	Solve for <i>x</i> :	$\frac{3}{x-2} = \frac{2}{x+3}.$

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7 Problems on Inequalities

80 Problem Make a sublist of the numbers *x* on the list $\left\{10, 0, -5, -100, -4.001, -3.999, 4, -\frac{401}{100}, -\frac{401}{100}\right\}$ that satisfy the inequality *x* < -4.

81 Problem Solve the inequality and graph its solution set: $4x + 1 \le 5x$.

82 Problem Solve the inequality and graph its solution set: $5x + 1 \le 4x$.

8 Word Problems

83 Problem Rahnnya takes 20 minutes to solve 3 maths problems. At this rate, how many minutes will it take her to solve 2880 problems?

84 Problem On a certain map City A is 4 inches apart from City B. If the scale is such that 1.2 inches represent 30 miles, find the actual distance, in miles, between City A and City B.

85 Problem Nanette is going to make *crêpes*. Her recipe requires 3 fl oz of beer for every 2 eggs. If she is going to utilise 8 eggs, how many fluid ounces of beer does she need?

86 Problem Find 30% of 810.

87 Problem 840 is 28% of what number?

88 Problem What percent of 408 is 34?

89 Problem Hillary trades goats for a 40% profit. If the original price of a goat is \$90, find the new selling price.

90 Problem You bought some furniture for \$424.53, price which included a 6% sales tax. What was the price of piece, before sales tax?

91 Problem After a night of beer and pizza, the bill comes to a total of \$34.20, including sales tax. If you are leaving a 15% tip, how much will you end up paying?

92 Problem A number is trebled and then the result is increased by 20, obtaining 107 as the final answer. What was the original number?

93 Problem An amount of \$493 is split between Peter, Paul and Mary so that Mary has six times as Peter, and Mary has four times as Paul. How much money does Peter have?

94 Problem A piece of equipment is bought by a factory. During its first year, the equipment depreciates a fifth of its original price. During its second year, it depreciates a sixth of its new value. If its current value is \$56, 000, what was its original price?

95 Problem The sum of five consecutive integers is 665. Which one is the middle number?

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9 True or False Questions

96 Problem True or false: $(2+3)^{10} = 2^{10} + 3^{10}$.

97 Problem True or false: $2x + 3x = 5x^2$ for all values of *x*.

98 Problem True or false: $-3^2 = 9$.

99 Problem True or false: $(-3)^2 = 9$.

100 Problem True or false: *A*% of *B* is the same as *B*% of *A*.

10 Equivalent Expression Problems

101 Problem Circle all which are equivalent to
$$\frac{a-b}{b-a}$$
.
(A) -1 (B) $\frac{b-a}{a-b}$ (C) 1 (D) $\frac{u-v}{v-u}$ (E) $2^3 - 3^2$

102 Problem Circle all which are equivalent to $10x\left(\frac{y}{5}+x\right)$. (A) $\frac{xy}{2}+10x^2$ (B) 2xy+10x (C) $10\left(\frac{y}{5}+x\right)x$ (D) $5x\cdot\left(x+\frac{y}{5}\right)\cdot 2$ (E) $2xy+10x^2$

103 Problem Circle all which are equivalent to $(2a + b)^2$. (A) (b+2a) (B) $4a^2 + b^2$ (C) $4a^2 + 4ab + b^2$ (D) $b^2 + 2(2ab) + (2a)^2$ (E) $(-2a - b)^2$

11 Answers

- $1 \quad 1 2 + 3 4 + 5 6 + 7 8 + 9 10 = -1 1 1 1 1 = -5.$
- $\mathbf{2} \quad (2 \cdot (-3) + 1)^2 ((-5) \cdot 4 + 2 \cdot 9)^3 = (-6 + 1)^2 (-20 + 18)^3 = (-5)^2 (-2)^3 = 25 (-8) = 33.$
- $3 \quad \frac{\frac{3}{5} + \frac{5}{3}}{\frac{3}{5} + \frac{5}{3}} = \frac{\frac{3}{5} \cdot \frac{3}{5}}{\frac{9}{15} + \frac{25}{15}} = \frac{\frac{9}{25}}{\frac{34}{15}} = \frac{9}{25} \cdot \frac{15}{34} = \frac{27}{170}$
- $4 \quad \left(\frac{2}{5}\right)^2 + 2 \cdot \left(\frac{2}{5}\right) \left(\frac{3}{10}\right) + \left(\frac{3}{10}\right)^2 = \frac{4}{25} + \frac{12}{50} + \frac{9}{100} = \frac{16}{100} + \frac{24}{100} + \frac{9}{100} = \frac{49}{100}$
- $5 \quad (2+3+4)(2^2+3^2+4^2-2\cdot 3-3\cdot 4-4\cdot 2) = (9)(4+9+16-6-12-8) = (9)(3) = 27 \\$
- $\textbf{6} \quad 10(0.1)(-0.02) + 20(0.1)^2 = (1)(-0.02) + 20(0.1)(0.1) = -0.02 + 2(0.1) = -0.02 + 0.2 = 0.18$
- $7 \quad \frac{100^3 + 100^3 + 100^3 + 100^3 + 100^3 + 100^3 + 100^3}{100^2 + 100^2 + 100^2} = \frac{6 \cdot 100^3}{3 \cdot 100^2} = 2 \cdot 100 = 200.$
- $8 \quad 123456^2 123455^2 = (123456 123455)(123456 + 123455) = 1(246911) = 246911.$
- $9 \quad \frac{1}{2 + \frac{3}{4 + \frac{1}{5}}} = \frac{1}{2 + \frac{3}{21}} = \frac{1}{2 + 3 \cdot \frac{5}{21}} = \frac{1}{2 + \frac{5}{7}} = \frac{1}{2 + \frac{5}{7}} = \frac{1}{\frac{19}{7}} = \frac{7}{19}.$
- $\mathbf{10} \quad (123^2 \cdot 456^2 789)^0 + 3 \cdot 2^3 = 1 + 3 \cdot 2^3 = 1 + 3 \cdot 8 = 1 + 24 = 25.$

 $11 - \frac{1}{2}$

12 $\frac{32}{105}$

 $\frac{14}{45}$ 13

 $\frac{51}{250}$ 14

21 $x^2 - 3x$

in total. **23** $(a+b)^2$ **24** $a^2 + b^2$

equal to 5.

Page 491 $\frac{D}{p}$; $\frac{D}{p-x}$ a+5b+10c+20d**20** a+b+c+d22 You start with A, your uncle Bob gives you A more so that you now have A + A = 2A. Your Aunt Rita gives you 10 more, so that now you have 2A + 10. You pay B dollars in fines, hence you are left with 2A + 10 - B. You spent 12 in fuel, so now you have 2A + 10 - B - 12 = 2A - B - 2, 25 1+6(n-1)E - OA, since OA eggs are used in frying O omelettes. 27 On the first step you have x. On the second step you have 2x. On the third step you have 2x + 10. On the four step you have $\frac{2x + 10}{2}$. On the fifth step you have $\frac{2x + 10}{2} - x$. You are asserting that $\frac{2x + 10}{2} - x$ is identically $(a+b)^2 + (a-b)^2 = (3+(-2))^2 + (3-(-2))^2 = 1^2 + 5^2 = 26$ $\frac{a}{b} + \frac{b}{a} - \frac{a+b}{a-b} = \frac{3}{-2} + \frac{-2}{3} - \frac{3+(-2)}{3-(-2)} = -\frac{71}{30}$ $a^{-b} + b^a = 3^{-(-2)} + (-2)^3 = 3^2 + (-8) = 9 + (-8) = 1$ $xy^2 + x^2y - 2xy = \left(-\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) + \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{2}{3}\right) - 2\left(-\frac{1}{2}\right)\left(\frac{2}{3}\right) = -\frac{2}{9} + \frac{1}{6} + \frac{2}{3} = \frac{11}{18}$ $(x^2 + 2xy + y^2)^0 = 1$ $\frac{x^2+1}{1-x^2} = \frac{1^2+1}{1-1^2} = \frac{2}{0}$, this is undefined. $\mathbf{34} \quad x+2x^2+3x^3=-0.1+2(-0.1)(-0.1)+3(-0.1)(-0.1)(-0.1)=-0.083$ a-2b+3c-4d = x-2(2x)+3(3x)-4(4x) = x-4x+9x-16x = -10x. $\frac{u}{v} + \frac{v}{u} + 2 = \frac{-\frac{x}{2}}{\frac{3x}{2}} + \frac{\frac{3x}{2}}{-\frac{x}{2}} + 2 = -\frac{x}{2} \cdot \frac{2}{3x} - \frac{3x}{2} \cdot \frac{2}{x} + 2 = -\frac{1}{3} - 3 + 2 = -\frac{4}{3}.$ $U^2 + (U+V)^2 - (U-V)^2 + V^2 = (2x)^2 + (-x)^2 - (5x)^2 + (-3x)^2 = 4x^2 + x^2 - 25x^2 + 9x^2 = -11x^2$. 2(a+2b+3c) - 2(a-3b+c) = 2a+4b+6c-2a+6b-2c = 10b+4c

- **39** $\frac{a}{7} + \frac{a}{5} = \frac{5a}{35} + \frac{7a}{35} = \frac{12a}{35}$.
- **40** $x^2 2x + 1$

Community College of Philadelphia Maths 017 Fall 2007 Mock Final Exam Page 492 **41** $x(x-1) - (x-1)^2 = x^2 - x - x^2 + 2x - 1 = x - 1$ **42** $(x+2)(x^2-2x+4) - (x-2)(x^2+2x+4) = (x^3-2x^2+4x+2x^2-4x+8) - (x^3+2x^2+4x-2x^2-4x-8) = 16$ **43** $3(2x-4) - \frac{24x-12}{4} = 6x - 12 - (6x - 3) = 6x - 12 - 6x + 3 = -9.$ $44 \quad \frac{6x^3 - 12x^2}{-2x^2} = \frac{6x^3}{-2x^2} - \frac{12x^2}{-2x^2} = -3x + 6$ 45 $\frac{(6x^3)(-12x^2)}{-2x^2} = \frac{-72x^5}{-2x^2} = -36x^3$ **46** $\frac{(x^3)^3 x^7}{(x^3)(x^2)^2} = \frac{x^9 x^7}{x^3 x^4} = \frac{x^{16}}{x^7} = x^9$, so n = 9. **47** *n* = 6 **48** 1-2x = -1(-1+2x). **49** $1-2x = 2\left(\frac{1}{2}-x\right)$. 50 $x^3y^3 + x^4y^2 - 2x^3y^2 = x^3y^2(y+x-1)$. 51 $\frac{12x^3}{y^5} + \frac{3y}{x} = \frac{4x}{3y} \left(\frac{9x^2}{y^4} + \frac{9y^2}{4x^2} \right).$ 52 $\frac{x^2}{4} - \frac{4}{x^2} = \left(\frac{x}{2} - \frac{2}{x}\right)\left(\frac{x}{2} + \frac{2}{x}\right).$ 53 $a^2b^2 - x^2y^2 = (ab - xy)(ab + xy).$ 54 $a^{3}b^{3} - x^{3}y^{3} = (ab - xy)(a^{2}b^{2} + abxy + x^{2}y^{2})$ 55 $\frac{x^3}{8} - \frac{8}{r^3} = \left(\frac{x}{2} - \frac{2}{x}\right) \left(\frac{x^2}{4} + 1 + \frac{4}{r^2}\right).$ 56 $x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1)$. 57 $x^4 - 1 = (x^2 - 1)(x^2 + 1) = (x - 1)(x + 1)(x^2 + 1).$ 58 I is a tautology. II and IV are conditional. III is a contradiction. **59** $4x - 3 = 9 \implies 4x = 9 + 3 \implies 4x = 12 \implies x = 3$ Cross-multiplying and using the distributive law, 60 $\frac{3-x}{2} = \frac{1-2x}{3} \implies 3(3-x) = 2(1-2x) \implies 9-3x = 2-4x$ Transposing, $9-3x=2-4x \implies -3x+4x=2-9 \implies x=-7.$ 61 The least common denominator of all fractions is 6, hence, multiplying through by 6 $6\left(\frac{3-x}{2}-1\right) = 6\left(\frac{1-2x}{3}\right) \implies 3(3-x) - 6 = 2(1-2x).$ Distributing and transposing, $3(3-x) - 6 = 2(1-2x) \implies 9-3x-6 = 2-4x \implies -3x+4x = 2-9+6 \implies x = -1.$ 62 Cross-multiplying and using the distributive law, $\frac{ax-b}{a} = \frac{b-cx}{c} \implies c(ax-b) = a(b-cx) \implies cax-cb = ab-acx.$ Transposing, $cax + acx = ab + cb \implies 2acx = ab + cb \implies x = \frac{ab + cb}{ac}$ 63 We have $F-32 = \frac{9}{5}C \Longrightarrow \frac{5}{9} \cdot (F-32) = \frac{5}{9} \cdot \frac{9}{5}C \Longrightarrow C = \frac{5}{9} \cdot (F-32).$ Thus, when F = 77, $C = \frac{5}{9} \cdot (77 - 32) = \frac{5}{9} \cdot 45 = 25,$ hence 77° F are 25° C.

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64 $4x - b = a \implies 4x = a + b \implies x = \frac{a+b}{4}$ $\mathbf{65} \quad \frac{bx}{a} + \frac{ax}{b} = 1 \implies \frac{b^2x}{ab} + \frac{a^2x}{ab} = 1 \implies \frac{b^2x + a^2x}{ab} = 1 \implies b^2x + a^2x = ab \implies x(b^2 + a^2) = ab \implies x = \frac{ab}{a^2 + b^2}$ **66** $\frac{x}{3} + \frac{1}{2} = \frac{1}{6} \implies \frac{x}{3} = -\frac{1}{2} + \frac{1}{6} \implies \frac{x}{3} = -\frac{3}{6} + \frac{1}{6} \implies \frac{x}{3} = -\frac{2}{6} \implies x = -\frac{3}{6} = -\frac{1}{2}.$ 67 $\frac{x-2}{3} = 5 \implies x-2 = 3(5) \implies x-2 = 15 \implies x = 15+2 \implies x = 17.$ **68** $x = \frac{17}{18}$ **69** *x* = 6 **70** $x = \frac{12}{5}$ **71** x = 3a**72** x = b73 $x = \frac{c-b}{a}$ 74 $x = \frac{a-2}{3}$ **75** *x* = 1 76 $x = \frac{a}{b}$ 77 $x = \frac{cd}{ab}$ **78** *x* = 5 **79** *x* = −13 **80** $\left\{-5, -100, -4.001, -\frac{401}{100}\right\}$ **81** $4x + 1 \le 5x \implies 4x - 5x \le -1 \implies -x \le -1 \implies x \ge 1$. 82 $5x+1 \le 4x \implies 5x-4x \le -1 \implies x \le -1$. -2 $^{-1}$ 83 Let *x* be the number of minutes she will take to solve 2880 problems. Then $\frac{20}{3} = \frac{x}{2880} \implies x = 19200.$ She will take 19200 minutes. **84** Let *x* be the actual distance in miles. Then $\frac{30}{4} = \frac{x}{12} \implies x = 1.2 \cdot \frac{30}{4} = 1.2 \cdot \frac{15}{2} = 0.6 \cdot 15 = 9.$ City A is 9 miles apart from City B.

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85 Let <i>x</i> be the number of fluid ounces of beer needed. Then	$\frac{3}{2} = \frac{x}{8}$	$\implies x = 8 \cdot \frac{3}{2} = 12.$	
She will need 12 fluid ounces of beer.			
86 Let <i>x</i> be 30% of 810. Then	$x = \frac{1}{1}$	$\frac{30}{00} \cdot 810 = 243.$	
87 Let the number be <i>x</i> . Then	$840 = x \cdot \frac{28}{100} =$	$\implies x = 840 \cdot \frac{100}{28} = 3000.$	
88 Let <i>x</i> be the percent. Then	$\frac{x}{100} \cdot 408 = 34 \implies \frac{40}{1}$	$\frac{8x}{00} = 34 \implies x = 34 \cdot \frac{100}{408} = \frac{25}{3}.$	
Thus 34 is $\frac{25}{3}$ % of 408.			
89 Let <i>x</i> be the new selling price. Then $x = 1.4 \cdot 90 = 126$. The new selling	price is \$126.		
90 Let <i>x</i> be the original price. Then	1.06x = 424.53	$\implies x = \frac{424.53}{1.06} = 400.50.$	
The piece costs \$400.50 before sales tax.			
91 Let <i>x</i> be the amount you will end up paying. Then	<i>x</i> = (1.1	5)(34.20) = 39.33.	
You will end up paying \$39.33.			
92 Let x be number. Then $3x + 20 = 107 \implies 3x = 107 - 20 \implies 3x = 87 =$	$\Rightarrow x = \frac{87}{3} = 29$. The original num	ber was 29.	
93 Let <i>x</i> be Peter's amount. Then Mary's amount is $6x$ and Paul's $\frac{6x}{4} = \frac{3}{2}$	$\frac{x}{2}$. We have		

 $x + 6x + \frac{3x}{2} = 493 \implies 7x + \frac{3x}{2} = 493 \implies \frac{14x}{2} + \frac{3x}{2} = 493 \implies \frac{17x}{2} = 493 \implies x = 493 \cdot \frac{2}{17} = (29)(2) = 58.$

Peter has \$58, Mary has \$348, and Paul has \$87.

94 Let the original price be *P*. After a year, the equipment costs $\frac{4P}{5}$. After the second year it costs $\frac{5}{6}\left(\frac{4P}{5}\right)$. Hence

 $\frac{5}{6}\left(\frac{4P}{5}\right) = 56000 \implies \frac{4P}{6} = 56000 \implies P = \frac{6}{4} \cdot 56000 \implies P = 84000.$

95 Let x, x + 1, x + 2, x + 3, x + 4 be the integers. Then

 $x + x + 1 + x + 2 + x + 3 + x + 4 = 665 \implies 5x + 10 = 665 \implies 5x = 655 \implies 131.$

The middle number is thus 131 + 2 = 133.

96 False.

- 97 False.
- 98 False.
- 99 True.
- 100 True.
- 101 A, B, D, E.
- 102 C, D, E.
- 103 A, C, D, E.

CEMEC Section

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Appendix G - Math 016 Materials

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Math 016 Lessons

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MATH 016 Lessons

LESSON 1 Natural Numbers

The set of *natural numbers* is given by

$$\mathbb{N} = \{0, 1, 2, 3, 4...\}.$$

Natural numbers are used for two main reasons:

1. counting, as for example, "there are 10 sheep in the herd",

2. or ordering, as for example, "Los Angeles is the second largest city in the USA."

Before we talk about the arithmetic of natural numbers we need to have a basic understanding of the way our number system works. To form any natural number, we use the 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. We can represent numbers as large as we want by using their place position to represent different values (place value). The first digit right to left corresponds to the ones place, the next digit (**right to left**) corresponds to the tens place, the next digit to the hundreds place, and so on. We use addition to explain the place value.

For example, consider the number 45, 321. Then we have

 $\underbrace{4}_{\text{ten-thousands thousands}}, \underbrace{5}_{\text{hundreds}}, \underbrace{3}_{\text{tens}}, \underbrace{2}_{\text{ones}}, \underbrace{1}_{\text{ones}}$ 45,321 = 40,000 + 5000 + 300 + 20 + 1

Addition and Multiplication of Natural Numbers.

When adding or multiplying natural numbers, the result is always a natural number.

The symbols '=' (is equal to) and ' \neq ' (is not equal to) are used between two objects in order to indicate whether they are or are not the same.

1.3)Examples:

- 1. 3 + 10 = 13
- 2. $3 \times 10 = 30$

- 3. $3 + 10 \neq 3 \times 10$
- 4. 5000 + 400 + 90 + 2 = 5492 (Notice the place value of the digits)

Notice that:

- 1. Any object is equal to itself, for example:
 - (a) 12 = 12
 - (b) a = a
- 2. Equals can always be substituted for equals.
 - (a) 2+3=5 and 1+4=5, then 2+3=1+4
 - (b) $3 \times 10 = 30$ and $30 = 6 \times 5$, then $3 \times 10 = 6 \times 5$

Some important properties of addition and multiplication of natural numbers are the following:

- 1. If \mathfrak{a} is a natural number then:
 - (a) a + 0 = 0 + a = aFor example: 5 + 0 = 0 + 5 = 5
 - (b) $1 \times a = a \times 1 = a$ For example: $1 \times 21 = 21 \times 1 = 21$
 - (c) $0 \times a = a \times 0 = 0$ For example: $0 \times 1231 = 1231 \times 0 = 0$
- 2. Addition and multiplication of natural numbers are commutative; in other words, changing the order of the numbers does not change the result. That is, if **a** and **b** are natural numbers, then:
 - (a) a + b = b + a
 - (b) $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$
- 1.4)Examples:
- 1. 15 + 123 = 123 + 15; 15 + 123 = 138, then 123 + 15 = 138
- 2. $2 \times 3 = 3 \times 2, 3 \times 2 = 6$, then $2 \times 3 = 6$

When performing operations many times grouping symbols are used. Grouping symbols are symbols used to group together a combination of numbers and operation symbols. Examples of grouping symbols are parentheses (); brackets [] or braces {}. When performing arithmetic operations, the expression inside the grouping symbol must be computed first.

1.5) Examples:

1. (3+2) + 6 = 5 + 6 = 11 (Notice the correct use of the '=' sign).

WARNING: It is incorrect to write

$$(3+2)+6=5=11$$

Because $5 \neq 11$!

- 2. 3 + (2 + 6) = 3 + 8 = 11
- 3. $(2 \times 3) \times 4 = 6 \times 4 = 24$
- 4. $2 \times (3 \times 4) = 2 \times 12 = 24$

Another important property of addition and multiplication is the associative property. Basically it means that when adding or multiplying more than two numbers, no matter how we group them, the result will be the same. Formally: If $a, b \in \mathbb{N}$ then:

- 1. (a + b) + c = a + (b + c)
- 2. $(a \times b) \times c = a \times (b \times c)$

1.6)Examples:

- 1. $(2 \times 5) \times 10 = 2 \times (5 \times 10)$ (Check it!).
- 2. (4+8)+8=4+(8+8) (Check it!).

Notice that when we need to add or multiply more than two numbers and we realize that it can be done either from left to right or from right to left, we are actually making use of the associative property. 1.7)Examples:

1. 1 + 2 + 3 = (1 + 2) + 3 = 3 + 3 = 62. 1 + 2 + 3 = 1 + (2 + 3) = 1 + 5 = 63. $2 \times 5 \times 10 = (2 \times 5) \times 10 = 10 \times 10 = 100$ 4. $2 \times 5 \times 10 = 2 \times (5 \times 10) = 2 \times 50 = 100$

1.1) *Exercises*

- 1. Fill in the blank using either '=' or ' \neq ' as appropriate.
 - (a) $3 \quad 3 + 0$ (b) $1 \times 3 \quad 2 \times 2$ (c) $64 \quad 604$ (d) $56 \quad 7 \times 8$ (e) $4 + 3 + 2 \quad (4 + 3) + 2$ (f) $3 \times 2 \times 4 \quad 8 \times 4$
- 2. Determine, without calculating, which of the following are true and in that case determine the property or properties of addition or multiplication that makes the statement true.
 - (a) 1298 + 7459 = 7459 + 1298
 - (b) 1298 + 7459 = 7460 + 1298
 - (c) $358 \times 498 = 498 \times 359$
 - (d) $987 \times 988 = 988 \times 987$
 - (e) (547 + 1250) + 3 = 547 + (1250 + 3)
 - (f) (547 + 1250) + 3 = 547 + (3 + 1250)

Let's concentrate for a little while on multiplication. It is important to mention that the numbers in a multiplication are called "*factors*". Notice that some multiplications are actually very easy, in fact we can guess the result without any computations. One particular case of this is the multiplication of a natural number and 10, 100, 1000, 10000 and so on (in other words we are considering natural numbers whose first digit left to right is one, followed by zeros). Basically the result of multiplying a natural number by 10 is the number resulting of adding one zero to the right of the number, if it were multiplying by 100 then we need to add two zeros and so on.

1.8) Examples

1.
$$58 \times 100 = 5800$$

2.
$$10 \times 345 = 3450$$

Related to multiplication we have the exponential notation.

Exponential Notation.

An exponential is an expression of the form

an

where, at this point both a and n are natural numbers. The number a is called the "base" of the exponential and n is called the "exponent". Notice the relative positions of the base and the exponent. To explain the meaning of the exponential notation let's consider the possibilities for n.

1. If n = 0, then $a^0 = 1$.

For example

- (a) $31^0 = 1$
- (b) $3^0 = 1$
- (c) $0^0 = 1$

2. $a^1 = a$.

For example

- (a) $3^1 = 3$
- (b) $1^1 = 1$
- (c) $0^1 = 0$

3. $a^2 = a \times a$. For example

- (a) $3^2 = 3 \times 3 = 9$
- (b) $1^2 = 1 \times 1 = 1$

- (c) $0^2 = 0 \times 0 = 0$
- 4. $a^3 = a \times a \times a$.

For example

- (a) $3^3 = 3 \times 3 \times 3 = 27$
- (b) $1^3 = 1 \times 1 \times 1 = 1$
- (c) $0^3 = 0 \times 0 \times 0 = 0$
- 5. In general, if n is more than 0, then

$$a^n = \underbrace{a \times a \times \dots \times a}_{n \text{ times}}$$

WARNING: When evaluating an exponential it is incorrect to multiply the base times the exponent, it is actually the base times itself as many times as the exponent says.

For example

- (a) $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$
- (b) $1^6 = 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$
- (c) $0^4 = 0 \times 0 \times 0 \times 0 = 0$

1.2) Exercises

Compute (use the '=' sign).

- 1. 1000×512
- 2. 10⁴
- 3. 15^1
- 4. 6⁰
- 5. 100^{3}
- 6. 325×100
- 7. Which one is larger, 2^3 or 3^2 ? Explain your answer.
- 8. Which one is larger, 10^0 or $0^{10}?\,$ Explain your answer.
- 9. Which one is larger, 1^3 or 1 + 1 + 1? Explain your answer.

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LESSON 2

Order of Operations

We have talked about addition and multiplication as well as exponential notation. Now let's consider the situation where these operations are combined, including the possible presence of grouping symbols. In that case the operations should be performed in the following order:

- 1. Operations inside grouping symbols.
- 2. Exponentials.
- 3. Multiplications.
- 4. Additions.

Also, be aware that when a number is followed by a grouping symbol (or vice versa) without an operation symbol in between, then the operation is actually multiplication.

 $(3)(5) \neq 35$

WARNING:

In fact

 $(3)(5) = 3 \times 5 = 15$ 2.1)Examples
1. $3(5) = 3 \times 5 = 15$ 2. $(7 \times 3)2 = (7 \times 3) \times 2 = 21 \times 2 = 42$ 3. $3 \times (4+2) = 3 \times 6 = 18$ 4. $3+4 \times 2 = 3+8 = 11$ 5. $2+3^{0} = 2+1 = 3$ 6. $(2+3)^{0} = 5^{0} = 1$ 7. $3 \times 2^{3} = 3 \times 8 = 24$

8. $(3 \times 2)^3 = 6^3 = 6 \times 6 \times 6 = 216$

WARNING: Do not confuse an expression like

$$3 \times 3^{\circ}$$

with

$$(3 \times 3)^{0}$$

They are not the same!

In fact,

 $3 \times 3^0 = 3 \times 1 = 3$, where the exponential is evaluated first, as there are no grouping symbols.

On the other hand,

 $(3\times3)^0=9^0=1$ $\,$, evaluating the expression inside the grouping symbol first.

2.1) Exercises

- 1. Compute. Use the '=' sign and when performing more than one operation show one step for each operation.
 - (a) $1 + 0 \times 4$
 - (b) $3 + 3^2$
 - (c) 3(5+2)
 - (d) 4×10^2
 - (e) $(4 \times 10)^2$
 - (f) $(3+3)^2$
 - (g) $5 \times 10^4 + 3 \times 10^4$
 - (h) $342 + 23 \times 2$
 - (i) $2 \times 200 + 2$
 - (j) 398 + 2 + 10
 - $(\mathrm{k}) \ 45\times100+3\times15$
 - (l) 0×278
 - (m) 100×981
 - (n) $2345 \times 678 \times 91 \times 0 \times 77$
- (o) 1⁸⁷³¹
- (p) $3^{0} \times 1^{3298}$
- (q) 6×10^2
- (r) $20^4 \times 10$
- 2. Determine for which of the following removing the parentheses would not change the value of the expression.
 - (a) (2+3)+4
 - (b) $2 \times (3+4)$
 - (c) $2 \times (3 \times 4)$
 - (d) $2 + (3)^2$
 - (e) $2 + (3+3)^2$
 - (f) $(3 \times 3)^{0}$
 - (g) $2 + (3 \times 4)$
- 3. Fill in the blank using either '=' or ' \neq ' as appropriate. Justify your answer.
 - (a) 2^{3} 2×3 (b) 3^{2} 2^{3} (c) 3×3^{2} 3^{3} (d) 4×2^{5} $(4 \times 2)^{5}$ (e) 92×34^{3} $34^{3} \times 92$ (f) 4^{3} $3 \times 3 \times 3 \times 3$ (g) 75^{5} $75 \times 75 \times 75 \times 75 \times 75$ (h) $12 + 45 \times 56$ $12 + (45 \times 56)$ (i) $9 \times 7 + 12$ $(9 + 7) \times 12$ (j) $(5 + 2) \times 4$ $5 + (2 \times 4)$ (k) (235 + 789) + 99 235 + (789 + 99)(l) 6543×548 548×6543

Subtraction of Natural Numbers and Order of Operations.

We now consider subtraction of natural numbers; however, the result would be another natural number as long as the first number in the subtraction is bigger than or equal to the second number. If the second number in the subtraction is bigger than the first number, the result is not a natural number

In other words, the result of subtracting two natural numbers is not necessarily another natural number.

If we need to perform more than one operation with natural numbers (additions, subtractions, multiplications, exponentials, and the possible presence of grouping symbols), we adopt the following convention regarding the order in which the operations are to be performed:

- 1. Operations inside grouping symbols.
- 2. Exponentials.
- 3. Multiplications.
- 4. Additions and subtractions are associated from LEFT TO RIGHT.

2.2)Examples

1. 6 - 4 + 2 = 2 + 2 = 4

WARNING: It is INCORRECT to say $6 - \underbrace{4+2}_{6} = 0$. Remember that when we are performing combinations of addition and subtractions we should go LEFT TO RIGHT.

- 2. 6 (4 + 2) = 6 6 = 0
- 3. $9 2^3 = 9 8 = 1$
- 4. $(9-2)^3 = 7^3 = 7 \times 7 \times 7 = 343$
- 5. $10 5 \times 2 = 10 10 = 0$
- 6. $(10-5) \times 2 = 5 \times 2 = 10$

2.2) *Exercises*

- 1. Compute. Use the '=' sign and show one step for each operation.
 - (a) 7 5 + 2

- (b) **8**-3-2
- (c) 7 + 3 10
- (d) 9 (5 + 4)
- (e) $(8-3) \times 10^2$
- (f) $12-2\times 3$
- (g) $(12-2) \times 3$
- (h) $8 2^3$
- (i) $10 \times (5 3 + 1)$
- (j) $10 \times 5 3 + 1$
- (k) $10 + 5 \times 3 3$
- 2. For each of the following determine if removing the parentheses would change the value of the expression. Justify your answer.
 - (a) (7-5)+2
 - (b) 7 (5 + 2)
 - (c) $(5-4)^{10}$
 - (d) $9 (2 \times 3)$
 - (e) $(9-2) \times 3$.

LESSON 3

REVIEW

- 1. Fill in the blank using either '=' or ' \neq ' as appropriate.
 - (a) 0+5 0×5
 - (b) 1² 2
 - (c) $2 \times 2 \times 2$ 2^3
 - (d) 1 + 1 + 1 1 + (1 + 1)
 - (e) 3×9 $3 \times (3 \times 3)$
- 2. Determine whether the following are true or false. Justify your answer.
 - (a) 948 + 3221 = 3221 + 948
 - (b) $(3+2) \times 4 = 3 + 2 \times 4$
 - (c) $218 + 49 \times 85 = 218 + (49 \times 85)$
 - (d) (9+15) + 18 = 9 + (15+18)
 - (e) $(35 \times 10) \times 41 = 35 \times (10 \times 41)$
 - (f) $(4 \times 3) + 5 = 4 \times (3 + 5)$

- 1. Fill in the blank using either '=' or ' \neq ' as appropriate.
 - (a) 3^3 3 × 3
 - (b) 1³ 2⁰
 - (c) 2×4 2+4
 - (d) $340 \quad 34 \times 10$
 - (e) 3(20) 320
- 2. Determine whether the following are true or false. Justify your answer.
 - (a) $585 \times 324 = 324 \times 585$
 - (b) $(3+2)^0 = (3+2)0$
 - (c) (98+25)+33=98+(33+25)
 - (d) $4 + (3 \times 7) = (4 + 3) \times 7$
 - (e) $5 + 6 \times 2 = 5 + 2 \times 6$
 - (f) $6 \times 6 \times 6 = 6^3$

- 1. Compute. Use the '=' sign.
 - (a) 10^3
 - (b) 35^{1}
 - (c) **9**⁰
 - (d) 100×21
 - (e) 2⁴
- 2. Which one is larger 1^4 or 4^1 ? Explain your answer.
- 3. Which one is larger 5^0 or 5×0 ? Explain your answer.
- 4. Compute. Use the '=' sign. Show one step for each operation.
 - (a) $2+3 \times 0$
 - (b) $2 \times 3 \times 0$
 - (c) $2 + 2^3$
 - (d) $(2+2)^3$
 - (e) 4(1+2)
 - (f) $3 + 10^2$
 - (g) 3×10^2

- 1. Compute. Use the '=' sign.
 - (a) 5^2
 - (b) 2^5
 - (c) 35⁰
 - (d) 1^5
 - (e) 18 × 100
- 2. Which one is larger 4^0 or 0^4 ? Explain your answer.
- 3. Which one is larger 2^1 or 1^2 ? Explain your answer.
- 4. Compute. Use the '=' sign. Show one step for each operation.
 - (a) $1 + 0 \times 2$
 - (b) $1 \times 0 + 2$
 - (c) $4 + 4^1$
 - (d) $(4+4)^1$
 - (e) (3+2)2
 - (f) $3 + 3^2$
 - (g) $10^3 \times 4$

- 1. Determine for which of the following removing the parentheses would change the value of the expression. Explain your answer.
 - (a) (4+5)+3
 - (b) $4 \times (5+3)$
 - (c) $4 \times (5 \times 3)$
 - (d) $(2 \times 4)^{0}$
 - (e) $1 + (3 \times 5)$
- 2. Compute. Use the '=' sign. Show one step for each operation.
 - (a) $3 \times 100 + 2 \times 5$
 - (b) $93 \times 0 \times 81$
 - (c) $2^3 \times 1^{23}$
 - (d) 9-5-4
 - (e) $9 5 \times 1$
 - (f) $10 \times (7 4 + 2)$
 - (g) $10 \times 7 4 + 2$

- 1. Determine for which of the following removing the parentheses would change the value of the expression. Explain your answer.
 - (a) $7 \times (3+2)$
 - (b) (9-3)+1
 - (c) 9 (3 + 1)
 - (d) $(2 \times 3)^1$
 - (e) $(2 \times 3)^2$
- 2. Compute. Use the '=' sign. Show one step for each operation.
 - (a) $3 \times 200 + 3$
 - (b) $3 \times 2 \times 100$
 - (c) $2 \times 10^2 + 1 \times 10^2$
 - (d) $1 + 0^{321}$
 - (e) $(1+0)^{321}$
 - (f) $19 4^2$
 - (g) $10-3 \times 2$

Compute. Use the '=' sign. Show one step for each operation. 1. $5(2 \times 5)$

2. $5+2\times 5$

- 3. $9^{0} \times 10^{3}$
- 4. $(9 \times 10)^{0}$
- 5. $2 \times 10 + 10 \times 3$
- 6. $2 \times (10 + 2) \times 3$
- 7. 7 3 1
- 8. 7 (3 1)
- 9. $10 2 \times 5$
- 10. $(10 2) \times 5$
- 11. $12 2^2$
- 12. $(9-2) \times 10^3$
- 13. $4 + 5 \times 2 10$

LESSON 4

INTEGERS

We already mentioned that when subtracting natural numbers, the result may not be a natural number. To fix this problem we will extend the set of natural numbers to the set of integers.

The set of integers is

 $\mathbb{Z} = \{..., -4, -3, -2, -1, 0, 1, 2, 3, 4...\}$

It is important to be aware of the following:

- 1. Any natural number is an integer.
- Any natural number different from zero will be called a "positive integer". The set of positive integers is

$$\{1, 2, 3, 4, 5, 6, ...\}$$

 Any integer that is not a natural number will be called a "negative integer". The set of negative integers is

$$\{\dots -7, -6, -5, -4, -3, -2, -1\}$$

- 4. According with our definitions of positive integer and negative integer, 0 is the only integer that is neither positive nor negative.
- 5. For any integer **a**, we have that

$$a = +a$$

For example:

- (a) 3 = +3
- (b) -2 = +(-2) Notice the use of parentheses.
- (c) 0 = +0
- 6. If a is a positive integer, then -a is a negative integer and also

$$-a = -(a) = -(+a)$$

4.1) Examples

- 1. -2 is a negative integer.
- 2. -2 is not a natural number.
- 3. +124 = 124
- 4. -124 = +(-124)
- 5. -10 = -(+10)
- 6. $-6 \neq 6$
- 7. +9 = 9

In order to have a graphic picture idea of the integers, we use a number line. It is a straight line that can be made horizontal, with equally spaced points corresponding to integers.



4.2) Examples.

Plot and label the numbers on a number line



The number line gives us a sense of a number being to the left or to the right of another. For example, 0 is to the right of any negative integer and to the left of any positive integer. This idea

can be used to introduce an "order" on the integers. This order would be expressed by using the symbols '<' (less than) or '>' (greater than).

If a and b are integers then a < b (a is less than b) or b > a (b is greater than a) means that if we locate a and b on a number line, then a is to the left of b or b is to the right of a.

Notice that

- 1. If a and b are integers then either
 - (a) a = b or
 - (b) a < b or
 - (c) a > b.
- 2. If a is a positive integer then a > 0 (or 0 < a).
- 3. If a is a negative integer then a < 0 (or 0 > a).
- 4. If a is a negative integer and b is a positive integer, then a < b.

Notice:

If a and b are positive integers and a < b then -a > -b. Also, if a > b then -a < -b. For example, 3 < 5



- 2. -2 < 0
- 3. -100 < 1

4. 2 < 67

5. -67 < -2

ADDITION OF INTEGERS

We understand that addition of integers should be an extension of addition of natural numbers. Adding two integers will result in another integer.

We treat grouping symbols with integers exactly the same way that we treat them with natural numbers.

Let's set some basic rules:

1. If a is an integer, then

$$a + 0 = 0 + a = a$$

2. If **a** is a positive integer then

$$\mathbf{a} + (-\mathbf{a}) = (-\mathbf{a}) + \mathbf{a} = \mathbf{0}$$

Here it is important to notice the use of parentheses.

Two numbers are *opposites* or additive inverses of each other if when adding the two numbers, the result is zero. In particular:

1. If **a** is an integer, then

- (a) -a is the opposite of a
- (b) \mathbf{a} is the opposite of $-\mathbf{a}$.

In view of this, we can say that the opposite of the opposite of a number is the original number or

$$-(-a) = a$$

- 2. Also notice that if \mathbf{a} is a negative integer then $-\mathbf{a}$ is a positive integer.
- 3. 0 + 0 = 0, therefore the opposite of 0 is 0, or

 $-\mathbf{0}=\mathbf{0}$

4.4)Examples

- 1. 0 + (-3) = -3
- 2. -2 + 2 = 0
- 3. -53 + 0 = -53
- 4. 100 + (-100) = 0
- 5. -(-8) = 8
- 6. 0 = +0 = -0
- 7. -(-7) > -7
- 8. +(-3) < -(-3)

Exercises

- 1. Fill in the blank using either '=', '<', or '>', as appropriate.
 - (a) -101 100 (b) -(-6) 6 (c) +125 125 (d) -450 0 (e) -(-5) 0 (f) -(-7) -7(g) -5 -4

2. Plot and label the numbers on a number line.



(d) -9(e) 5 3. Write the following numbers from the smallest to the largest.

$$-2, 10, -7, -9, 5, 0$$

4. Among the following numbers, identify all pairs of opposite numbers.

$$9, 7, -9, -(-8), 8, -8, 0, 12, -(+3), +(+3)$$

5. Find the opposite of each of the following integers. Then plot both numbers on a number line.



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LESSON 5

We can add integers by using "movement on a number line". Namely, if we start on the point corresponding to zero, we will move on the line using the numbers in the addition. If we have a positive integer in the addition, that indicates movement to the right. If we have a negative integer, it indicates movement to the left. If we have zero, then we do not move.

Let's see some examples.

1. 2+3

STEP ONE

Starting on 0 we move to the right 2 units



2. (-2) + (-3)

STEP ONE



STEP TWO

Now we move again to the left 3 units



Therefore,

(-2) + (-3) = -5

Notice the use of parentheses. Also, since 2 + 3 = 5 and equals can be substituted for equals then -2 + (-3) = -(2 + 3)

3. 2 + (-3)

STEP ONE

Starting on 0 we move to the right 2 units



STEP TWO

Now we move again to the left 3 units



Therefore,

2 + (-3) = -1

Notice that since 3 - 2 = 1 (this a subtraction of natural numbers!) and equals can be substituted for equals, then 2 + (-3) = -(3 - 2)

4. -2+3

STEP ONE



STEP TWO

Now we move again to the right 3 units



Therefore,

-2 + 3 = 1

Notice that, since 3-2 = 1 and equals can be substituted for equals, then -2+3 = 3-2.

In light of these examples, we can find the following rules reasonable:

If a and b are positive integers and b > a (in the previous example a = 2, b = 3 and it is true that 3 > 2), then:

- 1. -a + (-b) = -(a + b)
- 2. a + (-b) = -(b a)

3.
$$-a+b=b-a$$

Commutative and associative properties are valid for addition of integers. That is, if x, y and z are integers, then:

- 1. x + y = y + x
- 2. (x + y) + z = x + (y + z)

5.1) Examples 1. -13 + 0 = -132. 3 + 5 = 83. -3 + (-5) = -(3 + 5) = -84. -5 + 3 = 3 + (-5) = -(5 - 3) = -25. 5 + (-5) = 06. 2 + (-7) = -(7 - 2) = -57. 0 + (-7) = -78. -3 + 5 = 5 - 3 = 29. 5 + (-3) = 5 - 3 = 210. -3 + 3 = 0

Exercises

- 1. Determine which of the following are true and in that case determine the property of addition that makes the statement true.
 - (a) -235 + 789 = 789 + (-235)
 - (b) -87 + (-56) = 87 + 56
 - (c) -99 + (-33) = -33 + (-99)
 - (d) 54 + (-67) = 67 + (-54)
 - (e) (-6+4) + (-2) = -6 + [4 + (-2)]
 - (f) -1 + (-2 + 3) = (-1 + 2) + 3
- 2. Compute
 - (a) -6 + (-2)(b) -8 + 0
 - (c) -135 + 135
 - (d) 0 + (-189)

(e)
$$-15 + 9$$

(f) $15 + (-9)$
(g) $-1 + 1$
(h) $-1 + (-1)$
(i) $6 + (+5)$
(j) $-(-4) + 0$
(k) $-18 + 8$
(l) $-25 + 29$
(m) $+1 + (+2)$
(n) $(-9) + (-11)$
(o) $-9 + 11$

(p) 9 + (-11)

LESSON 6

Multiplication of Integers.

Multiplication of integers will also be a consistent extension of the multiplication of natural numbers. Therefore, we already know how to multiply integers greater than or equal to zero. We need to explain how to multiply if any of the numbers in the multiplication is negative.

First notice that the following properties will remain:

1. For any integer a

```
a \times 1 = 1 \times a = a
a \times 0 = 0 \times a = 0
```

In fact, the only way the multiplication of two integers could be equal to zero is when one of them is equal to zero.

In particular $(-1) \times 1 = 1 \times (-1) = -1$. This suggests that the multiplication of a negative number and a positive number should be negative. We codify this as a rule:

$$(-) \times (+) = (+) \times (-) = (-)$$

2. Multiplication is commutative:

 $a \times b = b \times a$ for any integers a and b

3. Multiplication is associative:

$$(a \times b) \times c = a \times (b \times c)$$

We understand that the presence of parentheses indicates that the multiplication inside should be performed first.

Notice also that one way to think about the associative property is that now, if we want to multiply more than two numbers, we can group them in any way we want. For example, we can perform the multiplications left to right or right to left.

Now let's try to summarize some of what we know about multiplication of integers, keeping in mind that we are trying to extend the multiplication of natural numbers.

1. Since positive integers are natural numbers, multiplying two positive integers will result in another positive integer.

- 2. Multiplying a positive integer and a negative integer will result in a negative integer.
- 3. Now we wonder what should happen if we multiply two negative integers. To try to figure it out, let's recall that we said that the statement that claims that "the opposite of the opposite of a number is the original number", will be expressed by writing:

$$-(-\alpha) = \alpha$$

In particular -(-1) = 1. We also said before that when a number is in front of a grouping symbol and there is no operation sign in between, it means that the operation is actually multiplication. Then, it sounds reasonable to think that we can interpret -(-1) as indication that to eliminate the parentheses, we should "multiply" the signs. Since we know that the result is a positive number, this is leading us to think that the rule should be that multiplication of negative numbers will result in a positive one. In order to remember this rule, we can write:

$$(-) \times (-) = +$$

Now we should be ready to state how to multiply integers if any of the numbers in the multiplication is not a natural number.

If a and b are positive integers (therefore -a and -b are negative integers), then

1.
$$-\mathbf{a} \times \mathbf{b} = \mathbf{a} \times (-\mathbf{b}) = -(\mathbf{a} \times \mathbf{b})$$

In particular, notice that

$$-1 \times a = -(1 \times a) = -a$$

2. $-a \times (-b) = a \times b$

6.1) Examples.

Compute.

- 1. $3 \times (-5) = -(3 \times 5) = -15$
- 2. $-21 \times 10 = -(21 \times 10) = -210$
- 3. $-4 \times (-7) = 4 \times 7 = 28$
- 4. $0 \times (-310) = 0$
- 5. $-1 \times 529 = -529$

6. $9(-8) = -(9 \times 8) = -72$ 7. -(-35) = 358. $-1 \times 3 \times 7 = -3 \times 7 = -21$ 9. $-10 \times (-1) \times (-15) = 10 \times (-15) = -150$ 10. $(-6) \times (-7) \times 100 = 42 \times 100 = 4200$

Exponential Notation.

Now, for the exponential notation we will allow the base to be an integer, keeping the same definition we gave before.

6.2) Examples

1. $(-3)^2 = (-3) \times (-3) = 9$

Notice the presence of the parentheses indicating that the base is -3.

WARNING: $(-3)^2 \neq -3^2$. In fact

$$-3^2 = -(3^2) = -(3 \times 3) = -9$$

In other words, without the parentheses the negative sign is "external" to the exponential.

- 2. $(-2)^3 = (-2) \times (-2) \times (-2) = 4 \times (-2) = -8$
- 3. $(-1)^4 = (-1)(-1)(-1)(-1) = 1(-1)(-1) = (-1)(-1) = 1$

Notice that an even exponent with a negative base will result in a positive result and an odd exponent with a negative base will result in a negative result.

- 4. $0^{210} = 0$
- 5. $1^{325} = 1$
- 6. $(-1)^{424} = 1$
- 7. $(-1)^{1231} = -1$

Exercises

Compute

1. $-1 \times (-2)$

- 2. $3 \times (-9)$
- 3. $-5 \times 0 \times 9$
- 4. $(-2)^5$
- 5. -2^5
- 6. -10×251
- 7. $(-10)^4$
- 8. -10^4
- 9. $(-100)^3$
- 10. $(-1)^{53}$
- 11. (**-1**)⁶²
- 12. -1⁴⁸
- 13. -1⁴⁹
- 14. $2 \times (-3) \times 3$
- 15. $-5 \times 10 \times (-3)$

LESSON 7

Subtraction of Integers.

When we talked about addition of integers we stated that if a and b are positive integers with b > a, then

$$\mathbf{b} + (-\mathbf{a}) = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$$

So we can say that

 $\mathbf{b} - \mathbf{a} = \mathbf{b} + (-\mathbf{a})$

In other words, subtraction of natural numbers can be understood as addition of integers. We will extend this idea to define subtraction of integers.

If a and b are integers, then

$$a - b = a + (-b)$$

7.1) Examples.

1. -3 - 2 = (-3) + (-2) = -5

WARNING: Do not confuse -3 - 2 with -3(-2).

-3(-2) indicates multiplication not subtraction!

2.
$$5 - 8 = 5 + (-8) = -3$$

3. 9 - 5 = 4

This example can be understood as subtraction of natural numbers where the result will be another natural number, so we don't need to rewrite it as an addition.

Also notice that

$$a - (-b) = a + [-(-b)] = a + [b] = a + b$$

One way we can think of what happens here is that, in order to **eliminate** parentheses we need to multiply the sign in front of the parentheses times the sign inside. Since $(-) \times (-) = (+)$, then

$$a - (-b) = a + b$$

7.2) Examples.

- 1. 8 (-4) = 8 + 4 = 12
- 2. -8 (-4) = -8 + 4 = -4
- 3. 0 (-5) = 0 + 5 = 5

Exercises

Compute

- 1. **1**2 3
- 2. **3 12**
- 3. 12 (-3)
- 4. −3 − 12
- 5. -3 (-12)
- 6. 1 (-1)
- 7. (+1) [+(-1)]
- 8. -5 0
- 9. 5 + (-0)
- 10. 0−2
- 11. 0 (-2)
- 12. +(-4) 2

Division of Integers and Fraction Notation.

The only operation that we have not mentioned so far is division. Let's start with natural numbers.

If a and b are natural numbers with $b \neq 0$ we can perform the ordinary process of long division of a divided by b. In this case a is called the dividend and b the divisor. The process ends with a remainder r (smaller than b), and a quotient q. Both r and q are natural numbers.

We can express the result of the division in the form

$$a = b \times q + r$$

For example, when dividing 14 by 3, the quotient is 4 and the remainder is 2.

That means that:

$$14 = 3 \times 4 + 2$$

In the case when the remainder is zero (for example when dividing 12 by 3), then we have

```
a = b \times q
```

 $a \div b = q$

 $12 = 3 \times 4$

In this case we say that

For example

therefore

 $12 \div 3 = 4$

What we are trying to show is that we can say that the result of dividing one natural number by another natural number is a natural number, only when remainder of the division is zero. Since this is not always true, then we can conclude that the result of dividing natural numbers is not always another natural number.

This problem is not fixed by considering the integers. To divide integers we must extend the division of natural numbers. In other words, if a and b are integers with $b \neq 0$, and q is another integer, then $a \div b = q$ if and only if $a = b \times q$. Again, having such an integer q is not always possible. This means that the result of dividing two integers is not always another integer.

LESSON 8

Understanding that we have limitations, let's mention some things regarding division of integers. Based on the fact that $a \div b = q$ if and only if $a = b \times q$, then when a and b are positive, so is q (of course, it is the same division of natural numbers!). When a and b are both negative, then q has to be positive. If a is positive and b is negative(or vice versa), then q has to be negative.

We can write the following rules for "division of signs":

- 1. $(+) \div (+) = (-) \div (-) = (+)$
- 2. $(+) \div (-) = (-) \div (+) = (-)$

In particular, it happens that:

1. $-a \div b = a \div (-b)$

2.
$$-a \div (-b) = a \div b$$

Also notice that:

- 1. $0 \div b = 0$ ($b \neq 0$), since $0 = b \times 0$.
- 2. $a \div 0$ is "undefined".
- 3. $b \div b = 1$ ($b \neq 0$), since $b = b \times 1$.
- 8.1)Examples.
- 1. $-250 \div (-10) = 250 \div 10 = 25$
- 2. $5 \div (-5) = -5 \div 5 = -1$
- 3. $-12 \div 4 = 12 \div (-4) = -3$
- 4. $0 \div 98 = 0$
- 5. $98 \div 0$ is undefined.
- 6. $1 \div (-1) = -1$
- 7. $-1 \div (-1) = 1$
- 8. $0 \div 0$ is undefined.

Fraction Notation.

So far we have been using the symbol ' \div ' to indicate division. Another way to express it is using fraction notation, that is, to indicate $a \div b$ we can write instead

 $\frac{a}{b}$

. In this expression a (the dividend) is called the "numerator", and b (the divisor) is called the "denominator".

So we are saying:

$$a \div b = \frac{a}{b}$$
 $b \neq 0$

8.2) Examples

1.
$$\frac{0}{4} = 0 \div 4 = 0$$

2. $\frac{-3}{0}$ is undefined.
3. $\frac{-4}{2} = \frac{4}{-2} = -\frac{4}{2} = -2$
4. $\frac{-12}{-3} = \frac{12}{3} = 4$
5. $\frac{68}{-68} = -\frac{68}{68} = -1$

Exercises

Compute if possible or write undefined.

1. $0 \div (-3)$

2.
$$\frac{16}{-2}$$

3. $+(-18) \div (-3)$

4.
$$\frac{7}{0}$$

5. $[-(-5)] \div (+1)$

6. $\frac{250}{-10}$

7.
$$-(+6) \div [+(-6)]$$

8.
$$\frac{-5}{+(-5)}$$

9.
$$\frac{0}{-(-8)}$$

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LESSON 9 REVIEW.

- 1. Fill in the blank using either '=', '<', or '>', as appropriate.
 - (a) -9 1
 - (b) 0 3241
 - (c) -(-3) + 3
 - (d) +(-9) 0
 - (e) -3 -1
- 2. Write the following numbers from the smallest to the largest.

$$-134, 71, -1, -2, -(-1), +2$$

- 3. Find the opposite of each of the following integers. Then plot both numbers on a number line.
 - (a) -(-3)

Opposite:



- 1. Fill in the blank using either '=', '<', or '>', as appropriate.
 - (a) -35 -1
 - (b) +(-7) -(-7)
 - (c) 0 (-4)
 - (d) +(-4) -4
 - (e) -0 + 0
- 2. Plot and label the numbers on a number line.



3. Among the following numbers, identify all pairs of opposite numbers.

$$-2, 0, +(+2), +0, -(-4) - 3, -(+4) - (-3)$$

- 1. Find the opposite of each of the following integers. Then plot both numbers on a number line.
 - (a) +(−**1**)

Opposite:



- 2. Determine which of the following are true and in that case determine the property of addition that makes the statement true.
 - (a) -23 + 38 = 38 + (-23)

(b)
$$15 + (-21) = 21 + (-15)$$

- (c) (-5+12) + (-9) = -5 + [12 + (-9)]
- (d) -2 + [(-4) + 1] = (-2 + 4) + 1
- 3. Compute.
 - (a) 0 + (-11)
 - (b) -7 + 7
 - (c) 3 + (+3)
 - (d) 4 + (-4)
 - (e) 5 + (-0)
1. Write the following numbers from the smallest to the largest.

$$-(+5), +(-1), -240, 0, -(-50), +5$$

- 2. Compute.
 - (a) -9 + 0
 - (b) -1 + (-1)
 - (c) -0 + (-0)
 - (d) -4 + (-3)
 - (e) -6+6
 - (f) -5+4
 - (g) 5 + (-4)
 - (h) -3+2
 - (i) -2 + (-2)
 - (j) 2 + (-2)

Compute and simplify.

1. $(-3)^2$ 2. -3^2 3. $-(-2)^4$ 4. $-(-2)^3$ 5. -1(-3) 6. $3 \times (-3)$ 7. -2(2) 8. -1×4 9. $(-10)^2$ 10. $(-0)^7$ 11. $(-1)^{3}1$

12. $2 \times (-1) \times 3$

Compute and simplify.

1. $(-1)^{42}$ 2. -1⁴² 3. $-(-100)^2$ 4. -1 + (-2)5. -1(-2) 6. $31 \times (-10)$ 7. $-5 \times 2 \times (-15)$ 8. $28 \times (-45) \times 0$ 9. -5+610. -5×6 11. -5+412. -5(+4)

Compute and simplify.

1.	-1 + 0
2.	-1(+0)
3.	-6 + (+6)
4.	$-3 \times (-3) \times (-1)$
5.	0 ⁵
6.	$(-1)^{13}$
7.	-9 + 5
8.	-9 + (-5)
9.	-9(+5)
10.	-9(-5)
11.	-5 + (-5)
12.	+5 + (-5)
13.	-7 ²
14.	$(-6)^2$
15.	-1+3
16.	$-(-1)^{12}$

Compute if possible or write 'undefined'.

1. **-1 - 4** 2. -1(-4) 3. $\frac{-4}{-1}$ 4. $0 \div (-4)$ 5. -2 + (-3)6. -2 - (-3)7. $\frac{5}{0}$ 8. -1+3 9. **-1**(+3) 10. $(-12) \div (-4)$ 11. $\frac{0}{2}$ 12. **-1 - 1** 13. -1 + (-1)14. **2**-**3**

Compute if possible or write 'undefined'.

1. $(-1)^5$ 2. (-1)5 3. -1+54. 5 + (-5)5. $5 \div (-5)$ 6. 5(-5) 7. -9-9 8. **-9**+**8** 9. -9 + 1010. $35 \times (-10)$ 11. (**-10**)⁰ 12. $\frac{9}{0}$ 13. $8 \div (-2)$ 14. $-2 \times (-2)$

Compute if possible or write 'undefined'.

1.	-3 + (+7)
2.	-3(+7)
3.	-3-(+7)
4.	-3-(-7)
5.	$\frac{10}{-5}$
6.	$(-5) \div 0$
7.	$0 \div (-5)$
8.	$(-1) \times (-1) \times (-3)$
9.	$-(-1)^{38}$
10.	$(-5)^3$
11.	-2-2
12.	4-5
13.	-4 - 5
14.	-30

Compute if possible or write 'undefined'.

1. -3 + (+3)2. -7 - (-2)3. 9 + (-8)4. -5-55. $(-1)^9$ 6. $-(-1)^{10}$ 7. $-(-10)^3$ 8. (85)⁰ 9. $-9 \times (-10) \times (-1)$ 10. $8 \times (-8) \times 2$ 11. $(-49) \div 7$ 12. $0 \div 51$ 13. $-8 \div 0$ 14. $1 \div (-1)$

LESSON 10 Order of Operations.

Let's talk again about the order of operations if we include subtraction and division. Operations should be performed in the following order:

1. Operations inside grouping symbols. If there are grouping symbols inside grouping symbols, then we need to work from inside out.

WARNING: If the result coming from a grouping symbol is a negative number, keep the number in a grouping symbol.

- 2. Exponentials.
- 3. Multiplications and divisions are associated LEFT TO RIGHT.

WARNING: There is not priority of multiplication over division or the other way around. You perform the one that comes first from left to right, regardless of whether it is multiplication or division.

4. Additions and subtractions are associated LEFT TO RIGHT.

Be aware that if within grouping symbols there is more than one operation then, inside the grouping symbol we should also follow the order, starting with exponentials.

9.1) Examples.

1.
$$(-3 + 2 - 2) \times 10 = (-1 - 2) \times 10 = (-3) \times 10 = -30$$

2. $(2 - 5)^3 = (-3)^3 = -3 \times (-3) \times (-3) = -27$
3. $2 - 5^3 = 2 - 125 = -123$
4. $-3 - 3 + 2 \times 4 = -3 - 3 + 8 = -6 + 8 = 2$
5. $(-2)^2 - 36 \div (-6) \times 2 = 4 - 36 \div (-6) \times 2 = 4 + 6 \times 2 = 4 + 12 = 16$
6. $-3^2 + 9 \times 6 \div 3 - 20 = -9 + 9 \times 6 \div 3 - 20 = -9 + 54 \div 3 - 20 = -9 + 18 - 20 = 9 - 20 = -11$
7. $-2 - (-3 + 1) \div (-2) \times 2 = -2 - (-2) \div (-2) \times 2 = -2 + 2 \div (-2) \times 2 = -2 - 1 \times 2 = -2 - 2 = -4$
8. $3 + 2(5 - 7) = 3 + 2(-2) = 3 - 4 = -1$

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9. (3+2)(5-7) = 5(-2) = -10

WARNING: Notice that if you do not use a parentheses for the result of the second parentheses, you will be writing 5-2 which is actually a subtraction and not a multiplication which is the right operation.

10. $-(3-2 \times 2) - (8 \div 2 \div 2) = -(3-4) - (4 \div 2) = -(-1) - 2 = 1 - 2 = -1$ 11. $2[4-3(1-2 \times 2) + 1] = 2[4-3(1-4) + 1] = 2[4-3(-3) + 1] = 2[4+9+1] = 2 \times 14 = 28$

Exercises

Compute if possible, or write that the expression is undefined. When computing, show one step for each operation. Make sure that you use the '=' sign correctly.

1. -5-3 2. (-5)(-3)3. -1 - 1(-1)4. $(4 - 2 \times 2)^{0}$ 5. $[5+5(-1)] \div [4-5]$ 6. $(4-5) \div [3+3(-1)]$ 7. $-1^2 - 10 \div (-5) \times 2$ 8. $7 - 3 \times 2$ 9. (7-3)210. $-2 + 4 \times (-1) + 3$ 11. -3 - 3(4 - 5)

12. (-3-3)(4-5)13. $5 - 2^2 + 3(-2)$ 14. $(5-2)^3 + 3(-2)$ 15. $-[5-2(1+1\times 3)] - [2-(2^2-8\div 2)]$ 16. (-1)(-2)(-5)17. $(-10)10^3 \times 10^0$ 18. -(-3)[-(-7)]19. $(-8^2 - 3^4 - 2^5)(-6 - 4)$ 20. -12 - (-4) + 3 - (4 + 3)21. $-125 \div 25 + [(-6 - 3) - 6 - 3]$ 22. -[(-7-5)(-7+5)(7-5)(7+5)]23. $-876 \times (-954)^0 - 3^1$

LESSON 11 REVIEW

Compute if possible or write 'undefined'. Show one step of each operation. Make sure that you use the '=' sign correctly.

1. $(1-3)^3$

- 2. $1 3^3$
- 3. $(-1 1 + 1) \times 10$
- 4. $4 3 \times 2 1$
- 5. $(-1)^2 9 \div (-3)$
- 6. 1 + 1(-2 2)
- 7. (2+1)(-1+1)(8-3)
- 8. $7 3^2 + 2(-1)$

Compute if possible or write 'undefined'. Show one step of each operation. Make sure that you use the '=' sign correctly.

1. $2 - (2 - 3)^{20}$

2. $-5 + 9^{\circ}$

3. $[2+2(-2)] \div (3-5)$

4. $-2^3 + 0 \div 2$

5. 3 + 2(-1 - 1)

6. $9 - 10 \div 5 \times 2$

7. $-[4 - 3(-1 - 1 \times 2)] + 2 - (-1)^3$

8. $(-10)^3 \div 10^2 \times 10^3$

Compute if possible or write 'undefined'. Show one step of each operation. Make sure that you use the '=' sign correctly.

1. -(-2)[-(-5)]

- 2. $(-3^2 2^3 5^0)(-1 2)$
- 3. 9 3 3 (-5 + 1)
- 4. $36 \div (-2) + [-(-7 3) \times 2]$
- 5. $-535 \times (536)^0 + 2^1$
- 6. $-3 + 5 \times (-2) \div 2 \times 5$
- 7. $-2[3-3(2-1\times 3)] [4-(1^3-9\div (-9))]$

LESSON 12

We already mentioned that sometimes division of two integers is not equal to an integer. This means that we need more numbers (those representing the result of a division when the result is not an integer). We will write those numbers as the **fraction** representing the division.

A fraction is an expression of the form

$$\frac{a}{b}$$
 where **a** and **b** are integers and $b \neq 0$

a is the numerator of the fraction and b the denominator of the fraction.

Since we know that a fraction also indicates a division, then some fractions are actually integers. For example:

1.
$$\frac{-6}{3} = -2$$

2. $\frac{10}{1} = 10$

Any fraction with denominator 1 is actually an integer. In fact, it is equal to the numerator of the fraction. This also means that any integer can be written as a fraction. Any integer is equal to the fraction with numerator equal to the same integer and denominator equal to 1.

For example:

(a)
$$7 = \frac{7}{1}$$

(b) $-14 = \frac{-14}{1}$

3. $\frac{-5}{-5} = 1$

When the numerator and denominator are equal (and both are different from zero!), then the fraction is equal to 1.

4.
$$\frac{0}{3} = 0$$

When the numerator is zero (and the denominator is different from zero) then the fraction is equal to zero.

5. $\frac{4}{0}$

An expression like this is considered **undefined**.

Now let's try to understand the meaning of a fraction that is not an integer. Let's start with the case when the numerator and denominator are positive integers: If \mathfrak{m} and \mathfrak{n} are positive integers, we can use the part-whole interpretation for the fraction $\frac{\mathfrak{m}}{\mathfrak{n}}$. That is, we can think that units of something are divided into \mathfrak{n} equal pieces and then we take \mathfrak{m} of those pieces. Notice that if $\mathfrak{m} > \mathfrak{n}$, in order to be able to take \mathfrak{m} pieces, we will need more than one unit. If $\mathfrak{m} < \mathfrak{n}$ one unit will be enough to cover the \mathfrak{m} pieces.

12.1)Examples

1. To understand the meaning of the fraction $\frac{3}{4}$ we consider a "unit" (for example, a square) divided into 4 equal pieces. Then we take 3 pieces.



If we think of the "unit" as the number 1 then we can conclude that $\frac{3}{4}$ is a number between 0 and 1, closer to 1 than to 0. Notice that the numerator is less than the denominator.

2. If we want to understand the fraction $\frac{5}{2}$ now we need to divide units into 2 equal pieces and then take 5 pieces. Therefore, not only we need more than one "unit", we actually need more than 2 "units".



This interpretation is telling us that $\frac{5}{2}$ is a number between 2 and 3 and in fact, it is right in the middle between 2 and 3. Now, when we say "middle", we are basically thinking of the location of the fraction as a number on a number line. If the fraction is not an integer, then the fraction corresponds to a point on the line that happens to be between two integers.

Location of a fraction on a number line.

Based on the rules for division of signs, we can conclude that when a fraction is not an integer, it can be written as either $\frac{m}{n}$ (positive fraction) or $-\frac{m}{n}$ (negative fraction) where m and n are positive integers. One way to understand the fraction as a number is by finding a location on a number line. To do that, we will consider the segment between two consecutive integers as a "unit". For example, the segment between -3 and -2 is a unit, the segment between -2 and -1 is a unit, the segment between 0 and 1 is a unit, etc.

We will understand that each unit is divided into n equal pieces. To find the location of the fraction on a number line we start on 0. If the fraction is positive, we will move to the right till we cover m pieces. If the fraction is negative, then we move to the left till we cover m pieces.

12.2) Examples.

Locate the fractions on a number line.

1. $\frac{1}{2}$



In this case, each unit is divided into two equal pieces (because the denominator is 2). Since the fraction is positive, we start on "0" and move to the right until we cover one piece (because the numerator is 1). Notice that this location is showing that $\frac{1}{2}$ is the number in the "middle" between 0 and 1.



To locate this fraction, each unit is divided into three equal pieces (because the denominator is 3). Since the fraction is positive, we start on 0 and move to the right until we cover five pieces (because the denominator is 5). This location shows that $\frac{5}{3}$ is a number between 1 and 2 closer to 2 than to 1.

3. $-\frac{9}{4}$



Now we need to divide each unit into four equal pieces (the denominator is equal to 4). Since this fraction is negative, we start on 0 and move to the left until we cover nine pieces (the numerator is equal to 9). This location shows that $-\frac{9}{4}$ is a number between -3 and -2, closer to -2 than to -3.

The set of numbers that can be written as fractions (and that includes the integers!) is called the set of **Rational Numbers**. We will use \mathbb{Q} to denote this new set.

Rational numbers can be located on a number line. Therefore, we have an "order" in \mathbb{Q} that we will express using the symbols '<'(is less than) or '>' (is greater than).

12.3) Examples.

Based on the examples in 12.2, now we can say that

1. $\frac{1}{2} > 0$ 2. $\frac{1}{2} < 1$ 3. $0 < \frac{1}{2} < 1$ 4. $\frac{5}{3} > 1$ 5. $\frac{5}{3} < 2$ 6. $1 < \frac{5}{3} < 2$ 7. $-\frac{9}{4} > -3$ 8. $-\frac{9}{4} < -2$ 9. $-3 < -\frac{9}{4} < -2$

It is also important to notice that if a and b are integers, with $b \neq 0$, then:

- 1. $\frac{-a}{b} = -\frac{a}{b}$
For example, $\frac{-3}{7} = -\frac{3}{7}$
2. $\frac{a}{-b} = -\frac{a}{b}$
 - For example, $\frac{3}{-7} = -\frac{3}{7}$
- 3. Based on the two previous items, we can write:

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

For example,

$$\frac{-3}{7} = \frac{3}{-7} = -\frac{3}{7}$$

4.
$$\frac{-a}{-b} = \frac{a}{b}$$

For example, $\frac{-3}{-7} = \frac{3}{7}$
Exercises

1. Locate the number on a number line.

(a)
$$\frac{7}{6}$$



(d)
$$-\frac{1}{3}$$



2. Fill in the blank using either '=', '<', or '>' as appropriate.



LESSON 13

Multiplication of Fractions.

We would like the multiplication of fractions to extend the multiplication of integers. In particular, if we write integers as fractions and we multiply them as fractions, the result should be the same as when we use the multiplication of integers.

For example $3 \times 2 = 6$, as fractions $\frac{3}{1} \times \frac{2}{1} = \frac{6}{1}$. This would make us think that the way to multiply positive fractions is by multiplying the numerators and multiplying the denominators. That is in fact the way to do it. When multiplying fractions that could be either positive or negative, we will use the rules of multiplication of signs, along with multiplying numerators and denominators.

In general, we can set the following rules regarding multiplication of fractions:

- 1. $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$ $b \neq 0; d \neq 0$
- 2. Notice that as a consequence of the previous point,

$$\frac{a}{b} \times 0 = 0 \times \frac{a}{b} = 0 \qquad b \neq 0$$

Let's check it

$$\frac{a}{b} \times 0 = \frac{a}{b} \times \frac{0}{1} = \frac{a \times 0}{b \times 1} = \frac{0}{b} = 0$$

3. Also as a consequence of the first point we have

$$\frac{a}{b} \times 1 = 1 \times \frac{a}{b} = \frac{a}{b} \qquad b \neq 0$$
$$\frac{a}{b} \times 1 = \frac{a}{b} \times \frac{1}{1} = \frac{a \times 1}{b \times 1} = \frac{a}{b}$$

13.1) Examples.

1. $\frac{3}{4} \times \frac{7}{2} = \frac{3 \times 7}{4 \times 2} = \frac{21}{8}$ 2. $\frac{5}{2} \times 9 = \frac{5}{2} \times \frac{9}{1} = \frac{5 \times 9}{2 \times 1} = \frac{45}{2}$

WARNING: When multiplying a fraction times an integer we need to write the integer as a fraction first, and then apply what we know about multiplying fractions. In particular

$$\frac{5}{2} \times 9 \neq \frac{45}{18}$$

We are not supposed to invent our own rules!!!

We should not multiply both numerator and denominator by 9!

3.
$$11 \times \frac{10}{7} = \frac{11}{1} \times \frac{10}{7} = \frac{11 \times 10}{1 \times 7} = \frac{110}{7}$$

4. $\frac{8}{5} \times 1 = \frac{8}{5}$

Also notice the properties:

1. Commutative property.

$$\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$$

2. Associative property.

$$\left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right)$$

13.1) Exercises.

Compute. If the resulting fraction is an integer, indicate it.

1. $\frac{7}{3} \times \frac{2}{5}$ 2. $3 \times \frac{1}{5}$ 3. $\frac{2}{7} \times 5$ 4. $\frac{11}{12} \times \frac{0}{5}$ 5. $\frac{9}{2} \times 1$ 6. $1 \times \frac{3}{5}$ 7. $\frac{1}{4} \times \frac{7}{4}$ 8. $\frac{1}{4} \times 4$ 9. $\frac{3}{2} \times \frac{4}{3}$ 10. $0 \times \frac{2}{3}$

Equivalent Fractions.

Two fractions may look different and still represent the same number. In that case, we say that they are equivalent, and we can state that they are equal.

For example, $\frac{1}{2} = \frac{2}{4}$. Let's see it from the part-whole interpretation point of view:

$\frac{1}{2}$			
2			
$\frac{2}{4}$			
From the arithmetic poin	nt of view, we can	take advantage of the rul	es for multiplication of

 $\frac{1}{2} = \frac{1}{2} \times 1$

 $1 = \frac{2}{2}$

Since

fraction. That is:

and equals can be substituted for equals, then:

$$\frac{1}{2} = \frac{1}{2} \times 1 = \frac{1}{2} \times \frac{2}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

In general for any fraction, we have:

$$\frac{a}{b} = \frac{a}{b} \times 1$$

For any integer $n \neq 0$, we have that

$$1 = \frac{n}{n}$$

and since equals can be substituted by equals, then:

$$\frac{a}{b} = \frac{a}{b} \times 1 = \frac{a}{b} \times \frac{n}{n} = \frac{a \times n}{b \times n}$$

This shows that if we multiply both numerator and denominator by the same integer (different from zero!), the "new" fraction is equivalent to the original one.

13.2) Examples.

1. Find a fraction equivalent to $\frac{7}{3}$

$$\frac{7}{3} = \frac{7}{3} \times 1 = \frac{7}{3} \times \frac{5}{5} = \frac{7 \times 5}{3 \times 5} = \frac{35}{15}$$

Therefore, one possible fraction equivalent to $\frac{7}{3}$ is $\frac{35}{15}$. Notice that replacing 1 with $\frac{5}{5}$ was just a choice. We could have picked any other fraction equal to 1.

2. Find a fraction equivalent to $\frac{7}{3}$ with denominator 18.

In this case we are looking for a particular new denominator. It is possible to make it be 18 because $18 = 3 \times 6$. If would not be possible, for example, to have 19 for the new denominator.

$$\frac{7}{3} = \frac{7}{3} \times 1 = \frac{7}{3} \times \frac{6}{6} = \frac{7 \times 6}{3 \times 6} = \frac{42}{18}$$

Therefore, the fraction we are looking for is $\frac{42}{18}$.

3. Find a fraction equivalent to $\frac{5}{2}$ with denominator 25.

This is not possible because 25 can not be the result of multiplying 2 by another integer!

4. Find a fraction equivalent to $\frac{11}{6}$ with denominator 24.

$$\frac{11}{6} = \frac{11}{6} \times 1 = \frac{11}{6} \times \frac{4}{4} = \frac{11 \times 4}{6 \times 4} = \frac{44}{24}$$

So, the fraction that we are looking for is $\frac{44}{24}$.

13.2) *Exercises*

- 1. Find 2 fractions equivalent to $\frac{5}{2}$. Show all the steps.
- 2. Find a fraction equivalent to $\frac{3}{5}$ with denominator 100. Show all the steps.
- 3. Find a fraction equivalent to $\frac{7}{10}$ with denominator 50. Show all the steps.
- 4. Determine which of the following pairs of fractions are equivalent. Justify your answer.

(a)
$$\frac{3}{7}, \frac{12}{28}$$

(b) $\frac{2}{3}, \frac{5}{6}$
(c) $\frac{0}{5}, \frac{0}{3}$
(d) $\frac{7}{7}, \frac{345}{345}$
(e) $\frac{14}{2}, \frac{21}{3}$
(f) $\frac{1}{16}, \frac{1}{32}$
(g) $\frac{1}{4}, \frac{2}{8}$

LESSON 14

Simplification of Fractions.

Notice that the process that we explained in the previous lesson to find an equivalent fraction, will make the "new" fraction have a bigger numerator and denominator.

Now, let's consider what happens if we do this process backwards. In other words, given a fraction, we want to find an equivalent fraction with a smaller numerator and denominator (if possible). This process is called "simplifying the fraction" or "reducing the fraction to lowest terms".

Take for instance the last example we discussed. In this case, the original fraction is $\frac{44}{24}$ and we want to show that $\frac{11}{6}$ is equivalent. We can just copy the steps we did before but backwards; however, we need to understand the meaning of the steps.

Simplify $\frac{44}{24}$.

FIRST STEP

$$\frac{44}{24} = \frac{11 \times 4}{6 \times 4}$$

Notice that we expressed both numerator and denominator as a multiplication. These multiplica-

tions have a factor in commom (in this case it is 4). We will call this factor a **common factor**. In this example, we factored 44 and 24 using the common factor 4.

SECOND STEP

$$\frac{44}{24} = \frac{11 \times 4}{6 \times 4} = \frac{11}{6} \times \frac{4}{4}$$

Here we are using the rule for multiplication of fractions "backwards". We certainly know that

$$\frac{11}{6} \times \frac{4}{4} = \frac{11 \times 4}{6 \times 4}$$

so we can say

$$\frac{11\times4}{6\times4} = \frac{11}{6}\times\frac{4}{4}$$

Also, we are making sure to use the fraction with numerator and denominator given by the common factor. Such fraction will be equal to 1.

THIRD STEP

$$\frac{44}{24} = \frac{11 \times 4}{6 \times 4} = \frac{11}{6} \times \frac{4}{4} = \frac{11}{6} \times 1$$

We know that

 $\frac{4}{4} = 1$

and since equals can be substituted for equals, then:

$$\frac{11}{6} \times \frac{4}{4} = \frac{11}{6} \times 1$$

FOURTH STEP

$$\frac{44}{24} = \frac{11 \times 4}{6 \times 4} = \frac{11}{6} \times \frac{4}{4} = \frac{11}{6} \times 1 = \frac{11}{6}$$

Let's comment on some things regarding this example.

Based on the last step we could write:

$$\frac{11\times4}{6\times4} = \frac{11}{6}$$

We can say that the effect was of "cancelling" the common factor 4. Once we advance some more in this class, we will show simplification without providing all the steps. We will go straight from the fraction that shows the common factor (or factors) to the simplified fraction, thinking about "cancelling". However, it is very important that we understand what allows us to do the "cancelling". That is, the fact that we have a multiplication in the numerator and denominator showing a common factor.

WARNING:

 $\frac{11+4}{6+4} \neq \frac{11}{6} \quad 4 \text{ can not be cancelled}$

In order to cancel, we must have **multiplication** in the numerator and denominator showing a common factor.

14.1)Examples.

Simplify the fractions as much as possible. Show all the steps of the simplification.

1. $\frac{18}{27}$

$$\frac{18}{27} = \frac{2 \times 9}{3 \times 9} = \frac{2}{3} \times \frac{9}{9} = \frac{2}{3} \times 1 = \frac{2}{3}$$

Notice that we could have done:

$$\frac{18}{27} = \frac{6 \times 3}{9 \times 3} = \frac{2 \times 3 \times 3}{3 \times 3 \times 3} = \frac{2}{3} \times \frac{3}{3} \times \frac{3}{3} = \frac{2}{3} \times 1 \times 1 = \frac{2}{3} \times 1 = \frac{2}{3}$$

2. $\frac{84}{132}$

$$\frac{84}{132} = \frac{7 \times 12}{11 \times 12} = \frac{7}{11} \times \frac{12}{12} = \frac{7}{11} \times 1 = \frac{7}{11}$$

Or

$$\frac{84}{132} = \frac{42 \times 2}{66 \times 2} = \frac{6 \times 7 \times 2}{11 \times 6 \times 2} = \frac{6}{6} \times \frac{7}{11} \times \frac{2}{2} = 1 \times \frac{7}{11} \times 1 = \frac{7}{11} \times 1 = \frac{7}{11}$$

Notice that since multiplication of rational numbers is commutative and associative, we don't need the factors to be written in any particular order and when having more than two numbers in a multiplication we can multiply left to right or right to left.

Or

$$\frac{84}{132} = \frac{42 \times 2}{66 \times 2} = \frac{21 \times 2 \times 2}{33 \times 2 \times 2} = \frac{7 \times 3 \times 2 \times 2}{11 \times 3 \times 2 \times 2} = \frac{7}{11} \times \frac{3}{3} \times \frac{2}{2} \times \frac{2}{2} = \frac{7}{11} \times 1 \times 1 \times 1 = \frac{7}{11}$$

3. $\frac{63}{8}$

This fraction can not be simplified because 63 and 8 do not have any common factors.

Exercises

Simplify the fractions. Show all the steps of the simplification.

- 1. $\frac{9}{21}$ 2. $\frac{81}{54}$
- 3. $\frac{0}{5}$
- 4. $\frac{88}{4}$

5.	$\frac{1230}{1230}$
6.	$\frac{9240}{220}$
7.	$\frac{5\times45}{45\times3}$
8.	$\frac{7 \times 100}{14 \times 10}$
9.	$\frac{28}{5 \times 28}$
10.	$\frac{9\times10\times25}{25\times9\times10}$

LESSON 15 REVIEW



1. Locate the number on a number line.

2. Fill in the blank using either '=', '<', or '>' as appropriate.

(a)
$$0 -\frac{2}{3}$$

(b) $\frac{-3}{10} -\frac{3}{-10}$
(c) $\frac{5}{7} - 0$
(d) $\frac{-4}{5} -\frac{-7}{-8}$

1. Locate
$$\frac{4}{3}$$
 and $-\frac{5}{3}$ on a number line.



2. Fill in the blank using either '=', '<', or '>' as appropriate.

(a)
$$0 \frac{0}{-8}$$

(b) $\frac{7}{8} 1$
(c) $\frac{-7}{-3} \frac{7}{3}$
(d) $\frac{1000}{-1000} \frac{1}{1000}$

3. Compute. If the resulting fraction is an integer, indicate it.

(a)
$$\frac{5}{4} \times \frac{1}{3}$$

(b)
$$7 \times \frac{7}{8}$$

(c) $\frac{1}{3} \times \frac{1}{3}$

- 1. Compute. If the resulting fraction is an integer, indicate it.
 - (a) $\frac{2}{5} \times \frac{7}{3}$
 - (b) $1 \times \frac{8}{7}$
 - (c) $\frac{0}{3} \times \frac{1}{10}$
 - ${\rm (d)}~\frac{3}{8}\times7$
- 2. Find two fractions equivalent to $\frac{2}{7}$
- 3. Find a fraction equivalent to $\frac{1}{10}$ with denominator 800.
- 4. Determine which of the following pair of fractions are equivalent.

(a)
$$\frac{3}{13}; \frac{7}{17}$$

(b) $\frac{1}{5}; \frac{5}{25}$
(c) $\frac{0}{7}; \frac{0}{8}$
(d) $\frac{1}{15}; \frac{1}{30}$

1. Find a fraction equivalent to
$$\frac{2}{15}$$
 with denominator 60.

- 2. Determine which of the following pair of fractions are equivalent.
 - (a) $\frac{54}{2}; \frac{81}{3}$ (b) $\frac{6}{7}; \frac{12}{21}$ (c) $\frac{8}{8}; \frac{1000}{1000}$ (d) $\frac{3}{8}; \frac{12}{32}$
- 3. Simplify the fractions. Show all the steps of the simplification.

(a)
$$\frac{45}{5}$$

(b) $\frac{31}{93}$

${\rm Homework/Class}~{\rm Work/Quiz}~26$

1.	<u>90</u> 12
2.	$\frac{0}{7}$
3.	$\frac{232}{4}$
4.	<u>820</u> 220
5.	$\frac{9 \times 37}{37 \times 6}$
6.	$\frac{14 \times 22}{33 \times 7}$
7.	$\frac{9 \times 6}{36}$
8.	$\frac{8 \times 17 \times 11}{11 \times 16 \times 17}$
9.	$\frac{19 \times 36 \times 42}{21 \times 5 \times 12}$

Simplify the fractions. Show all the steps of the simplification.
LESSON 16

Multiplication of Fractions and Simplification.

We explained that simplifying a fraction is possible when we can express, both the numerator and denominator, as a product with common factors. On the other hand, multiplying fractions will result in having the numerators and denominators multiplied. This means that the numerators of the fractions will be factors of the resulting fraction, and the denominators will be factors of the denominator of the resulting fraction. Therefore, we will be able to simplify before actually performing the multiplication.

16.1) Examples.

1.

$$\frac{7}{3} \times \frac{3}{8} = \frac{7 \times 3}{3 \times 8} = \frac{7}{8} \times \frac{3}{3} = \frac{7}{8} \times 1 = \frac{7}{8}$$

We can think that we "cancelled" the 3 in the denominator of the first fraction with the 3 in the numerator of the second fraction. We understand that $3 = 3 \times 1$, so when cancelling, what we have left at the denominator of the first fraction and numerator of the second, is 1.

$$\frac{7}{3} \times \frac{3}{8} = \frac{7}{1 \times 3} \times \frac{3 \times 1}{8} = \frac{7 \times 3 \times 1}{1 \times 3 \times 8} = \frac{7}{1} \times \frac{1}{8} \times \frac{3}{3} = \frac{7}{1} \times \frac{1}{8} \times 1 = \frac{7}{1} \times \frac{1}{8} = \frac{7}{8}$$

Most of the time we go straight from the multiplication to what is left after cancelling the common factor. In this case, it would be:

$$\frac{7}{3} \times \frac{3}{8} = \frac{7}{1} \times \frac{1}{8} = \frac{7}{8}$$

2.

$$\frac{7}{3} \times \frac{9}{8} = \frac{7}{1 \times 3} \times \frac{3 \times 3}{8} = \frac{7}{1} \times \frac{3}{8} = \frac{21}{8}$$

As a result of the associative property, we can multiply more than two fractions, grouping in any way we want. We will end up multiplying all numerators and all denominators. Also, keep in mind what it means to have a number followed by an expression in parentheses, or the other way around. It means that the operation in between is multiplication.

If we include negative fractions in the multiplication, we should follow the rules for multiplication of signs and again multiply numerators and denominators. Another important point is that when multiplying fractions, it is convenient to try to simplify before multiplying. Multiplication is the only operation with fractions that allows this procedure.

16.2) Examples.

Compute and simplify. Show all the factors that can be cancelled.

$$1. \frac{6}{5} \times \frac{9}{4} \times \frac{15}{18} = \frac{3 \times 2}{5 \times 1} \times \frac{9 \times 1}{2 \times 2} \times \frac{3 \times 5}{2 \times 9} = \frac{3}{1} \times \frac{1}{2} \times \frac{3}{2} = \frac{3 \times 1 \times 3}{1 \times 2 \times 2} = \frac{9}{4}$$

$$2. \frac{4}{5} \left(-\frac{10}{3}\right) = -\left(\frac{4}{5} \times \frac{10}{3}\right) = -\left(\frac{4}{1 \times 5} \times \frac{5 \times 2}{3}\right) = -\left(\frac{4}{1} \times \frac{2}{3}\right) = -\frac{8}{3}$$

$$3. -\frac{7}{4} \times \left(-\frac{1}{14}\right) = \frac{7 \times 1}{4} \times \frac{1}{2 \times 7} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$$4. -\frac{9}{2} \left(-\frac{3}{7}\right) \left(-\frac{14}{30}\right) = -\left(\frac{9}{1 \times 2} \times \frac{3 \times 1}{7 \times 1} \times \frac{7 \times 2}{3 \times 10}\right) = -\left(\frac{9}{1} \times \frac{1}{1} \times \frac{1}{10}\right) = -\frac{9}{10}$$

$$5. -\frac{121}{90} \left(\frac{45}{10}\right) \left(-\frac{2}{121}\right) = \frac{121}{45 \times 2 \times 1} \times \frac{45 \times 1}{10} \times \frac{2 \times 1}{121 \times 1} = \frac{1}{1} \times \frac{1}{10} \times \frac{1}{1} = \frac{1}{10}$$

$$6. 3 \left(-\frac{4}{9}\right) \frac{7}{2} = -\left(\frac{3}{1} \times \frac{4}{9} \times \frac{7}{2}\right) = -\left(\frac{3 \times 1}{1} \times \frac{2 \times 2}{3 \times 3} \times \frac{7}{2 \times 1}\right) = -\left(\frac{1}{1} \times \frac{2}{3} \times \frac{7}{1}\right) = -\frac{14}{3}$$

$$7. \frac{1}{2}(-5) = -\left(\frac{1}{2} \times \frac{5}{1}\right) = -\frac{5}{2}$$

Exponential Notation.

Let's talk again about exponential notation. This time we will allow the base to be a fraction. In that case, we need to use a parentheses for the base and the exponent should be written outside the parentheses. The reason for this is that, if the parentheses is not there, it would look like if the exponent is only for the numerator and not for the whole fraction. Also, if a negative sign is inside the parentheses (and in front of the fraction), then the negative sign is part of the base. If we see a negative sign outside the parentheses that is used for the base, then it would be "external" to the exponential.

16.3) Examples. Compute.

$$1. \left(\frac{1}{2}\right)^{3} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$2. \left(\frac{3}{4}\right)^{2} = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

$$3. \left(-\frac{2}{3}\right)^{4} = \left(-\frac{2}{3}\right) \left(-\frac{2}{3}\right) \left(-\frac{2}{3}\right) \left(-\frac{2}{3}\right) = \frac{16}{81}$$

$$4. - \left(\frac{1}{3}\right)^{4} = -\left(\frac{1}{3}\right) \left(\frac{1}{3}\right) \left(\frac{1}{3}\right) \left(\frac{1}{3}\right) = -\frac{1}{81}$$

$$5. \left(-\frac{4}{3}\right)^{3} = \left(-\frac{4}{3}\right) \left(-\frac{4}{3}\right) \left(-\frac{4}{3}\right) = -\frac{64}{27}$$

$$6. \left(-\frac{2}{5}\right)^{2} = \left(-\frac{2}{5}\right) \times \left(-\frac{2}{5}\right) = \frac{2}{5} \times \frac{2}{5} = \frac{2 \times 2}{5 \times 5} = \frac{4}{25}$$

$$7. \left(-\frac{1}{3}\right)^{4} = \left(-\frac{1}{3}\right) \times \left(-\frac{1}{3}\right) \times \left(-\frac{1}{3}\right) \times \left(-\frac{1}{3}\right) = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1 \times 1 \times 1 \times 1 \times 1}{3 \times 3 \times 3 \times 3} = \frac{1}{81}$$

$$8. \left(-\frac{2}{3}\right)^{3} = \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) = -\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = -\frac{2 \times 2 \times 2}{3 \times 3 \times 3} = -\frac{8}{27}$$

$$9. - \left(\frac{1}{2}\right)^{2} = -\left(\frac{1}{2} \times \frac{1}{2}\right) = -\frac{1 \times 1}{2 \times 2} = -\frac{1}{4}$$

$$10. - \left(-\frac{1}{2}\right)^{4} = -\left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) = -\frac{1}{16}$$

$$11. - \left(-\frac{1}{2}\right)^{3} = -\left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) = \frac{1}{8}$$

Exercises

Compute and simplify. Show all the factors that can be cancelled.

1. $\frac{10}{3} \times \frac{9}{2}$ 2. $\frac{16}{5} \times \frac{9}{7} \times \frac{70}{12}$ 3. $\frac{100}{9}\left(-\frac{6}{20}\right)$ 4. $-\frac{2}{7}\left(-\frac{21}{10}\right)\left(-\frac{15}{4}\right)$ 5. $-\frac{36}{25} \times \frac{90}{7} \times \left(-\frac{35}{120}\right)$ 6. $\frac{5}{8}(-16)$ 7. $9 \times \left(-\frac{4}{15}\right) \times \frac{1}{2}$ 8. $-\left(-\frac{1}{3}\right)^3$ 9. $\left(\frac{2}{5}\right)^2$ 10. $-\left(-\frac{1}{2}\right)^4$ 11. $\left(\frac{3}{5}\right)^2$

$$12. - \left(\frac{1}{5}\right)^2$$

$$13. \left(-\frac{1}{3}\right)^2$$

$$14. - \left(-\frac{1}{9}\right)^2$$

$$15. \left(\frac{2}{3}\right)^3$$

$$16. \left(-\frac{3}{4}\right)^3$$

$$17. - \left(-\frac{2}{3}\right)^3$$

$$18. \left(-\frac{1}{2}\right)^4$$

LESSON 17

In order to figure out what would be the "right" rule for division of fractions, let's notice some facts.

1. Division of integers can be written as **multiplication of an integer and a fraction**. For example

(a)

$$10 \div 2 = \frac{10}{2} = 10 \times \frac{1}{2}$$

(b)
 $3 \div 5 = \frac{3}{5} = 3 \times \frac{1}{5}$

2. Division of integers can be written as multiplication of fractions. For example

(a)

$$10 \div 2 = \frac{10}{2} = \frac{10}{1} \times \frac{1}{2}$$

(b)
 $3 \div 5 = \frac{3}{5} = \frac{3}{1} \times \frac{1}{5}$

3. Division of integers can be written as division of fractions. For example

(a)

$$10 \div 2 = \frac{10}{1} \div \frac{2}{1}$$

The previous examples showed that

$$10 \div 2 = \frac{10}{1} \times \frac{1}{2}$$

And, since equals can be substituted for equals, we can write

$$\frac{10}{1} \div \frac{2}{1} = \frac{10}{1} \times \frac{1}{2}$$

(b) Similarly we can also write

$$\frac{3}{1} \div \frac{5}{1} = \frac{3}{1} \times \frac{1}{5}$$

4. If a and b are integers different from zero then

$$\frac{a}{b} \times \frac{b}{a} = \frac{a \times b}{b \times a} = 1$$

When the product of two numbers is equal to 1, we say that the numbers are **reciprocals**. Therefore,

the reciprocal of
$$\frac{a}{b}$$
 is $\frac{b}{a}$, and the reciprocal of $\frac{b}{a}$ is $\frac{a}{b}$.

Also notice that

- (a) The reciprocal of a positive number is also positive.
- (b) The reciprocal of a negative number is also negative.
- (c) Zero is the only number that does not have a reciprocal.

17.1)Examples

1. Write the division of integers as a fraction, then as product of an integer and a fraction, and then as product of fractions.

(a)
$$81 \div 3 = \frac{81}{3} = 81 \times \frac{1}{3} = \frac{81}{1} \times \frac{1}{3}$$

(b) $-7 \div 5 = -\frac{7}{5} = -7 \times \frac{1}{5} = -\frac{7}{1} \times \frac{1}{5}$
(c) $10 \div (-3) = -\frac{10}{3} = -10 \times \frac{1}{3} = -\frac{10}{1} \times \frac{1}{3}$

2. Determine the reciprocal of the number.

(a) 5
Since
$$5 = \frac{5}{1}$$
, then its reciprocal is $\frac{1}{5}$
(b) $-\frac{3}{5}$
The reciprocal is $-\frac{5}{3}$
(c) 1

The reciprocal is 1

(d) -1The reciprocal is -1(e) $\frac{1}{9}$ The reciprocal is 9

Everything we just discussed leads us to the following rule for division of fractions:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

In words, to divide two fractions we have to multiply the first one times the reciprocal of the second one.

Once again, fraction notation can be used to indicate division of fractions:

$$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}}$$

17.2) Examples.

Divide and simplify. When simplifying show all the factors that are cancelled.
1.
$$\frac{2}{9} \div \frac{4}{3} = \frac{2}{9} \times \frac{3}{4} = \frac{2 \times 1}{3 \times 3} \times \frac{3 \times 1}{2 \times 2} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

2. $\frac{10}{3} \div 5 = \frac{10}{3} \div \frac{5}{1} = \frac{10}{3} \times \frac{1}{5} = \frac{2 \times 5}{3} \times \frac{1}{5 \times 1} = \frac{2}{3} \times \frac{1}{1} = \frac{2}{3}$
3. $9 \div \frac{6}{5} = \frac{9}{1} \div \frac{6}{5} = \frac{9}{1} \times \frac{5}{6} = \frac{3 \times 3}{1} \times \frac{5}{2 \times 3} = \frac{3}{1} \times \frac{5}{2} = \frac{15}{2}$
4. $\frac{-\frac{3}{10}}{\frac{6}{25}} = -\frac{3}{10} \div \frac{6}{25} = -\frac{3}{10} \times \frac{25}{6} = -\frac{3 \times 1}{2 \times 5} \times \frac{5 \times 5}{2 \times 3} = -\frac{1}{2} \times \frac{5}{2} = -\frac{5}{4}$
5. $\frac{11}{5} \div \left(-\frac{3}{10}\right) = \frac{11}{5} \times \left(-\frac{10}{3}\right) = -\frac{11}{5 \times 1} \times \frac{5 \times 2}{3} = -\frac{11}{1} \times \frac{2}{3} = -\frac{22}{3}$
6. $\frac{-7}{-\frac{7}{9}} = \frac{7}{1} \div \frac{7}{9} = \frac{7}{1} \times \frac{9}{7} = \frac{7 \times 1}{1} \times \frac{9}{7 \times 1} = \frac{1}{1} \times \frac{9}{1} = \frac{9}{1} = 9$

Exercises

Divide and simplify. When simplifying show all the factors that are cancelled.

1.
$$\frac{1}{5} \div \frac{5}{4}$$

2. $\frac{-\frac{8}{25}}{\frac{2}{5}}$
3. $-\frac{16}{9} \div \left(-\frac{3}{2}\right)$
4. $\frac{-\frac{49}{64}}{-\frac{7}{8}}$
5. $-\frac{36}{25} \div 12$
6. $\frac{3}{-\frac{27}{2}}$
7. $-\frac{1}{3} \div (-3)$
8. $\frac{\frac{4}{9}}{8}$

LESSON 18

Comparing Fractions.

If we consider two fractions that are not equivalent, then, one of them is less than the other one. How do we know which one? Let's first mention some situations that are predictable.

1. Any negative fraction is less than zero. For example

$$-\frac{5}{2} < 0$$

2. Zero is less than any positive fraction. For example

$$0 < \frac{1}{1000}$$

3. Any negative fraction is less than any positive fraction. For example

$$-\frac{101}{5} < \frac{1}{2}$$

4. A positive fraction with the numerator less than the denominator is less than 1. For example

$$\frac{99}{100} < 1$$

5. A positive fraction with the numerator greater than the denominator is greater than 1. For example

$$\frac{1001}{1000} > 1$$

6. Any positive fraction less than one (numerator less than denominator) is less than any fraction greater than 1 (numerator greater than denominator). For example

$$\frac{235}{236} < \frac{8}{7}$$

If we were dealing with, for example, two positive fractions that are both less than 1 or both greater than 1, then we need to look at the situation more carefully.

Based on the part-whole interpretation, a positive fraction is related to equal pieces of a "unit". The denominator determines what "kind of pieces" we are using. Most of the time, in real life, we compare only objects of the same kind. We compare two chairs, maybe in terms of which one is more comfortable, but there is no point in comparing say a chair and an apple!.

We can think that fractions are of the "same kind" when they have the same denominator. So, in order to understand how to compare two fractions that are not equivalent, we would like them to have the same denominator.

Consider, for example $\frac{2}{5}$ and $\frac{3}{5}$, we can say immediately that $\frac{2}{5} < \frac{3}{5}$.

However, it would take some thinking to compare $\frac{3}{5}$ and $\frac{4}{7}$ (notice that both fractions are less than 1). We would like them to be the "same kind" of fractions, meaning, having the same denominator.

Remember that we can change the denominator of a fraction without changing the number represented by the fraction. We can do that by multiplying the numerator and the denominator by the same integer different from zero. In this case we have denominators 5 and 7, and we want to change them to the same number (we want a common denominator). One possibility is to change them to be $35 (35 = 5 \times 7)$, in the following way:

$$\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}$$

also

$$\frac{4}{7} = \frac{4 \times 5}{7 \times 5} = \frac{20}{35}$$

Since $\frac{21}{35} > \frac{20}{35}$, then we can conclude that $\frac{3}{5} > \frac{4}{7}$.

Let's now compare the fractions $\frac{7}{12}$ and $\frac{11}{18}$.

First, since they are both positive fractions less than one, they require some thinking. We are going to make them have the same denominator. In other words, we need a common denominator. One possibility for a common denominator is 12×18 . This number would work, but it is fairly large, so we might prefer to work with a smaller number. Keep in mind that any common denominator would result of multiplying each denominator by an integer greater or equal to one. Therefore a common denominator can not be less than the largest denominator.

WARNING: In this example we should not think that 2 could be a common denominator. In fact it can not be any number less than 18.

To try to find a smaller common denominator, we should start considering the largest denominator. For this particular example it is 18. If it were possible to multiply the other denominator (12) by an integer and get 18 as the result, that would mean that 18 is a possible common denominator. In fact it would be the smallest possibility. However, since it is not possible, then we need to consider a number greater than 18. If we notice that

 $36 = 12 \times 3$ and $36 = 18 \times 2$

we can conclude that 36 is a possible common denominator. Now, let's compare the fractions:

$$\frac{7}{12} = \frac{7 \times 3}{12 \times 3} = \frac{21}{36}$$

$$\frac{11}{18} = \frac{11 \times 2}{18 \times 2} = \frac{22}{36}$$

Since $\frac{21}{36} < \frac{22}{36}$ then $\frac{7}{12} < \frac{11}{18}$.

18.1)Examples.

1. For each pair of fractions, find equivalent fractions having the same denominator.

(a)
$$\frac{7}{6}; \frac{5}{2}$$

We should start by considering the largest denominator: 6. Since the other denominator is 2 and $2 \times 3 = 6$, then 6 is not only a possible common denominator, but in fact it is the smallest possibility.

$$\frac{5}{2} = \frac{5 \times 3}{2 \times 3} = \frac{15}{6}$$

 $\frac{7}{6}$ and $\frac{15}{6}$ have the same denominator. (b) $\frac{9}{7}$; $\frac{11}{6}$

The largest denominator, in this case 7, is not a possible common denominator. However, since the other denominator is 6 and $6 \times 7 = 42$, then we can use 42.

$$\frac{9}{7} = \frac{9\times6}{7\times6} = \frac{54}{42}$$

$$\frac{11}{6} = \frac{11 \times 7}{6 \times 7} = \frac{77}{42}$$

 $\frac{54}{42}$ and $\frac{77}{42}$ have the same denominator.

(c)
$$\frac{7}{12}$$
; $\frac{9}{16}$

The largest denominator is 16. We can not use it as a common denominator. We could use the result of 12×18 . However, if we realize that

$$16 \times 3 = 48$$
 and $12 \times 4 = 48$

then we can conclude that 48 can be used as a common denominator.

$$\frac{7}{12} = \frac{7 \times 4}{12 \times 4} = \frac{28}{48}$$
$$\frac{9}{16} = \frac{9 \times 3}{16 \times 3} = \frac{27}{48}$$

 $\frac{28}{48}$ and $\frac{27}{48}$ have the same denominator.

(d) $\frac{43}{50}$; $\frac{97}{105}$

The largest denominator is 105. It can not be used as a common denominator. Let's use the result of 50×105

 $50 \times 105 = 5250$

$$\frac{43}{50} = \frac{43 \times 105}{50 \times 105} = \frac{4515}{5250}$$

$$\frac{97}{105} = \frac{97 \times 50}{105 \times 50} = \frac{4850}{5250}$$

 $\frac{4515}{5250}$ and $\frac{4850}{5250}$ have the same denominator.

2. Fill in the blank using either '<' or '>' as appropriate. Explain your answer.

(a)
$$\frac{9}{10} = \frac{13}{12}$$

 $\frac{9}{10} < 1$ and $1 < \frac{13}{12}$

Then

$$\frac{9}{10} < \frac{13}{12}$$

(b)
$$-\frac{1}{10}$$
 $\frac{1}{1000}$
 $-\frac{1}{10} < 0 \text{ and } 0 < \frac{1}{1000}$

Then

$$-\frac{1}{10} < \frac{1}{1000}$$

(c)
$$\frac{9}{10} = \frac{91}{100}$$

 $\frac{9}{10} = \frac{9 \times 10}{10 \times 10} = \frac{90}{100}$

Then

$$\frac{9}{10} < \frac{91}{100}$$

(d)
$$\frac{7}{12} = \frac{9}{16}$$

 $\frac{7}{12} = \frac{7 \times 4}{12 \times 4} = \frac{28}{48}$
 $\frac{9}{16} = \frac{9 \times 3}{16 \times 3} = \frac{27}{48}$

Then

$$\frac{7}{12} > \frac{9}{16}$$

(e)
$$\frac{43}{50} = \frac{89}{105}$$

 $\frac{43}{50} = \frac{43 \times 105}{50 \times 105} = \frac{4515}{5250}$
 $\frac{89}{105} = \frac{89 \times 50}{105 \times 50} = \frac{4450}{5250}$
Then

$$\frac{43}{50} > \frac{89}{105}$$

Exercises

- 1. For each pair of fractions find equivalent fractions having the same denominator.
 - (a) $\frac{2}{3}; \frac{5}{9}$ (b) $\frac{7}{12}; \frac{3}{5}$ (c) $\frac{7}{24}; \frac{11}{30}$ (d) $\frac{33}{20}; \frac{65}{56}$
- 2. Fill in the blank using either '<' or '>' as appropriate. Explain your answer.

(a)
$$\frac{235}{233}$$
 $\frac{498}{501}$
(b) $-\frac{143}{4}$ 0
(c) -1 $-\frac{7}{8}$
(d) -1230 $\frac{1}{10}$
(e) $\frac{7}{12}$ $\frac{3}{5}$
(f) $\frac{9}{24}$ $\frac{11}{30}$

LESSON 19 REVIEW

${\rm Homework/Class}~{\rm Work/Quiz}~27$

Compute and simplify. When simplifying show all the factors that can be cancelled.

1.
$$\frac{3}{8} \times \frac{5}{7}$$

2. $\frac{14}{5} \times \frac{10}{7}$
3. $\frac{11}{3} \times 5$
4. $\frac{10}{9} \left(-\frac{12}{25}\right)$
5. $8 \times \frac{7}{16}$
6. $\left(-\frac{25}{18}\right) \left(-\frac{27}{10}\right)$
7. $\left(\frac{3}{2}\right)^3$
8. $\frac{7}{3}(-8)$

Compute and simplify. When simplifying show all the factors that can be cancelled.

1. $\frac{16}{9} \times \frac{1}{5} \times \frac{2}{5}$ 2. $\frac{32}{14} \times \frac{2}{25} \times \frac{35}{16}$ 3. $-\frac{81}{8} \times \frac{10}{9} \times \left(-\frac{10}{3}\right)$ 4. $-\frac{1}{3}\left(-\frac{2}{7}\right)\left(-\frac{5}{9}\right)$ 5. $-\left(\frac{2}{3}\right)^4$ 6. $\left(-\frac{2}{3}\right)^3$ 7. $-\left(-\frac{1}{4}\right)^2$ 8. $-\left(-\frac{1}{4}\right)^3$

Compute and simplify. When simplifying show all the factors that can be cancelled.

1. $\frac{3}{14} \div \frac{2}{3}$ 2. $\frac{12}{5} \div \frac{9}{10}$ 3. $\frac{15}{2} \div 3$ 4. $7 \div \frac{21}{4}$ 5. $\frac{\frac{21}{8}}{\frac{9}{4}}$ 6. $\frac{\frac{12}{5}}{16}$ 7. $\frac{100}{\frac{20}{3}}$

Compute and simplify. When simplifying show all the factors that can be cancelled.

- 1. $\frac{9}{7} \div \left(-\frac{7}{3}\right)$ 2. $-\frac{3}{4} \div \frac{9}{10}$ 3. $-\frac{14}{15} \div \left(-\frac{21}{10}\right)$ 4. $\frac{7}{9} \div (-9)$ 5. $(-6) \div \left(-\frac{9}{7}\right)$ 6. $\frac{-\frac{36}{25}}{\frac{6}{100}}$ 7. $\frac{\frac{11}{13}}{-\frac{22}{39}}$
- 8. $\frac{-\frac{8}{7}}{-14}$

Compute and simplify. When simplifying show all the factors that can be cancelled. 1. $\frac{18}{35} \times \left(-\frac{14}{9}\right)$

- $2. \quad \frac{18}{35} \div \left(-\frac{14}{9}\right)$
- 3. 8 $\left(\frac{10}{12}\right)$
- 4. $8 \div \frac{10}{12}$
- 5. $\left(-\frac{1}{5}\right)^2$
- $6. \left(-\frac{2}{5}\right)^2$
- $7. \left(-\frac{1}{6}\right)^3$
- 8. $\frac{-\frac{9}{10}}{2}$

9. $\frac{-\frac{5}{8}}{-\frac{25}{12}}$

${\rm Homework/Class}~{\rm Work/Quiz}~32$

- 1. For each pair of fractions find equivalent fractions having the same denominator.
 - (a) $\frac{7}{27}$; $\frac{5}{3}$
 - (b) $\frac{2}{7}$; $\frac{4}{5}$

(c)
$$\frac{21}{100}$$
; $\frac{12}{150}$

2. Fill in the blank using either '<' or '>' as appropriate. Explain your answer.

(a)	$\frac{532}{535}$	$\frac{451}{448}$
(b)	$\frac{9}{8}$	$\frac{25}{26}$
(c)	$\frac{0}{25}$	$\frac{38}{-7}$
(d)	$\frac{13}{16}$	$\frac{19}{24}$

- 1. For each pair of fractions find equivalent fractions having the same denominator.
 - (a) $\frac{8}{21}$; $\frac{7}{6}$
 - (b) $\frac{15}{39}$; $\frac{25}{26}$

(c)
$$\frac{5}{80}$$
; $\frac{3}{40}$

2. Fill in the blank using either '<' or '>' as appropriate. Explain your answer.

(a)	$\frac{-19}{-18}$	$\frac{-21}{-22}$
(b)	$\frac{10}{-10}$	$-\frac{10}{9}$
(c)	$\frac{5}{3}$	$-\frac{10}{6}$
(d)	$\frac{4}{21}$	$\frac{3}{14}$

LESSON 20

Addition and Subtraction of Fractions.

In our day to day life we are constantly adding and subtracting. If we have two pencils, and then we buy three more pencils, then we say that we have five pencils (5 = 2 + 3). If we have \$100 and we spend \$35 in groceries, then we say that we have \$65 left (65 = 100 - 35).

However, if we have two pencils and then we buy three erasers, we will still say that we have two pencils and three erasers!. In this case, we don't add because we are not talking about the same kind of object.

In general we add or subtract quantities related to the same kind of object. That same thing happens when we want to add or subtract fractions. We need them to be the same "kind" of fractions, In other words we need them to have the same denominator. If that is the case, the denominator indicates the "kind of object" and the numerator says "how many " of them. The resulting fraction should be the same "kind of fraction", meaning a fraction with the same denominator as the original ones. The numerator of the resulting fraction will be the result of adding or subtracting the original numerators.

20.1) Examples.

1.
$$\frac{5}{3} + \frac{4}{3} = \frac{3}{3} = \frac{3}{3} = 3$$

keep denominator

2.
$$\frac{10}{14} - \frac{3}{14} = \frac{10}{14} = \frac{10}{14} = \frac{7 \times 1}{14} = \frac{7}{7 \times 2} = \frac{1}{2}$$
keep denominator

These examples show the general rule, that is:

when adding or subtracting fractions with the same denominator, we keep the denominator and add or subtract the numerators.

If the fractions do not have the same denominator, we need to "make" them have a common denominator, in other words, we need to replace the fractions in the sum or difference by equivalent fractions with a **common denominator**. 20.2) Examples.

$$1. \quad \frac{7}{6} - \frac{13}{12} = \frac{7 \times 2}{6 \times 2} - \frac{13}{12} = \frac{14}{12} - \frac{13}{12} = \frac{14 - 13}{12} = \frac{1}{12}$$

$$2. \quad \frac{11}{2} + \frac{1}{6} = \frac{11 \times 3}{2 \times 3} + \frac{1}{6} = \frac{33}{6} + \frac{1}{6} = \frac{33 + 1}{6} = \frac{34}{6} = \frac{17 \times 2}{3 \times 2} = \frac{17}{3}$$

$$3. \quad \frac{7}{12} + \frac{7}{16} = \frac{7 \times 4}{12 \times 4} + \frac{7 \times 3}{16 \times 3} = \frac{28}{48} + \frac{21}{48} = \frac{28 + 21}{48} = \frac{49}{48}$$

$$4. \quad \frac{7}{12} - \frac{5}{18} = \frac{7 \times 3}{12 \times 3} - \frac{5 \times 2}{18 \times 2} = \frac{21}{36} - \frac{10}{36} = \frac{21 - 10}{36} = \frac{11}{36}$$

$$5. \quad \frac{11}{10} + \frac{1}{15} = \frac{11 \times 3}{10 \times 3} + \frac{1 \times 2}{15 \times 2} = \frac{33}{30} + \frac{2}{30} = \frac{33 + 2}{30} = \frac{35}{30} = \frac{7 \times 5}{6 \times 5} = \frac{7}{6}$$

$$6. \quad \frac{7}{5} + 2 = \frac{7}{5} + \frac{2}{1} = \frac{7}{5} + \frac{2 \times 5}{1 \times 5} = \frac{7}{5} + \frac{10}{5} = \frac{7 + 10}{5} = \frac{17}{5}$$

$$7. \quad 9 - \frac{5}{3} = \frac{9}{1} - \frac{5}{3} = \frac{9 \times 3}{1 \times 3} - \frac{5}{3} = \frac{27}{3} - \frac{5}{3} = \frac{27 - 5}{3} = \frac{22}{3}$$

Addition of fractions is also commutative and associative. When adding more than two fractions, we can group the fractions in any way. In particular we can add left to right or right to left. However, since we know that when we add two fractions we need them to have the same denominator, when adding more than two, it is convenient to start by making sure that all fractions have the same denominator. It can be shown than we can now add by keeping the common denominator and adding all numerators.

20.3) Examples.

$$1. \quad \frac{2}{5} + \frac{1}{10} + \frac{1}{6} = \frac{2 \times 6}{5 \times 6} + \frac{1 \times 3}{10 \times 3} + \frac{1 \times 5}{6 \times 5} = \frac{12}{30} + \frac{3}{30} + \frac{5}{30} = \frac{12 + 3 + 5}{30} = \frac{20}{30} = \frac{2 \times 10}{3 \times 10} = \frac{2}{3}$$
$$2. \quad \frac{3}{2} + \frac{1}{4} + 1 = \frac{3}{2} + \frac{1}{4} + \frac{1}{1} = \frac{3 \times 2}{2 \times 2} + \frac{1}{4} + \frac{1 \times 4}{1 \times 4} = \frac{6}{4} + \frac{1}{4} + \frac{4}{4} = \frac{6 + 1 + 4}{4} = \frac{11}{4}$$

Now let's include negative fractions in additions or subtractions. We will take advantage of the fact that

$$-\frac{a}{b} = \frac{-a}{b}$$

20.4) Examples.

$$1. -\frac{3}{5} + \frac{1}{3} = -\frac{3 \times 3}{5 \times 3} + \frac{1 \times 5}{3 \times 5} = -\frac{9}{15} + \frac{5}{15} = \frac{-9}{15} + \frac{5}{15} = \frac{-9+5}{15} = \frac{-4}{15} = -\frac{4}{15}$$

$$2. -\frac{7}{5} - \frac{1}{10} = -\frac{7 \times 2}{5 \times 2} - \frac{1}{10} = -\frac{14}{10} - \frac{1}{10} = \frac{-14}{10} - \frac{1}{10} = \frac{-14-1}{10} = \frac{-15}{10} = -\frac{3 \times 5}{2 \times 5} = -\frac{3}{2}$$

$$3. \frac{1}{3} - \frac{1}{2} = \frac{1 \times 2}{3 \times 2} - \frac{1 \times 3}{2 \times 3} = \frac{2}{6} - \frac{3}{6} = \frac{2-3}{6} = \frac{-1}{6} = -\frac{1}{6}$$

$$4. \frac{5}{9} + \left(-\frac{1}{2}\right) = \frac{5 \times 2}{9 \times 2} + \left(-\frac{1 \times 9}{2 \times 9}\right) = \frac{10}{18} + \left(-\frac{9}{18}\right) = \frac{10}{18} + \left(-\frac{9}{18}\right) = \frac{10+(-9)}{18} = \frac{1}{18}$$

$$5. -\frac{12}{7} + \left(-\frac{2}{3}\right) = -\frac{12 \times 3}{7 \times 3} + \left(-\frac{2 \times 7}{3 \times 7}\right) = -\frac{36}{21} + \left(-\frac{14}{21}\right) = \frac{-36}{21} + \left(-\frac{14}{21}\right) = \frac{-36+(-14)}{21} = \frac{-50}{21} = -\frac{50}{21}$$

$$6. -5 + \frac{6}{5} = -\frac{5}{1} + \frac{6}{5} = -\frac{5 \times 5}{1 \times 5} + \frac{6}{5} = -\frac{25}{5} + \frac{6}{5} = -\frac{25+6}{5} = -\frac{19}{5} = -\frac{19}{5}$$

$$7. \frac{2}{3} - \left(-\frac{5}{6}\right) = \frac{2}{3} + \frac{5}{6} = \frac{2 \times 2}{3 \times 2} + \frac{5}{6} = \frac{4}{6} + \frac{5}{6} = \frac{4+5}{6} = \frac{9}{6} = \frac{3 \times 3}{3 \times 2} = \frac{3}{2}$$

Exercises.

Compute and simplify. Show all the steps of the computation. When simplifying show the factors that are cancelled.

1.
$$\frac{5}{9} + \frac{4}{9}$$

2. $\frac{5}{9} - \frac{2}{9}$ 3. $\frac{2}{3} + 4$ 4. $\frac{12}{5} - 1$ 5. $\frac{9}{5} + \frac{3}{10}$ 6. $\frac{7}{9} - \frac{11}{6}$ 7. $\frac{1}{2} + \frac{5}{3} + \frac{7}{6}$ 8. $\frac{2}{5} + 3 + \frac{5}{6}$ 9. $-\frac{5}{9} + \frac{4}{9}$ 10. $-\frac{5}{9}-\frac{2}{9}$ 11. $\frac{2}{3} - 4$ 12. $\frac{12}{5} - 3$ 13. $\frac{9}{5} + \left(-\frac{3}{10}\right)$

14. $\frac{7}{9} - \frac{5}{6}$

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LESSON 21

Now we will combine addition and subtraction of fractions. One way to proceed is to make all fractions have the same denominator. Then, keep the denominator and perform the additions or subtractions of the numerators from left to right.

21.1)Examples.

$$1. -\frac{2}{3} + \frac{4}{9} - \frac{1}{2} = -\frac{2 \times 6}{3 \times 6} + \frac{4 \times 2}{9 \times 2} - \frac{1 \times 9}{2 \times 9} = -\frac{12}{18} + \frac{8}{18} - \frac{9}{18} = \frac{-12}{18} + \frac{8}{18} - \frac{9}{18} = \frac{-13}{18} = -\frac{13}{18} = -\frac{13}{18}$$

$$2. \frac{5}{4} + \frac{3}{5} - \frac{5}{6} = \frac{5 \times 15}{4 \times 15} + \frac{3 \times 12}{5 \times 12} - \frac{5 \times 10}{6 \times 10} = \frac{75}{60} + \frac{36}{60} - \frac{50}{60} = \frac{75}{60} + \frac{36}{60} - \frac{50}{60} = \frac{61}{60}$$

$$3. \frac{1}{8} - \frac{1}{6} - \frac{1}{9} = \frac{1 \times 9}{8 \times 9} - \frac{1 \times 12}{6 \times 12} - \frac{1 \times 8}{9 \times 8} = \frac{9}{72} - \frac{12}{72} - \frac{8}{72} = \frac{9}{72} - \frac{12}{72} - \frac{8}{72} = -\frac{11}{72} = -\frac{11}{72}$$

$$4. \frac{1}{4} - \frac{2}{3} + \frac{3}{2} + 3 = \frac{1 \times 3}{4 \times 3} - \frac{2 \times 4}{3 \times 4} + \frac{3 \times 6}{2 \times 6} + \frac{3 \times 12}{1 \times 12} = \frac{3}{12} - \frac{8}{12} + \frac{18}{12} + \frac{36}{12} = \frac{3}{12} - \frac{8}{12} + \frac{18}{12} + \frac{36}{12} = \frac{3}{12} - \frac{8}{12} + \frac{18}{12} + \frac{36}{12} = \frac{-16}{12} - \frac{8}{12} - \frac{49}{12} = \frac{49}{12}$$

$$5. -2 - \frac{1}{6} - \frac{1}{2} = -\frac{2 \times 6}{1 \times 6} - \frac{1}{6} - \frac{1 \times 3}{2 \times 3} = -\frac{12}{6} - \frac{1}{6} - \frac{3}{6} = \frac{-13}{6} = -\frac{16}{6} = -\frac{8 \times 2}{3 \times 2} = -\frac{8}{3}$$

Now let's combine all operations, keeping in mind the order of operations. For example,

$$\frac{1}{2} + \frac{1}{2} \times \frac{1}{3} = \frac{1}{2} + \frac{1}{6} = \frac{1 \times 3}{2 \times 3} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{3+1}{6} = \frac{4}{6} = \frac{2 \times 2}{2 \times 3} = \frac{2}{3}$$

Also notice that when we have a fraction with operations in the numerator and in the denominator, it means the same as having the expression at the numerator in parentheses, divided by the expression in the denominator also in parentheses. That is:

$$\frac{A}{B} = (A) \div (B)$$

Therefore, this indicates that the first thing to do is the operations in the numerator and in the denominator and then work with the resulting fraction.

For example,

$$\frac{\frac{3}{5} - \frac{1}{5}}{1 + \frac{2}{5}} = \frac{\frac{2}{5}}{\frac{7}{5}} = \frac{2}{5} \times \frac{5}{7} = \frac{2}{7}$$

However, in the case where the operations at the numerator and at the denominator are ONLY multiplications, then we are in the situation that allows us to simplify before performing the multiplications.

For example,

$$\frac{2 \times 5 \times 7}{6 \times 2 \times 14} = \frac{5 \times 7}{6 \times 14} = \frac{5 \times 7}{6 \times 7 \times 2} = \frac{5}{6 \times 2} = \frac{5}{12}$$

21.2)Examples.

1.
$$\left(\frac{1}{2} + \frac{1}{2}\right) \times \frac{1}{3} = \frac{1+1}{2} \times \frac{1}{3} = \frac{2}{2} \times \frac{1}{3} = 1 \times \frac{1}{3} = \frac{1}{3}$$

2. $1 + \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = \frac{1}{1} - \frac{1}{2} + \frac{1}{2} = \frac{1 \times 2}{1 \times 2} - \frac{1}{2} + \frac{1}{2} = \frac{2}{2} - \frac{1}{2} + \frac{1}{2} = \frac{2-1+1}{2} = \frac{2}{2} = 1$
3. $1 - \frac{1}{2} + \left(-\frac{1}{2} + 1\right) = 1 - \frac{1}{2} + \left(-\frac{1}{2} + \frac{2}{2}\right) = 1 - \frac{1}{2} + \left(-\frac{1+2}{2}\right) = 1 - \frac{1}{2} + \frac{1}{2} = 1 + \left(-\frac{1}{2}\right) + \frac{1}{2} = 1 + \frac{1}{2} + \frac$

4.
$$\frac{2}{5} - \left(\frac{3}{2}\right)^2 = \frac{2}{5} - \frac{9}{4} = \frac{2 \times 4}{5 \times 4} - \frac{9 \times 5}{4 \times 5} = \frac{8}{20} - \frac{45}{20} = \frac{8 - 45}{20} = \frac{-37}{20} = -\frac{37}{20}$$

9.
$$\frac{1-3\left(\frac{1+2}{4-1}\right)}{5(7-6)-\frac{10}{2}} = \frac{1-3\left(\frac{3}{3}\right)}{5\times1-\frac{10}{2}} = \frac{1-3\times1}{5-\frac{10}{2}} = \frac{1-3}{5-5} = \frac{-2}{0}$$
 This is undefined!

Exercises.

Compute and simplify. Show all the steps of the computation and when simplifying show the factors that are cancelled.

1. $\frac{1}{2} - \frac{5}{3} - \left(-\frac{7}{6}\right)$ 2. $-2 + \frac{5}{7} - \frac{1}{3}$ 3. $-\frac{2}{9} - \frac{1}{3} - \frac{3}{2}$ 4. $1 + \frac{1}{2} - \frac{3}{2} - \frac{5}{2}$ 5. $\frac{2}{3} - \frac{3}{5} + \frac{1}{2}$ 6. $\frac{1}{3} + 2 - \frac{1}{2} - \frac{1}{4}$



$$19. \ \frac{2}{3} + \frac{1}{6} \left(2 + \frac{2}{3}\right)$$

$$20. \ -\frac{1}{2} + \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right)$$

$$21. \ 1 + \left(\frac{1}{2}\right)^3 + \frac{1}{3}$$

$$22. \ \left(-\frac{1}{2}\right)^2 - \frac{1}{4}$$

$$23. \ 3 - \frac{2}{5} \div 5 \times 2$$

$$24. \ \left(1 - \frac{2}{5}\right) \times 10 - 10$$

$$25. \ -\left(-\frac{1}{3}\right)^3 + \frac{1}{3^2} - \frac{1}{3}$$

$$26. \ \frac{\frac{1}{3} - \frac{3}{2}}{-\left(-\frac{1}{2}\right) - \frac{1}{2}}$$

$$27. \ \frac{1200 \times 625}{100 \times 25 \times 50}$$

$$28. \ \left(\frac{\frac{2}{3}}{\frac{4}{9}}\right)^4 \times \left(\frac{2}{3}\right)^3$$

$$29. \ \frac{1}{1 - 3 + 1} + \frac{5}{3 - 4 + 4}$$

$$30. \ \frac{2 - 3\left(\frac{3 - 2}{1 - 3}\right)}{2\left(\frac{1}{3} - \frac{1}{2}\right) - \frac{1}{2}}$$

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31.
$$1 - \left(\frac{\frac{1}{2}}{\frac{1}{2} - \frac{1}{4}}\right) \times \frac{1}{2}$$

32.
$$\frac{1 - \left(\frac{1}{2} - \frac{1}{3}\right) + \frac{1}{6}}{(1+1)\left(1 + \frac{1}{2}\right)}$$

LESSON 22 REVIEW

Compute and simplify. Show all the steps of the computation and when simplifying show the factors that are cancelled.

1.
$$\frac{7}{6} + \frac{8}{6}$$

2. $\frac{5}{3} - \frac{2}{3}$
3. $\frac{3}{5} + 2$
4. $\frac{9}{4} - 1$
5. $\frac{4}{5} + \frac{2}{15}$
6. $\frac{10}{7} - \frac{5}{21}$
7. $\frac{2}{5} + \frac{1}{3} + \frac{19}{15}$
8. $\frac{5}{2} + 2 + \frac{1}{3}$

Compute and simplify. Show all the steps of the computation and when simplifying show the factors that are cancelled.

1. $5 + \frac{4}{5} + \frac{3}{2}$ 2. $\frac{1}{2} + \frac{1}{3} + \frac{1}{5}$ 3. $\frac{5}{9} + \frac{4}{3} + \frac{8}{6}$ 4. $-\frac{3}{7} + \frac{17}{7}$ 5. $-\frac{8}{10}-\frac{4}{10}$ 6. $\frac{9}{5} - \frac{11}{5}$ 7. $3 - \frac{15}{7}$ 8. $\frac{16}{7} - 3$

Compute and simplify. Show all the steps of the computation and when simplifying show the factors that are cancelled.

1.
$$\frac{3}{4} - \frac{7}{3} - \left(-\frac{5}{6}\right)$$

2. $\frac{5}{3} - 4 - \frac{3}{2}$
3. $-\frac{1}{10} - \frac{1}{5} - \frac{1}{5}$
4. $\frac{5}{3}(-4)\left(-\frac{3}{2}\right)$
5. $\frac{1}{2} + \frac{1}{3} - 3 - \frac{5}{6}$
6. $-\frac{2}{5} + \frac{1}{3} - \frac{2}{15}$
7. $\frac{3}{7} - \left(-\frac{2}{3}\right) - 2$
1.
$$\frac{3}{4}\left(-\frac{12}{5}\right)$$

2. $\frac{3}{4} - \frac{12}{5}$
3. $\left(-\frac{5}{9}\right) \div \left(-\frac{2}{15}\right)$
4. $\frac{1}{2} - \frac{1}{4} + \frac{5}{2}$
5. $-\left(-\frac{1}{10}\right)^2$
6. $3\left(-\frac{7}{15}\right)$
7. $3 - \frac{7}{15}$
8. $\frac{7}{3} + \frac{1}{2} - \frac{1}{9} - \frac{1}{18}$

Compute and simplify. Show all the steps of the computation and when simplifying show the factors that are cancelled.

1. $\frac{2}{3} + \frac{2}{3} \times \frac{6}{5}$ 2. $\left(\frac{2}{3} + \frac{2}{3}\right) \times \frac{6}{5}$ 3. $-1 - \frac{2}{5} \div \frac{3}{10}$ 4. $-\frac{1}{3} - \left(-\frac{1}{3}\right)^2$ 5. $\frac{2-3}{\frac{1}{2}-\frac{1}{3}}$ 6. $-2\left(-\frac{1}{3}\right)^3$ 7. $-2 - \frac{3}{5} \div 5$

1.
$$\frac{1}{9} - \left(\frac{1}{5} - \frac{1}{5}\right)$$

2. $\frac{1}{9} - \frac{1}{5} - \frac{1}{5}$
3. $\left(\frac{\frac{1}{10}}{\frac{1}{5}}\right)^2 \times \left(-\frac{8}{7}\right)$
4. $\frac{15 - (-5)}{5(-5)}$
5. $\frac{1}{5} - \frac{2}{3}\left(2 - \frac{3}{4}\right)$
6. $\left(\frac{1}{5} - \frac{2}{3}\right)\left(2 - \frac{3}{4}\right)$
7. $\frac{25 \times 18}{81 \times 75} - \frac{1}{9}$

${\rm Homework/Class}~{\rm Work/Quiz}~40$

1.
$$-3\left(-\frac{1}{3}\right)^3 + \frac{1}{2} \times \frac{4}{3}$$

2.
$$-2 - \left(\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)$$

3.
$$\frac{\frac{1}{3} - \left(-\frac{1}{3}\right)^2}{\frac{1}{3}\left(-\frac{1}{3}\right) + 1}$$

4.
$$\frac{4+2\times 5-2^3}{1+32\div (-16)\div (-4)}$$

5.
$$-\left(\frac{-\frac{2}{3}}{3}\right)^2 \times \left(\frac{-3}{\frac{5}{2}}\right)^2$$

${\rm Homework/Class}~{\rm Work/Quiz}~41$

1.
$$\frac{2}{9} + \frac{10}{9} \div \frac{20}{9} - \frac{5}{3}$$

2.
$$\left(-\frac{1}{5}+\frac{2}{10}\right) \times \frac{2}{3}-2$$

3.
$$-2\left(\frac{\frac{3}{2}}{\frac{1}{3}-\frac{1}{9}}\right) \times \frac{2}{15}$$

4.
$$\frac{-2 - \left(\frac{1}{3} - \frac{1}{9}\right) + \frac{1}{9}}{(2+3)(-1+1)}$$

5.
$$\frac{-2 - \left(\frac{1}{3} - \frac{1}{9}\right) + \frac{1}{9}}{(2+3)(-1-1)}$$

LESSON 23

Mixed Numbers.

We already discussed that a positive fraction with the numerator bigger than the denominator represents a number bigger than one. If that is the case, then we may wonder if it is bigger than two or three, and so on. More precisely we want to know if the number is between one and two or between two and three, etc. One way to find this out, is by writing the fraction as a mixed number. A mixed number is a number written as a positive integer, followed by a positive fraction with numerator less than denominator (the fraction represents a number between zero and one).

For example: $2\frac{1}{3}$, $1\frac{4}{5}$, $3\frac{10}{11}$.

Now let's talk about the meaning of the mixed number. Take for example $2\frac{1}{3}$. This is a number between 2 and 3, and in fact:

$$2\frac{1}{3} = 2 + \frac{1}{3}$$

therefore, written as a fraction

$$2\frac{1}{3} = 2 + \frac{1}{3} = \frac{7}{3}$$

Any mixed number can be written as a fraction, the result of adding the positive integer and the fraction. Notice that the resulting fraction will represent a number greater than one. Also, any positive fraction **bigger than one** can be written as a mixed number. To do so, we can take advantage of the rule for addition of fractions "backwards".

23.1)Examples.

1. Write the fraction as a mixed number, if it applies.

(a)
$$\frac{7}{3} = \frac{6+1}{3} = \frac{6}{3} + \frac{1}{3} = 2 + \frac{1}{3} = 2\frac{1}{3}$$

Notice that we decide to "break" 7 into 6+1 because 6 is the largest number that is less than 7 and for which 3 is a factor.

(b) $\frac{13}{15}$

This fraction can not be written as a mixed number because it is actually a number between 0 and 1.

(c)
$$\frac{15}{4} = \frac{12+3}{4} = \frac{12}{4} + \frac{3}{4} = 3 + \frac{3}{4} = 3\frac{3}{4}$$

(d) $\frac{19}{17} = \frac{17+2}{17} = \frac{17}{17} + \frac{2}{17} = 1 + \frac{2}{17} = 1\frac{2}{17}$
(e) $\frac{19}{5} = \frac{15+4}{5} = \frac{15}{5} + \frac{4}{5} = 3 + \frac{4}{5} = 3\frac{4}{5}$

2. Write the mixed number as a fraction.

(a)
$$1\frac{2}{3} = 1 + \frac{2}{3} = \frac{3}{3} + \frac{2}{3} = \frac{5}{3}$$

(b) $5\frac{3}{5} = 5 + \frac{3}{5} = \frac{25}{5} + \frac{3}{5} = \frac{28}{5}$
(c) $4\frac{1}{7} = 4 + \frac{1}{7} = \frac{28}{7} + \frac{1}{7} = \frac{29}{7}$

3. For each fraction, write as a mixed number (if it applies) and then locate it on a number line.

(a)
$$\frac{13}{4}$$

 $\frac{13}{4} = \frac{12+1}{4} = \frac{12}{4} + \frac{1}{4} = 3 + \frac{1}{4} = 3\frac{1}{4}$

Now let's locate it on a number line.



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Exercises.

- 1. Write the mixed number as a fraction.
 - (a) $3\frac{4}{7}$ (b) $1\frac{5}{6}$ (c) $2\frac{3}{5}$ (d) $5\frac{1}{6}$
- 2. Write the fraction as a mixed number if possible. Then, locate it on a number line.



(c) $\frac{14}{5}$



LESSON 24

Operations on Mixed Numbers.

To perform operations on mixed number we can always write them as fractions and then perform the operations. If we want the answer as a mixed number, we just have to write the resulting fraction again as a mixed number, if possible.

There are some short cuts for addition or subtraction.

For example, for additions we can proceed as follows:

24.1)Examples.

1.
$$2\frac{2}{3} + 1\frac{3}{4} = 2 + \frac{2}{3} + 1 + \frac{3}{4} = (2+1) + (\frac{2}{3} + \frac{3}{4}) = 3 + \frac{8+9}{12} = 3 + \frac{17}{12} = 3 + \frac{12+5}{12} = 3 + \frac{12}{12} + \frac{5}{12} = 4\frac{5}{12}$$

2.
$$3\frac{1}{5} + 4\frac{1}{3} = 3 + \frac{1}{5} + 4 + \frac{1}{3} = (3+4) + \left(\frac{1}{5} + \frac{1}{3}\right) = 7 + \frac{8}{15} = 7\frac{8}{15}$$

It is important to understand that here we are applying both the commutative and associative property for addition, even though we are not showing the formal details.

24.2) Examples.

Compute. Give your answer both as a fraction and as a mixed number, if it applies.

1.
$$3\frac{1}{3} - 2\frac{4}{5} = \left(3 + \frac{1}{3}\right) - \left(2 + \frac{4}{5}\right) = \left(\frac{9+1}{3}\right) - \left(\frac{10+4}{5}\right) = \frac{10}{3} - \frac{14}{5} = \frac{50-42}{15} = \frac{8}{15}$$

The answer as a fraction is $\frac{8}{15}$.

Since this is a number between 0 and 1, it can not be written as a mixed number.

2.
$$2\frac{2}{5} \times 3\frac{1}{2} = \left(2 + \frac{2}{5}\right) \times \left(3 + \frac{1}{2}\right) = \left(\frac{12}{5}\right) \times \left(\frac{7}{2}\right) = \frac{6 \times 2}{5} \times \frac{7}{2} = \frac{6}{5} \times \frac{7}{1} = \frac{42}{5}$$

The answer as a fraction is $\frac{42}{5}$

As a mixed number,

$$\frac{42}{5} = \frac{40+2}{5} = \frac{40}{5} + \frac{2}{5} = 8 + \frac{2}{5} = 8\frac{2}{5}$$
3. $4\frac{2}{7} + 2\frac{2}{3} = \left(4 + \frac{2}{7}\right) \div \left(2 + \frac{2}{3}\right) = \frac{30}{7} \div \frac{8}{3} = \frac{30}{7} \times \frac{3}{8} = \frac{2 \times 15}{7} \times \frac{3}{2 \times 4} = \frac{15}{7} \times \frac{3}{4} = \frac{45}{28}$
The answer as a fraction is $\frac{45}{28}$
As a mixed number,

$$\frac{45}{28} = \frac{28 + 17}{28} = \frac{28}{28} + \frac{17}{28} = 1 + \frac{17}{28} = 1\frac{17}{28}$$
4. $1\frac{1}{5} + 2\frac{4}{5} = 1 + \frac{1}{5} + 2 + \frac{4}{5} = (1 + 2) + \left(\frac{1}{5} + \frac{4}{5}\right) = 3 + 1 = 4$
5. $\frac{4}{5} \times 2\frac{1}{4} = \frac{4}{5} \times \left(2 + \frac{1}{4}\right) = \frac{4}{5} \times \frac{9}{4} = \frac{9}{5}$
The answer as a fraction is $\frac{9}{5}$
As a mixed number,
 $\frac{9}{5} = \frac{5 + 4}{5} = \frac{5}{5} + \frac{4}{5} = 1 + \frac{4}{5} = 1\frac{4}{5}$
6. $1\frac{2}{5} \div 5 = \left(1 + \frac{2}{5}\right) \div 5 = \frac{7}{5} \div \frac{5}{1} = \frac{7}{5} \times \frac{1}{5} = \frac{7}{25}$
7. $2\frac{7}{8} + \frac{3}{4} = \left(2 + \frac{7}{8}\right) + \frac{3}{4} = 2 + \left(\frac{7}{8} + \frac{3}{4}\right) = 2 + \frac{13}{8} = \frac{29}{8}$
As a mixed number,
 $\frac{29}{8} = \frac{24 + 5}{8} = \frac{24}{8} + \frac{5}{8} = 3 + \frac{5}{8} = 3\frac{5}{8}$
8. $4\frac{3}{5} - \frac{9}{10} = \left(4 + \frac{3}{5}\right) - \frac{9}{10} = \frac{23}{5} - \frac{9}{10} = \frac{46}{10} - \frac{9}{10} = \frac{37}{10}$
As a mixed number,
 $\frac{37}{10} = \frac{30 + 7}{10} = \frac{30}{10} + \frac{7}{10} = 3 + \frac{7}{10} = 3\frac{7}{10}$

Exercises

Compute. Write your answer both as a fraction (or integer) and a mixed number (if it applies).

1. $3\frac{2}{5} + 7\frac{1}{3}$ 2. $1\frac{4}{5} + 2\frac{1}{2}$ 3. $5\frac{1}{3} - 2\frac{8}{9}$ 4. $4\frac{2}{7} - 3$ 5. $10 \div 2\frac{1}{2}$ 6. $4\frac{4}{7} + 3$ 7. $3\frac{2}{3} + \frac{3}{7}$ 8. $2\frac{1}{4} \times 3\frac{5}{9}$ 9. $5\frac{1}{5} \div 1\frac{3}{10}$ 10. $10 \times 7\frac{3}{5}$

LESSON 25 REVIEW

- 1. Write the mixed number as a fraction.
 - (a) $1\frac{4}{5}$ (b) $2\frac{3}{7}$ (c) $4\frac{1}{2}$ (d) $6\frac{3}{5}$
- 2. Write the fraction as a mixed number if possible. Then, locate it on a number line.
 - (a) $\frac{9}{5}$





1. Write the fraction as a mixed number if possible. Then, locate it on a number line.

2. Compute. Give your answer both as a fraction and as a mixed number, if it applies.

(a) $5\frac{2}{7} + 3\frac{1}{4}$ (b) $1\frac{7}{8} + 4\frac{2}{3}$ (c) $2\frac{1}{4} - 1\frac{4}{5}$ (d) $5\frac{1}{2} - 2\frac{1}{3}$

Compute. Give your answer both as a fraction (or an integer) and as a mixed number, if it applies.

1. $3\frac{1}{5} \times 4\frac{2}{3}$ 2. $3\frac{1}{5} + 4\frac{2}{3}$ 3. $6\frac{3}{7} \div 2\frac{5}{6}$ 4. $2\frac{3}{5} + 1\frac{5}{6}$ 5. $2\frac{2}{3} - 1\frac{5}{6}$ 6. $9 \times 5\frac{2}{3}$ 7. $9 \div 5\frac{2}{3}$ 8. $5\frac{5}{7}-\frac{3}{5}$

${\rm Homework/Class}~{\rm Work/Quiz}~45$

Compute. Give your answer both as a fraction (or an integer) and as a mixed number, if it applies.

1.
$$7\frac{3}{5} \div \frac{5}{2}$$

2. $-\frac{3}{2} + 2\frac{2}{5}$
3. $\frac{4}{5} \times 5\frac{1}{6}$
4. $3\frac{3}{5} \div 2$
5. $2\frac{4}{5} \times 10$
6. $5\frac{1}{3} + \frac{2}{5}$
7. $4\frac{2}{3} - \frac{1}{9}$

LESSON 26

Decimal Notation.

Decimal notation refers to writing a number using a horizontal arrangement of the ten digits 0,1,2,3,4,5,6,7,8 and 9. The position of the digit will determine its value based on powers of ten. For example:

$$232 = 2 \times 100 + 3 \times 10 + 2 = 2 \times 10^{2} + 3 \times 10^{1} + 2 \times 10^{0}$$

How do we write decimal notation for a fraction that is not an integer? Let's concentrate only on positive fractions.

First we need to understand decimal; notation for a particular kind of fractions, that is, those with numerator 1 and denominator given by a positive power of 10 (10, 100, 1000 and so on).

For such fractions, decimal notation will include a point called "decimal point". The first digit (left to right) after the decimal point is related to the fraction $\frac{1}{10}$, the second digit is related to the fraction $\frac{1}{100}$, the third digit is related to the fraction $\frac{1}{1000}$, etc. In particular:

1. $\frac{1}{10} = 0.1$ (or .1)

This is showing that

$$0.1 = 0 + 1 \times \frac{1}{10}$$

2.
$$\frac{1}{100} = 0.01$$
 (or .01)

This is showing that

$$0.01 = 0 + 0 \times \frac{1}{10} + 1 \times \frac{1}{100}$$

3.
$$\frac{1}{1000} = 0.001$$
 (or .001)

This is showing that

$$0.001 = 0 + 0 \times \frac{1}{10} + 0 \times \frac{1}{100} + 1 \times \frac{1}{1000}$$

Notice that these three fractions have a numerator less than the denominator. Therefore, they are numbers **between zero and one**. Also, notice that the decimal expression has the natural number zero before the decimal point.

26.1)Examples

1.
$$\frac{3}{10} = 3 \times \frac{1}{10} = 0.3$$

This is a number between zero and one.

2.
$$\frac{5}{1000} = 5 \times \frac{1}{1000} = 0.005$$

This is a number between zero and one.

3.
$$\frac{3201}{1000} = \frac{3000 + 200 + 1}{1000} = \frac{3000}{1000} + \frac{200}{1000} + \frac{1}{1000} = 3 + \frac{2}{10} + \frac{1}{1000} = 3 + 2 \times \frac{1}{10} + 0 \times \frac{1}{100} + 1 \times \frac{1}{1000} = 3.201$$

Two things to mention are that, first, this is a number between 3 and 4. Second, the decimal notation has the same digits as the number in the numerator of the original fraction. However, the decimal point is located such that there are three digits after the point. The denominator of the fraction is a power of 10, and in fact it is 10^3 ($10^3 = 1000$). In other words, the exponent for 10 (or the numbers of zeros in the denominator) is the number of digits after the point in the decimal notation.

4.
$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10} = 5 \times \frac{1}{10} = 0.5$$

5. $\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100} = \frac{20 + 5}{100} = \frac{20}{100} + \frac{5}{100} = \frac{2}{10} + \frac{5}{100} = 2 \times 10 + 5 \times \frac{1}{100} = 0.25$
6. $\frac{13}{5} = \frac{10 + 3}{5} = \frac{10}{5} + \frac{3}{5} = \underbrace{2 + \frac{3}{5}}_{\text{number between 2 and 3}} = 2 + \frac{3 \times 2}{5 \times 2} = 2 + \frac{6}{10} = 2.6$

These examples will make the following facts plausible (even though we are not giving a formal proof!):

For a positive fraction with the denominator a positive power of 10 (10, 100, 1000, and so on), the decimal notation will have the same digits as the natural number in the numerator. The decimal point is located such that there will be as many digits after the point as zeros in the denominator. If the digits of the natural number in the numerator do not provide the necessary number of digits after the point in the decimal notation, we must put zeros to the left of the natural number in the numerator. For example:

(a)
$$\frac{31}{10} = 3.1$$

(b) $\frac{31}{100} = 0.31$
(c) $\frac{31}{1000} = 0.031$
(d) $\frac{31}{10000} = 0.0031$

2. If a fraction does not have the denominator written as a power of 10, then, if possible, we could try using an equivalent fraction with denominator that is a power of 10. We can then determine the decimal notation as we did above. For example:

(a)
$$\frac{1}{2} = \frac{5}{10} = 0.5$$

(b) $\frac{1}{4} = \frac{25}{100} = 0.25$
(c) $\frac{1322}{500} = \frac{2644}{1000} = 2.644$

3. At this point, it is important to be aware of the following:

We know that for natural numbers (actually also for integers!) zeros on the left hand side do not have any value, for example

$$4 = 04 = 004 = 0000000004$$

Something similar happens for decimals regarding zeros on the right hand side, after the decimal point. For example:

$$2 = \frac{2}{1} = \frac{20}{10} = \frac{200}{100} = \frac{20000000}{100000000}$$

On the other hand

$$\frac{20}{10} = 2.0$$
$$\frac{200}{100} = 2.00$$
$$\frac{20000000}{10000000} = 2.00000000$$

therefore:

$$2 = 2.0 = 2.00 = 2.0000000$$

In general, any integer is equal to the decimal number that we get when adding a decimal point and zeros after it.

- 4. A positive number that is not an integer and is written in decimal notation, will be a number between the natural number before the decimal point and the same natural number plus one. For example:
 - (a) 0.003 is a number between 0 and 1. Therefore we can write:

(b) 2.999 is a number between 2 and 3. Then we can write:

2 < 2.999 < 3

(c) 36.12111 is a number between 36 and 37. Therefore:

26.2) Examples

1. Find decimal notation for each of the following fractions.

(a)
$$\frac{351}{100}$$

 $\frac{351}{100} = 3.51$

(b)
$$\frac{12}{5}$$

 $\frac{12}{5} = \frac{24}{10} = 2.4$
(c) $\frac{11}{4}$
 $\frac{11}{4} = \frac{11 \times 25}{4 \times 25} = \frac{275}{100} = 2.75$

2. Fill in the blank using either '=', '<', or '>' as appropriate:

(a) 3 3.01
Since 3.01 is between 3 and 4, then

$$3 < 3.01$$

(b) 4.0 3.9999
Since 4.0 = 4 and 3.9999 is between 3 and 4, then
 $4.0 > 3.9999$
(c) $\frac{86}{10}$ 8.6
Since the decimal notation for $\frac{86}{10}$ is 8.6, then
 $\frac{86}{10} = 8.6$

Exercises

1. Find decimal notation for each of the following fractions.

(a)
$$\frac{2391}{100}$$

(b) $\frac{91}{1000}$
(c) $\frac{9}{5}$
(d) $\frac{13}{4}$
(e) $\frac{7}{2}$

(f)
$$\frac{7}{100}$$

(g) $\frac{2}{25}$

(h)
$$\frac{501}{500}$$

2. Fill in the blank using either '=', '<', or '>' as appropriate:

- (a) $\frac{501}{500}$ 2
- (b) **5.0000** 5
- (c) 5.000001 5
- (d) 9 8.999999999

LESSON 27

Sometimes trying to change the denominator of a fraction into a power of ten could involve big numbers or it could be impossible! For example, if the denominator of the fraction is 3, we can not change the denominator into a power of 10 because 3 is not a factor for any power of 10. Therefore, we would like to count with an algorithm that will provide us with the decimal notation, without having to rewrite the fraction with denominator which is a power of 10. That algorithm is the "extended" division algorithm. We are using the word "extended" because we will go beyond the point of division with natural numbers, until we can make the remainder 0 or we conclude that we can never make it be 0. In the last case we need to understand the meaning of this situation.

11.2 Examples.

1. $\frac{1}{10}$

 $\frac{1}{10} = 1 \div 10$

Let's see the division step by step.

STEP ONE

$$\begin{array}{r} 0\\10)1\\-\underline{0}\\1\end{array}$$

In the first step the remainder is not zero but we do not have any more digits to bring down. We will write the dividend 1 as 1.0 and when bringing down the first digit after the point (in this case 0), we will also put a decimal point in the quotient and then proceed with the division.

STEP TWO

$$\begin{array}{r}
0.1 \\
10)1.0 \\
- 0 \\
10 \\
- 10 \\
- 10 \\
0
\end{array}$$

So the division is showing that

$$\frac{1}{10} = 0.1$$

2. $\frac{13}{8}$

$$\frac{13}{8} = 13 \div 8$$

Let's start the division:

$$\frac{1}{18)13}$$

- 8
5

In this case, we need to write 13 as 13.000 so we have enough digits to bring down until the remainder is zero. Again keep in mind that when bringing down the first digit after the point (in this case it is zero), we have to put a decimal point in the quotient before performing the division.

$$\begin{array}{r}
1.625 \\
8)13.000 \\
- 8 \\
- 4 8 \\
- 4 8 \\
20 \\
- 16 \\
40 \\
- 40 \\
0
\end{array}$$

Therefore we have that

$$\frac{13}{8} = 1.625$$

3. $\frac{1}{3}$

$$\frac{1}{3} = 1 \div 3$$

Let's start the division:

$$\begin{array}{r} 0\\3\overline{)1}\\-\underline{0}\\1\end{array}$$

Now let's write 1 as 1.0 and go on with the division. Now we get:

$$\begin{array}{r}
0.3 \\
3)1.0 \\
- 0 \\
10 \\
- 9 \\
1
\end{array}$$

Since the remainder is not zero now, we need to write 1 as 1.00 so we can go on with the division. Now we get:

$$\begin{array}{r} 0.33\\ 3)\overline{1.00}\\ - 0\\ 1 \\ 0\\ - 9\\ - 9\\ 10\\ - 9\\ 1\end{array}$$

It is not going to take long to arrive at the conclusion that the remainder will always be 1, meaning that there is no way to end the division with a remainder of zero! The meaning of this is that the decimal notation that we are looking for will **not** be **terminating**. In other words, the digits after the decimal point will never stop. However, there is a pattern that repeats infinitely (in this case the digit 3). For that reason, we say that the decimal expression is **repeating**. In order to indicate the pattern, we will use a horizontal bar on top of it.

So, in this case we write:

$$\frac{1}{3} = 0.\overline{3}$$

Examples 1 and 2 show decimal expressions that are terminating. Example 3 shows a decimal expression that is not terminating and repeating.

All fractions have decimal representations that are either **terminating** or **not terminating** and **repeating**.

27.1) Examples

Use division to find decimal notation for each of the following fractions.

1. $\frac{15}{16}$

$$\begin{array}{r} 0.9375\\ 16)\overline{15.0000}\\ - & 0\\ \hline 15 & 0\\ - & 14 & 4\\ \hline & 60\\ - & 48\\ \hline & 120\\ \hline & 112\\ \hline & 80\\ - & 80\\ \hline & 0\end{array}$$

Therefore $\frac{15}{16} = 0.9375$

$$\frac{1.66}{3)5.00} - \frac{3}{20} - \frac{18}{20} - \frac{18}{2}$$

Since the remainder will always be 2, then what we have is a decimal notation which is not terminating and repeating, where the pattern that repeats is 6.

In this case

$$\frac{5}{3} = 1.\overline{6}$$

From Decimal Notation to Fraction Notation.

Using the facts that we pointed out regarding decimal notation for a fraction with denominator given by a positive power of 10, we can write a terminating decimal as a fraction.

27.2) Examples.

1.
$$1.9 = \frac{19}{10}$$

2.
$$0.037 = \frac{37}{1000}$$

3. $5.3111 = \frac{53111}{10000}$

4.
$$3.15 = \frac{315}{100}$$

It is important to notice that for decimals, zeros on the right hand side **after the decimal point** do not have any value.

For example:

$$5.3 = \frac{53}{10} = \frac{530}{100} = \frac{5300}{1000} = \frac{53000}{10000}$$

therefore

$$5.3 = 5.30 = 5.300 = 5.3000$$

WARNING: It is incorrect to think that, for example, 2.50 is greater than 2.5 because 50 is greater than 5! According with what we just noticed, actually

$$2.50 = 2.5$$

Comparing Positive Decimals.

If we want to compare two different positive decimals that are not integers, it is convenient to first notice the consecutive positive integers having the decimals in between. In other words, we first look at the natural number before the point for each decimal. If they are different, then it should be clear how to compare the decimals. For example:

1. 3.312 > 1.5234

Since 3.312 is a number between 3 and 4 and 1.5234 is a number between 1 and 2, then it is easy to determine that 3.312 is greater than 1.5234

2. 4.0001 < 5.3

Since 4.0001 is a number between 4 and 5 and 5.3 is a number between 5 and 6, then it is easy to determine that 4.0001 is less than 5.3

However, if the natural number to the left of the point is the same, then we need to look at the digits after the point in order to compare the numbers. Let's take for example the numbers 3.12 and 3.2.

WARNING: it is **incorrect** to think that 3.12 is bigger than 3.2 because after the point in 3.12 there is a 12, and for 3.2, there is a 2 after the point. Although 12 > 2, 3.12 is not bigger than 3.2.

In fact,

	$3.12 = \frac{312}{100}$
	$3.2 = \frac{32}{10} = \frac{320}{100}$
Since	$\frac{312}{100} < \frac{320}{100}$

then we can conclude that

3.12 < 3.2

Comparing two positive decimals with the same natural number before the point requires comparing the digits after the point "digit by digit". Do not try to interpret the digits after the point as whole numbers. The bigger number will be the one with the bigger digit in the same position. In the case of 3.12 and 3.2, the first digit after the point for 3.12 is 1 and the first digit after the point for 3.2 is 2, so, since 1 < 2, then 3.12 < 3.2.

27.3) Examples.

Fill in the blank using either '=', '<' or '>' as appropriate.

1. 42.235 42.24

Since these are positive decimals with the same natural number before the point (42), then we need to look at the digits after the point. The first digit after the point is the same, but the second digit of the first number is less than the second digit of the second number. Therefore,

2. -1.4567 -1.5

Let's see first what happen with the positive decimals 1.4567 and 1.5. Again, the natural number before the point is the same (1), so, we need to look at the digits after the point. Since the first digit after the point for the first number is less than the first digit after the point for the second number, it follows that

This means that

$$-1.4567 > -1.5$$

3.
$$\frac{1}{3}$$
 0.333333
Since $\frac{1}{3} = 0.\overline{3}$, then $\frac{1}{3} > 0.33333$

4.
$$\frac{101}{100}$$
 1.0001
Since $\frac{101}{100} = 1.01$, then

$$\frac{101}{100} > 1.0001$$

5. 4.5000 4.5

Based on our previous discussions

$$4.5000 = 4.5$$

Exercises

1. Find decimal notation for each of the following numbers

(a)
$$\frac{139}{8}$$

(b) $-\frac{32}{25}$
(c) $\frac{31}{12}$
(d) $2\frac{103}{330}$
(e) $-\frac{5}{16}$

- 2. Write each of the following numbers as a fraction.
 - (a) **25.6655**
 - (b) 0.0043
 - (c) -12.01
 - (d) $(1.3)^2$
- 3. Fill in the blank using either '=', '<' or '>' as appropriate. Explain your answer.
 - (a) $\frac{2}{3}$ 0. $\overline{6}$ (b) 2.999 3.00 (c) 9.330 9.6 (d) -34.2230 - 34.24 (e) 0.0001 - 234
 - (f) $3.25 \qquad \frac{3250}{1000}$

LESSON 28

Addition and Subtraction of Decimals.

We know than when arranging natural numbers vertically to perform an addition, we need to make sure that digits in a particular place go under those in the same place: ones under ones, tens under tens and so on.

When adding decimals, we need to use the same idea. That means that when arranging them vertically, the decimal points need to be lined up so ones go under ones, tens under tens and so on. Tenths (the first digit after the decimal point) must also be lined up under tenths, hundredths (the second digit after the decimal point) under hundredths, and so on.

The same principle applies when subtracting a positive decimal number from a bigger positive decimal number.

Keep in mind that all natural numbers can be written as decimals by putting the decimal point at the right hand side, and then writing zeros after the decimal point (for example, 5 = 5.0 = 5.00, etc).

28.1) Examples Compute

1. 23.25 + 7.9

	23.25
+	7.9
	31.15

or

	23.25
+	7.90
	31.15

Then

23.25 + 7.9 = 31.15

2. **15** – **8**.7

	15.0
-	8.7
	6.3

Then

15 - 8.7 = 6.3

3. 9.456 + 11 + 5.9

	+	$9.456 \\ 11 \\ 5.9 \\ 26.356$
or		
		9.456
		11.000
	+	5.900
		26.356

Therefore

9.456 + 11 + 5.9 = 26.356

4. 132.52 - 14.7

	132.52
-	14.7
	117.82

or

Therefore

132.52 - 14.7 = 117.82

Now, let's do some additions and subtractions including negative numbers written in decimal notation. The rules regarding the signs are the same as in the case of integers.

28.2)Examples.

1. -3.15 + 2.7 = -(3.15 - 2.7) = -0.45

$$3.15$$

- 2.7
- 0.45

2. -11.3 - 134.47 = -(11.3 + 134.47) = -145.77

3. 9.55 + (-10.1) = -(10.1 - 9.55) = -0.55

$$\begin{array}{r}
 10.10 \\
 - 9.55 \\
 \hline
 0.55
 \end{array}$$

4. 0.91 - 1 = -(1 - 0.91) = -0.09

$$\begin{array}{r}
 1.00 \\
 - 0.91 \\
 \overline{0.09}
 \end{array}$$

5. -3.3 - (-2.22) = -3.3 + 2.22 = -(3.3 - 2.22) = -1.08

$$3.30$$

- 2.22
1.08

6. 9.151 - (-11.5) = 9.151 + 11.5 = 20.651

$$9.151$$

+ 11.5
20.651

Also if we combine additions and subtractions, they are associated from left to right. 28.3)Examples.

- 1. $\underbrace{9.73 11.7}_{-1.97} 3.59 + 5 = \underbrace{-1.97 3.59}_{-5.56} + 5 = -5.56 + 5 = -0.56$
- 2. $\underbrace{-1.1 1.1}_{-2.2} + 1 + 1 = \underbrace{-2.2 + 1}_{-1.1} + 1 = -1.1 + 1 = -0.1$
- 3. $\underbrace{-0.3 + 1.12}_{0.82} (-2.3) = 0.82 + 2.3 = 3.12$
- 4. 1.2 1.2 3.12 + 0.352 = 0 3.12 + 0.352 = -3.12 + 0.352 = -(3.12 0.352) = -2.768

Exercises

Compute

- 1. 17.3 + 9.77
- 2. 12.12 + 7 + 9.9
- 3. 221.321 + 45.9 + 99 + 5.35
- 4. 35.1 12.532
5. 9-7.351

- 6. **14.12 8**
- 7. -2.2 3.31
- 8. -1.15 + 5.321
- 9. 0.16 1.76
- 10. **3.41** − (−**1**)
- 11. **1.93 3**
- 12. -5 (-6.713)
- 13. 8.2 9.17 3 + 1.1
- 14. -(-1.1) (-1.1) 1.1
- 15. 0.351 1.23 + 2
- 16. -5.37 (-5.37) 4.65 + 7 3.3339
- 17. 8 8.001 + 1 (-0.3) + 1

LESSON 29

Multiplication of Decimals.

If we want to multiply two decimals, one thing that we could do is to write them as fractions and then multiply them.

For example:

$$2.5 \times 3.71 = \frac{25}{10} \times \frac{371}{100} = \frac{25 \times 371}{10 \times 100} = \frac{9275}{1000} = 9.275$$

The example shows that the digits of the result (9.275) are exactly the same ones we would get if we "eliminate" the decimal points from the original numbers and simply multiply the natural numbers $(25 \times 371 = 9275)$. Also the result (9.275) has three digits after the point. The first number in the multiplication (2.5) has one digit after the point, and the second number in the multiplication (3.71) has two digits after the point. In other words, the result of the multiplication has as many digits after the point as the total of digits after the point among the numbers multiplied.

In conclusion, we note that we could have done the multiplication by multiplying the original numbers just as we multiply natural numbers. Then, locate the decimal point, so that the number of digits after it is the same as the total number of digits after the decimal point for the numbers that we are multiplying.

$$\begin{array}{r}
2.5 \\
\times & 3.71 \\
\hline
25 \\
1750 \\
+ & 7500 \\
\hline
9.275
\end{array}$$

Notice that when arranging the numbers vertically to perform the multiplication, we do not need to line up the decimal points.

Also, it is important to realize that if we write the decimals as fractions and multiply, we would get the same result but written as a fraction. Therefore, all the properties that are true for operations with fractions are also true for operations with decimals. In particular, keep in mind commutative and associative properties.

Besides, we can also consider negative decimals. We will use the same rules for multiplication of signs that we discussed before. 29.1)Examples.

1. $4 \times (-5.31) = -(4 \times 5.31) = -21.24$

$$\begin{array}{r} 5.31 \\ \times & 4 \\ \hline 21.24 \end{array}$$

2. $-4.11 \times (-3.102) = 4.11 \times 3.102 = 12.74922$

	3.102
\times	4.11
	3102
	31020
+	1240800
	12.74922

3.
$$3.25 \times 10 = \frac{325}{100} \times 10 = \frac{325 \times 10}{10 \times 10} = \frac{325}{10} = 32.5$$

Notice that in this example one of the numbers is 10. Also, the result has the same digits as the other number in the multiplication (3.25), but the decimal point has been "moved" one place to the right.

In general, if we multiply a decimal by a positive power of $10 (10 = 10^1, 100 = 10^2, 1000 = 10^3,$ and so on), the result is what we get by "moving" the decimal point of the decimal number to the right, as many places as we have zeros in the power of 10. If we run out of places to continue "moving" the point to the right, then we start adding zeros, as it shows in the next example.

4.
$$-35.3 \times 100 = -\frac{353}{10} \times 100 = -353 \times 10 = -3530$$

5.
$$4.2 \times 0.1 = \frac{42}{10} \times \frac{1}{10} = \frac{42}{100} = 0.42$$

Notice that in this last example the decimal point of the first number in the multiplication "moved" one place to the left!

It happened because the other number in the multiplication is 0.1. Something similar will happen when multiplying a decimal by 0.01 or 0.001, and so on. In other words, when the other number in the multiplication is a negative integer power of 10 (0.1, 0.01, 0.001, etc), we must "move" the decimal point to the left.

6.
$$-53.1 \times (-0.001) = 53.1 \times 0.0001 = \frac{531}{10} \times \frac{1}{1000} = \frac{531}{10000} = 0.0531$$

7. $2.11 \times (-1.2) \times 3 = -2.532 \times 3 = 7.596$

8.
$$(-1.3) \times (-2.56) \times (-10) = 3.328 \times (-10) = -33.28$$

9.
$$4.51 \times 0.1 \times (-3.3) \times 10 = 4.51 \times (-3.3) \times \underbrace{0.1 \times 10}_{1} = 4.51 \times (-3.3) \times 1 = -14.883 \times 1 - -14.883$$

Notice that in this example we used the commutative and associative properties of multiplication.

Now we can also compute exponentials that have a decimal number for the base.

29.2) Examples.

Compute

- 1. $(1.2)^2 = 1.2 \times 1.2 = 1.44$
- 2. $(-2.12)^2 = (-2.12) \times (-2.12) = 2.12 \times 2.12 = 4.4944$
- 3. $(1.1)^3 = 1.1 \times 1.1 \times 1.1 = 1.21 \times 1.1 = 1.331$
- 4. $-(2.3)^4 = -(2.3 \times 2.3 \times 2.3 \times 2.3) = -(5.29 \times 2.3 \times 2.3) = -(12.167 \times 2.3) = -27.9841$

Division of Decimals.

Let's start with the case where the dividend is a positive decimal but the divisor is a positive integer.

We can always write the decimal as a fraction and perform the division. For example:

$$3.99 \div 3 = \frac{399}{100} \div \frac{3}{1} = \frac{399}{100} \times \frac{1}{3} = \frac{399}{100 \times 3} = \frac{3 \times 133}{100 \times 3} = \frac{133}{100} = 1.33$$

However, sometimes the numbers may not be "nice" enough to be handled in this way. Therefore, we would like to know how to perform the long division algorithm.

The first thing to do is to divide the natural number before the decimal point by the divisor (in the usual way). This basically means that in the first step of the division we can not go beyond the point. We go on with the usual process. When bringing down the first digit after the decimal point, before dividing, we need to put a decimal point in the quotient and then proceed with the division.

Keep in mind that if necessary, we can always add zeros at the end of the decimal number (it would not change its value), so we can go on with the division until either the remainder is zero, or we determine that the quotient will not be terminating but a repeating decimal.

29.3)Examples.

1. $1.21 \div 2 = 0.605$

Let's show the long division step by step.

FIRST STEP

We divide the natural number before the decimal point by the divisor.

$$\begin{array}{r}
 0 \\
 2)1.21 \\
 - 0 \\
 1
 \end{array}$$

SECOND STEP

Now we bring down the first digit after the decimal point (2), put a point at the quotient and then divide.

0.6			
2)1.	21	
-	0		
-	1	2	
-	1	2	
-		0	

THIRD STEP

Continue with the division bringing down the next digit (1).

$$\begin{array}{r} 0.60\\ 2)1.21\\ - 0\\ 1 2\\ - 1 2\\ - 0\\ - 0\\ 1\\ \end{array}$$

FOURTH STEP

Since the remainder is not zero and there are no more digits to bring down, we may add a zero at the end of the dividend and continue with the division

$$\begin{array}{r} 0.605\\ \hline 2)1.210\\ \hline -0\\ \hline 12\\ \hline -12\\ \hline 01\\ \hline -0\\ \hline 10\\ \hline -10\\ \hline 0\end{array}$$

2. $24.382 \div 12 = 2.0318\bar{3}$

$$\begin{array}{r} 2.031833\\12)24.382000\\-\underline{24}\\03\\-\underline{0}\\38\\-36\\\underline{22}\\-\underline{12}\\100\\-\underline{96}\\40\\-\underline{36}\\40\\-\underline{36}\\4\end{array}$$

We can see that from this point forward, the remainder will always be 4. That means that the decimal expression of the quotient is not terminating and repeating. In this case the pattern that repeats is the digit 3.

3. $135.36 \div 3 = 45.12$

$$\begin{array}{r}
45.12\\3)\overline{135.36}\\-\underline{12}\\15\\-\underline{15}\\0&3\\-\underline{3}\\06\\-\underline{6}\\0\end{array}$$

4. $23.5 \div 10 = 2.35$

$$23.10 \div 10 = \frac{235}{10} \div \frac{10}{1} = \frac{235}{10} \times \frac{1}{10} = \frac{235}{100} = 2.35$$

Notice that the result has the same digits as the dividend with the decimal point "moved" to the left one place. This is what happens when the divisor is 10.

In general, when dividing a decimal by a positive integer power of 10 (10, 100, 1000, and so on), the result will be the one corresponding to "moving" the decimal point of the dividend to the left, as many spaces as zeros we have in the power of 10. We understand that if we run out of places to continue "moving" the decimal point, then we can add zeros to the left hand side of the dividend as necessary.

5. $23.5 \div 1000 = 0.0235$

Now let's also consider negative decimals. In this case, all we need to do is to use the same rules for division of signs that we learned when working with integers.

29.4) Examples.

1. $31.5972 \div (-12) = -(31.5972 \div 12) = -2.6331$

$$\begin{array}{r}
2.6331 \\
12)31.5972 \\
- \underline{24} \\
75 \\
- \underline{72} \\
39 \\
-36 \\
37 \\
-36 \\
12 \\
-12 \\
0
\end{array}$$

2. $(-65.32) \div 1000 = -0.06532$

3. $(-4.328) \div (-0.01) = 432.8$

Exercises

Compute.

- 1. $3.52 \times (-4.111)$
- 2. $(-2.8) \times (-9.7)$
- 3. 53.41 × 1000
- 4. 9×0.01
- 5. $3.25 \times 5.5 \times (-2.1)$
- 6. $(-0.1) \times (-0.1) \times (-0.01)$
- 7. $(-6.63) \times 0.01 \times (-2.2) \times 1000$
- 8. $(-5.5)^3$
- 9. $-(-2.11)^3$
- 10. $-(-33.1)^2$
- 11. (0.1)⁵
- 12. $40.1951 \div (-11)$
- 13. $-5.932 \div (-12)$
- 14. −**893.16** ÷ 200
- 15. (-4.78999) ÷ 1000
- 16. $35.8722 \div (-0.001)$
- 17. $0.1 \div 200$
- 18. −69.797 ÷ 13
- 19. $0.35888 \div (-9)$

LESSON 30

Now let's consider the case where the divisor is a positive decimal. We can "change" the division into one where the divisor is a positive integer. For example:

$$2.351 \div 1.2 = \frac{2351}{1000} \div \frac{12}{10} = \frac{2351}{1000} \times \frac{10}{12} = \frac{2351}{100} \times \frac{1}{12} = \frac{2351}{100} \div \frac{12}{1} = 23.51 \div 12$$

We notice that in this example, the original division is equal to the division where the new numbers are the result of "moving" the decimal point of the original numbers one place to the right.

It could be checked that all rules that are valid for operations with fractions, are also valid when using fraction notation. In other words, even if the numerator and denominator are not integers, the rules remain valid.

In particular we can write:

$$2.351 \div 1.2 = \frac{2.351}{1.2} = \frac{2.351 \times 10}{1.2 \times 10} = \frac{23.51}{12}$$

Now we only need to perform the last division.

30.1)Examples.

1. $2.351 \div 1.2$

 $2.351 \div 1.2 = 23.51 \div 12 = 1.9591\overline{6}$

$$\begin{array}{r}
1.95916\\
12)23.51000\\
- 12\\
\overline{11}5\\
- 108\\
\overline{71}\\
-60\\
\overline{110}\\
-108\\
\overline{00}\\
-12\\
\overline{80}\\
- 72\\
\overline{8}
\end{array}$$

2. 59.7432 ÷ 1.1

$$59.7432 \div 1.1 = \frac{59.7432}{1.1} = \frac{59.7432 \times 10}{1.1 \times 10} = \frac{597.432}{11} = 54.312$$

$$\frac{54.312}{11} = \frac{54.312}{11} = \frac{54$$

3. $2.81373 \div (-2.13)$

$$2.81373 \div (-2.13) = -(2.81373 \div 2.13) = -\left(\frac{2.81373}{2.13}\right) = -\left(\frac{2.81373 \times 100}{2.13 \times 100}\right) = -\left(\frac{281.373}{213}\right) = -1.321$$

$$\begin{array}{r}
1.321\\
213)281.373\\
- 213\\
\overline{\ 68\ 3}\\
- 63\ 9\\
\overline{\ 4\ 47}\\
- 4\ 26\\
\overline{\ 213}\\
- 213\\
\overline{\ 0}
\end{array}$$

4. $(-16.12) \div (0.13)$

$$(-16.12) \div (0.13) = -(16.12 \div 0.13) = -\left(\frac{16.12}{0.13}\right) = -\left(\frac{16.12 \times 100}{0.13 \times 100}\right) = -\left(\frac{1612}{13}\right) = -124$$

5.
$$(-1.246) \div (-0.02) = \frac{1.246 \times 100}{0.02 \times 100} = \frac{124.6}{2} = 62.3$$

6. $5.1 \div 0.1$

$$5.1 \div 0.1 = \frac{5.1}{0.1} = \frac{5.1 \times 10}{0.1 \times 10} = \frac{51}{1} = 51$$

Now the decimal point "moved" to the right! This occurred because the divisor was 0.1, which is a negative integer power of 10.

7. $5.1 \div 0.01$

$$5.1 \div 0.01 = \frac{5.1}{0.01} = \frac{5.1 \times 100}{0.01 \times 100} = \frac{510}{1} = 510$$

Combining Operations with Decimals.

Once again let's combine all operations, this time using decimals and integers. Keep in mind the order of operations.

- 1. Operations inside grouping symbols. If there are grouping symbols inside grouping symbols, then we need to work from inside out.
- 2. Exponentials.
- 3. Multiplications and divisions are associated LEFT TO RIGHT.

30.2)Examples.

- 1. $1 + 2 \times 0.2 = 1 + 0.4 = 1.4$
- 2. $(1+2) \times 0.2 = 3 \times 0.2 = 0.6$

- 3. $3.12 \times (0.1)^3 = 3.12 \times 0.001 = 0.00312$
- 4. $-2.43 + 5 \times 1.3 + (-10)^3 (0.1)^2 = -2.43 + 5 \times 1.3 + (-1000)(0.01) = -2.43 + 6.5 10 = 4.07 10 = -5.93$
- 5. $(-2.43+5) \times 1.3 (-10)^3 \times (-0.1)^2 = 2.57 \times 1.3 (-10)^3 \times (-0.1)^2 = 2.57 \times 1.3 (-1000) \times (0.01) = 3.341 + 10 = 13.341$
- 6. $-1.15 + 10(7 8.3)^2 + 5 \div 0.1 \times 10 = -1.15 + 10(-1.3)^2 + 5 \div 0.1 \times 10 = -1.15 + 10 \times 1.69 + 5 \div 0.1 \times 10 = -1.15 + 16.9 + 5 \div 0.1 \times 10 = -1.15 + 16.9 + 50 \times 10 = -1.15 + 16.9 + 500 = 15.75 + 500 = 515.75$

$$\frac{(0.2)^2 \times (0.1)^3 \times 10^2}{0.2 \times 100} = \frac{0.2 \times 0.2 \times (0.1)^3 \times 100}{0.2 \times 100} = 0.2 \times 0.001 = 0.0002$$

8.

$$\frac{(1.2 - 0.2)(1.1 \times 0.1)}{0.11} = \frac{1 \times 0.11}{0.11} = 1$$

9.

$$\frac{11.1 + 0.1}{1.1 - 0.1} = \frac{11.2}{1} = 11.2$$

10.

$$0.3 - (0.7)(0.2) = 0.3 - 0.14 = 0.16$$

11.

$$\frac{1.21 - 2.3}{2 - 2 \times 6} = \frac{-1.09}{2 - 12} = \frac{-1.09}{-10} = \frac{1.09}{10} = 0.109$$

12.

$$\frac{1.21 - 2.3}{(2 - 2)6} = \frac{-1.09}{0 \times 6} = \frac{-1.09}{0}$$
 This is undefined!

13.

$$\frac{-3.1 + 0.1 \times 31}{5} = \frac{-3.1 + 3.1}{5} = \frac{0}{5} = 0$$

Exercises

Compute

- 1. $0.9435 \div 0.3$
- 2. $28.875 \div (-1.25)$
- 3. $(-3274.5 \div 1.11)$
- 4. $(-5.392) \div (-0.02)$
- 5. 85938 ÷ 1.2
- 6. $(-10.25) \div (-2.5)$
- 7. $63.212 \div (-0.01)$
- 8. (-9.21) ÷ (0.001)
- 9. $(-538) \div (-0.1)$
- 10. $0.02 \div 0.1$
- 11. $-1 + 3 \times (-0.1)$
- 12. $(-1+3) \times (-0.1)$
- 13. $1.12 \times (0.3)^2$
- 14. $10 5 \times 2.2 (0.2)^3 \times 10^3$
- 15. $(10-5) \times 2.2 (-0.2)^3 \times 10^2$
- 16. $-3 + 10(6 7.28) 3 \div 0.3 \times 10$

17.

$$\frac{(0.4)^3 \times (-0.1)^3 \times 10}{(0.4)^2 \times 10^2}$$

18.

$$\frac{(2.2 - 3.3)(2.5 - 2.50)}{2.567}$$
19.

$$\frac{2 - 0.1 \times 3}{4 - 0.1 \times 4}$$
20.

$$\frac{2(-2.11 + 1.1) \times 4}{2 \times 1.01}$$

1. Find decimal notation for each of the following fractions.

(a)
$$\frac{25}{1000}$$

(b)
$$\frac{17}{5}$$

(c)
$$\frac{9}{4}$$

(d)
$$\frac{435}{100}$$

(e)
$$\frac{5}{2}$$

2. Fill in the blank using either '=', '<', or '>' as appropriate:

(a)
$$\frac{468}{467}$$

(b) 7.0
$$\frac{700}{100}$$

1

1. Find decimal notation for each of the following fractions.

(a)
$$\frac{332}{10000}$$

(b)
$$\frac{7}{5}$$

(c)
$$\frac{21}{4}$$

(d)
$$\frac{11}{8}$$

(e)
$$\frac{4}{3}$$

2. Fill in the blank using either '=', '<', or '>' as appropriate:

(a)
$$\frac{2}{3}$$
 0.666666666
(b) 4.56 4.123
(c) -8 0.99
(d) -0.084 - 0.12
(e) 5.01 $\frac{5010}{1000}$

Compute. 1. 58.3 + 9.28 2. 27 - 9.91 3. 13.251 + 35 + 7.3 4. -4.19 + 1.3 5. **4.88** + (-15.9) 6. 351.73 - 38.9 7. -21.8 - 285.987

8. 0**.8**9 − 1

Compute.

1. -4.5 - (-3.33)

- 2. 8.295 (-25.3)
- 3. 8.75 12.9 2.88 + 4
- 4. -3.3 3.3 2 + 4.4
- 5. -0.7 + 3.35 (-0.189)
- 6. 3.35 3.35 4.27 + 0.991
- 7. 3.1×5.78
- 8. 17.12 × 10

Compute.

1. -8.71×7

- 2. $5.01 \times (-9.321)$
- 3. $-0.01 \times (-3.2)$

4. -12.1 × 3.12

5. -0.001×8

6. $0.01 \times (-4.31)$

7. $3.11 \times (-4) \times 1.22$

8. $-0.1 \times (-35.8) \times (-0.001)$

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Compute.

1. $(-8.3)^2$

2. $-(-1.1)^4$

3. $(3.1)^3$

4. $(-1.01)^3$

5. 3.63 ÷ 2

6. 48.764 ÷ 6

7. $-1.9 \div 20$

8. 48.6 ÷ (−3)

Compute.

1. $4.702 \div 2.4$

2. $3.52 \div 0.1$

3. $3.1 \div (-0.02)$

4. $-24.382 \div (-0.3)$

5. $-5.3 \div 100$

6. $6.57 \div (-0.09)$

7. $-52 \div 0.65$

8. 45 ÷ 72

Compute.

1. $2 - 4 \times 1.51$

2. $(2-4) \times 1.51$

3. $(0.1)^4 \times 51.35$

4. $-3.681 + 4 \times 2.1 + (-10)^4 (0.1)^3$

5. $(-3.681 + 4) \times 2.1 - (-10)^5 (0.1)^3$

6. $-3.89 + 10(6 - 7.31)^2 + 6 \div (-0.02) \times 10^{-10}$

Compute.

1. 0.4 - (0.9)(0.3)

2.
$$0.01 - 0.346 \div 0.4$$

3.
$$\frac{(0.5)^3 \times (0.1)^4 \times 10^3}{0.5 \times 1000}$$

4.
$$\frac{(2.3 - 4.3)(2.38 \times 0.1)}{0.0238 \times 10}$$

5.
$$\frac{35.4 + 0.01}{2.3 - 3 \times 0.1}$$

LESSON 1

Exercises 1.1 (page 4)

- 1. Fill in the blank using either '=' or '≠' as appropriate.
 - a. 3 = 3 + 0
 - b. $1 \times 3 \neq 2 \times 2$
 - c. $64 \neq 604$
 - d. $56 = 7 \times 8$
 - e. 4+3+2=(4+3)+2
 - f. $33 \times 2 \times 4 \neq 8 \times 4$
- 2. True or false and if true, name the property or properties
 - a. 1298 + 7459 = 7459 + 1298: true, commutative property of addition.
 - b. 1298 + 7459 = 7460 + 1298: false. The result on the right hand side is one more than the result on the left hand side (you don't need the actual computation to check this!).
 - c. $358 \times 498 = 498 \times 359$: false. The result on the right hand side is 498 more than the result on the left hand side (you don't need the actual computation to check this!).
 - d. $987 \times 988 = 988 \times 987$: true, commutative property of multiplication
 - e. (547 + 1250) + 3 = 547 + (1250 + 3): true, associative property of addition.
 - f. (547 + 1250) + 3 = 547 + (3 + 1250): true, associative and commutative properties of addition.

Exercises 1.2 (page 6)

- 1) $1000 \times 512 = 512000$
- 2) $10^4 = 10 \times 10 \times 10 \times 10 = 10000$
- 3) $15^1 = 15$
- 4) $6^0 = 1$
- 5) $100^3 = 100 \times 100 \times 100 = 1000000$
- 6) $325 \times 100 = 32500$
- 7) $2^3 = 2 \times 2 \times 2 = 8$; $3^2 = 3 \times 3 = 9$; therefore, 3^2 is larger.
- 9) $1^{3} = 1 \times 1 \times 1 = 1; 1+1+1=3$; therefore, 1+1+1 is larger.

LESSON 2

Exercises 2.1 (pages 8-9)

- 1. Compute . Use the equal sign and when performing more than one operation show one step for each operation.
 - a. $1+0 \times 4 = 1+0 = 1$ b. $3+3^2=3+9=12$ c. 3(5+2) = 3(7) = 21d. $4 \times 10^2 = 4 \times 100 = 400$ e. $(4 \times 10)^2 = 40^2 = 40 \times 40 = 1600$ f $(3+3)^2 = 6^2 = 6 \times 6 = 36$ g. $5 \times 10^4 + 3 \times 10 = 5 \times 10000 + 3 \times 10 = 50000 + 30 = 50030$ h. $342 + 23 \times 2 = 342 + 46 = 388$ i. $2 \times 200 + 2 = 400 + 2 = 402$ j. 398 + 2 + 10 = 400 + 10 = 410k. $45 \times 100 + 3 \times 15 = 4500 + 3 \times 15 = 4500 + 45 = 4545$ 1. $0 \times 278 = 0$ m. $100 \times 981 = 98100$ n. $2345 \times 678 \times 0 \times 77 = 0$ o. $1^{8731} = 1$ p. $3^0 \times 1^{3298} = 1 \times 1 = 1$
 - q. $6 \times 10^2 = 6 \times 100 = 600$
 - r. $20^4 \times 10 = 160000 \times 10 = 1,600,000$
- 2. Determine for which of the following removing the parentheses would not change the value of the expression.
 - a. No change ; (2+3)+4=5+4=9 and 2+3+4=5+4=9
 - b. Yes, the value changes $(2 \times (3 + 4)) = 2 \times 7 = 14$ whereas $2 \times 3 + 4 = 6 + 4 = 10$
 - c. No change; $2 \times (3 \times 4) = 2 \times 12 = 24$ and $2 \times 3 \times 4 = 6 \times 4 = 24$
 - d. No change i; $2 + (3)^2 = 2 + 9 = 11$ and $2 + 3^2 = 2 + 9 = 11$
 - e. Yes, the value changes; $2 + (3+3)^2 = 2 + (6)^2 = 2 + 36 = 38$ whereas $2+3+3^2 = 2+3+9 = 5+9 = 14$.
 - f. Yes, the value changes; $(3 \times 3)^0 = (9)^0 = 1$ whereas $3 \times 3^0 = 3 \times 1 = 3$
 - g. No change; $2 + (3 \times 4) = 2 + 12 = 14$ and $2 + 3 \times 4 = 2 + 12 = 14$

- 3. Fill in $= or \neq$ and justify
 - a. $2^3 \neq 2 \times 3$; Left side has value 8 but right side has value 6; also $2^3 = 2 \times 2 \times 2 \neq 2 \times 3$
 - b. $3^2 \neq 2^3$; left side has value 9 but right side has value 8; also $3 \times 3 \neq 2 \times 2 \times 2$
 - c. $3 \times 3^2 = 3^3$; both sides have value 27; also $3 \times 3^2 = 3 \times 3 \times 3 = 3^3$
 - d. $4 \times 2^5 \neq (4 \times 2)^5$; left side has value 128 but right side has value 32768; also left side is the multiplication problem of $4 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ which is not the same as the right sides multiplication problem of $(4 \times 2) \times (4 \times 2) \times (4 \times 2) \times (4 \times 2) \times (4 \times 2)$
 - e. $92 \times 34^3 = 34^3 \times 92$; commutative property of multiplication
 - f. $4^3 \neq 3 \times 3 \times 3 \times 3$; left side has value 64 but right side has value 81; also $4^3 = 4 \times 4 \times 4$ which is the left side and that is not the same as the right side of $3 \times 3 \times 3 \times 3$
 - g. $75^5 = 75 \times 75 \times 75 \times 75 \times 75$; the definition of the exponent 5 as repeated multiplication of 75, 5 times
 - h. $12 + 45 \times 56 = 12 + (45 \times 56)$; order of operations confirms that both sides yield 2532; multiplication must be performed before addition
 - i. $9 + 7 \times 12 \neq (9 + 7) \times 12$; order of operation would require multiplication to be performed first on the left side but the parentheses would take precedent on the right side and addition would be performed first; the left side value is 93 and the right side value is 192
 - j. $(5+2) \times 4 \neq 5 + (2 \times 4)$; the left side yields 28 but the right side yields 40; placement of parentheses makes a difference here
 - k. (235 + 789) + 99 = 235 + (789 + 99); this is an example of the associative property of addition.
 - 1. $6543 \times 548 = 548 \times 6543$; this is an example of the commutative property of multiplication.

Exercises 2.2 (pages 10, 11)

- 1. Compute. Use the equal sign and show one step for each operation.
 - a. 7 5 + 2 = 2 + 2 = 4
 - b. 8 3 2 = 5 2 = 3
 - c. 7+3-10=10-10=0
 - d. 9 (5 + 4) = 9 9 = 0
 - e. $(8-3) \times 10^2 = 5 \times 10^2 = 5 \times 100 = 500$
 - f. $12 2 \times 3 = 12 6 = 6$
 - g. $(12-2) \times 3 = 10 \times 3 = 30$
 - h. $8-2^3 = 8-8 = 0$
 - i. $10 \times (5 3 + 1) = 10 \times (2 + 1) = 10 \times 3 = 30$
 - j. $10 \times 5 3 + 1 = 50 3 + 1 = 47 + 1 = 48$
 - k. $10+5 \times 3-3 = 10+15-3 = 25-3 = 22$

- 2. For each of the following determine if removing the parentheses would change the value of the expression. Justify your answer
 - a. No change; (7-5)+2=2+2=4 and 7-5+2=2+2=4
 - b. Yes, the value changes; 7 (5 + 2) = 7 7 = 0 whereas 7 5 + 2 = 2 + 2 = 4
 - c. Yes, the value changes; $(5-4) \times 0 = 1 \times 0 = 0$ whereas $5-4 \times 0 = 5-0 = 5$
 - d. No change; $9 (2 \times 3) = 9 6 = 3$ and $9 2 \times 3 = 9 6 = 3$
 - e. Yes, the value changes; $(9-2) \times 3 = 7 \times 3 = 21$ whereas $9-2 \times 3 = 9-6 = 3$

LESSON 4

Exercises

- 1. Fill in the blank using either '=', '<', or '>', as appropriate.
 - (a) -101 < 100(b) -(-6) = 6(c) +125 = 125(d) -450 < 0(e) -(-5) > 0(f) -(-7) > -7(g) -5 < -4
- 2. Plot and label the numbers on a number line.



3. Write the following numbers from the smallest to the largest.

$$-2, 10, -7, -9, 5, 0$$

From the smallest to the largest:

$$-9, -7, -2, 0, 5, 10$$

4. Among the following numbers, identify all pairs of opposite numbers.

9,7,-9,-(-8),-8,0,12,-(+3),+(+3)

- (a) 9 and -9 are opposites
- (b) -(-8) = 8, therefore -(-8) and -8 are opposites
- (c) -(+3) = -3 and +(+3) = 3, therefore -(+3) and +(+3) are opposites
- 5. Find the opposite of each of the following integers. Then plot both numbers on a number line.

(a) **8**

Opposite: -8





Exercises

- 1. Determine which are true and if so, what property of addition makes the statement true
 - a. -235 + 789 = 789 + (-235): true; commutative property.
 - b. -87 + (-56) = 87 + 56; false; the results are opposites of each other:
 - **-**142 ≠142
 - c. -99 + (-33) = -33 + (-99); true; commutative property.
 - d. 54 + (-67) = 67 + (-54); false; the results are opposites of each other: $-13 \neq 13$
 - e. (-6+4)+(-2) = -6+[4+(-2)]; true; associative property
 - f. -1+(-2+3) = (-1+2)+3; false; $0 \neq 4$
- 2. Compute

a.
$$-6 + (-2) = -(6+2) = -8$$

b. $-8 + 0 = -8$
c. $-135 + 135 = 0$
d. $0 + (-189) = -189$
e. $-15 + 9 = -(15 - 9) = -6$
f. $15 + (-9) = 15 - 9 = 6$
g. $-1 + 1 = 0$
h. $-1 + (-1) = -(1 + 1) = -2$
i. $6 + (+5) = 6 + 5 = 11$
j. $-(-4) + 0 = 4 + 0 = 4$
k. $-18 + 8 = -(18 - 8) = -10$
l. $-25 + 29 = 29 - 25 = 4$
m. $+1 + (+2) = 1 + 2 = 3$
n. $(-9) + (-11) = -(9 + 11) = -20$
o. $-9 + 11 = 11 - 9 = 2$
p. $9 + (-11) = -(11 - 9) = -2$

Exercises

Compute

1.
$$-1 \times (-2) = 1 \times 2 = 2$$

2. $3 \times (-9) = -(3 \times 9) = -27$
3. $-5 \times 0 \times 9 = 0 \times 9 = 0$
4. $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = 4 \times (-2) \times (-2) = -8 \times (-2) \times (-2) = = 16 \times (-2) = -32$
5. $-2^5 = -(2^5) = -(2 \times 2 \times 2 \times 2 \times 2) = -32$
6. $-10 \times 251 = -(10 \times 251) = -2510$
(Note: affix a 0 to 251 since multiplying by 10.)
7. $(-10)^4 = (-10) \times (-10) \times (-10) \times (-10) = 1000 \times (-10) \times (-10) = -1000 \times (-10) = 10000$
8. $-10^4 = -(10^4) = -(10 \times 10 \times 10 \times 10) = -10000$
9. $(-100)^3 = (-100) \times (-100) \times (-100) = 10000 \times (-100) = = -(10000 \times 100) = -1000000$
10. $(-1)^{53} = -1$
(Note: exponent is odd; result will be negative for negative base.)
11. $(-1)^{62} = 1$
(Note: exponent is even; result will be positive for negative base.)
12. $-1^{48} = -(1^{48}) = -(1) = -1$
(Note: 1 raised to any exponent is equal to 1.)
13. $-1^{49} = -(1^{49}) = -(1) = -1$
(Note: 1 raised to any exponent is equivalent to 1.)
14. $2 \times (-3) \times 3 = -6 \times 3 = -18$
15. $-5 \times 10 \times (-3) = -50 \times (-3) = 50 \times 3 = 150$

Exercises

Compute

- 1. 12-3=92. 3-12=3+(-12)=-(12-3)=-93. 12-(-3)=12+3=154. -3-12=(-3)+(-12)=-(3+12)=-155. -3-(-12)=-3+12=12-3=96. 1-(-1)=1+1=27. (+1)-[+(-1)]=(+1)-[-1]=1+1=28. -5-0=-5(Note: subtracting 0 from a number does not change the number.) 9. 5+(-0)=5(Note: -0=0 and adding 0 to a number does not change the number.) 10. 0-2=-2+0=-211. 0-(-2)=0+2=2
- 12. + (-4) 2 = -4 2 = (-4) + (-2) = -(4 + 2) = -6

Exercises

Compute if possible or write undefined

1.
$$0 \div (-3) = 0$$

(Note: 0 divided by a non-zero number is always 0.)
2. $\frac{16}{-2} = 16 \div (-2) = -(16 \div 2) = -8$
3. $+(-18) \div (-3) = (-18) \div (-3) = 18 \div 3 = 6$
4. $\frac{7}{0} = 7 \div 0 =$ undefined (
(Note: division by 0 is undefined.)
5. $[-(-5)] \div (+1) = [5] \div (+1) = 5 \div 1 = 5$
6. $\frac{250}{-10} = \frac{-250}{10} = -(\frac{250}{10}) = -25$
7. $-(+6) \div [+(-6)] = -6 \div (-6) = 1$
8. $\frac{-5}{+(-5)} = \frac{-5}{-5} = 1$
9. $\frac{0}{-(-8)} = 0$

Exercises

Compute if possible, or write that the expression is undefined. When computing, show one step for each operation. Make sure that you use the '=' sign correctly.

1.
$$-5-3=-5+(-3)=-(5+3)=-8$$

2. $(-5)(-3)=(-5)\times(-3)=5\times 3=15$
3. $-1-1(-1)=-1+1=0$
4. $(4-2\times 2)^{0}=(4-4)^{0}=0^{0}=1$
5. $[5+5(-1)]+[4-5]=[5-5]+[-1]=0+[-1]=0$
6. $(4-5)+[3+3(-1)]=(-1)+[3-3]=(-1)+0$ which is undefined
7. $-1^{2}-10+(-5)\times 2=-1-10+(-5)\times 2=-1+2\times 2=-1+4=3$
8. $7-3\times 2=7-6=1$
9. $(7-3)2=(4)2=8$
10. $-2+4\times(-1)+3=-2+(-4)+3=-6+3=-3$
11. $-3-3(4-5)=-3-3(-1)=-3+3=0$
12. $(-3-3)(4-5)=(-6)(-1)=6$
13. $5-2^{2}+3(-2)=5-4+3(-2)=5-4-6=1-6=-5$
14. $(5-2)^{3}+3(-2)=3^{3}+3(-2)=27+3(-2)=27-6=21$
15. $-[5-2(1+1\times 3)]-[2-(2^{2}-8+2)]=-[5-2(1+3)]-[2-(4-8+2)]=$
 $-[5-2(4)]-[2-(4-4)]=-[5-8]-[2-0]=-[-3]-2=3-2=1$
16. $(-1)(-2)(-5)=2(-5)=-10$
17. $(-10)10^{3}\times 10^{0}=(-10)1000\times 10^{0}=(-10)1000\times 1=-10000\times 1=-10000$
18. $-(-3)[-(-7)]=3[7]=21$
19. $(-8^{2}-3^{4}-2^{2})(-6-4)=(-64-81-32)(-10)=(-177)(-10)=1770$
20. $-12-(-4)+3-(4+3)=-12-(-4)+3-7=-12+4+3-7=-8+3-7=-5-7=-12$
21. $-125+25+[(-6-3)-6-3]=-125+25+[-9-6-3]=-125+25+[-18]=-5-18=-23$
22. $-[(-7-5)(-7+5)(7-5)(7+5)]=-[(-12)(-2)(2)(12)]=-[24(2)(12)]=-[48(12)]=-576$

Math 016 Exercises - Solutions LESSON 12

1. Locate the number on a number line.


2. Fill in the blank using either '=', '<', or '>' as appropriate.

(a)
$$-\frac{24}{5} < 0$$

(b) $0 < \frac{1}{3}$
(c) $\frac{6}{-3} < 2$
(d) $\frac{-6}{3} = -2$
(e) $\frac{-3}{-5} > \frac{5}{-3}$
(f) $\frac{9}{8} > \frac{10}{10}$
(g) $1 < \frac{9}{10}$
(h) $\frac{1}{3} = \frac{5}{15}$
(i) $-1 < -\frac{9}{10}$
(j) $\frac{0}{3} > \frac{-1}{100}$.

Lesson 13 Exercises

1) $\frac{7}{3} \times \frac{2}{5} = \frac{7 \times 2}{3 \times 5} = \frac{14}{15}$	2) $3 \times \frac{1}{5} = \frac{3}{1} \times \frac{1}{5} = \frac{3 \times 1}{1 \times 5} = \frac{3}{5}$
3) $\frac{2}{7} \times 5 = \frac{2}{7} \times \frac{5}{1} = \frac{2 \times 5}{7 \times 1} = \frac{10}{7}$	4) $\frac{11}{12} \times \frac{0}{5} = \frac{11 \times 0}{12 \times 5} = \frac{0}{60} = 0$
5) $\frac{9}{2} \times 1 = \frac{9}{2}$ 6) $1 \times \frac{3}{5} = \frac{3}{5}$	7) $\frac{1}{4} \times \frac{7}{4} = \frac{1 \times 7}{4 \times 4} = \frac{7}{16}$
8) $\frac{1}{4} \times 4 = \frac{1}{4} \times \frac{4}{1} = \frac{1 \times 4}{4 \times 1} = \frac{4}{4} = 1$	9) $\frac{3}{2} \times \frac{4}{3} = \frac{3 \times 4}{2 \times 3} = \frac{12}{6} = 2$
10) $0 \times \frac{2}{3} = 0$	

13.1 Compute. If the resulting fraction is an integer, indicate it.

13.2 Exercises

1. Find 2 fractions equivalent to $\frac{5}{2}$. Show all the steps.
a) $\frac{5}{2} = \frac{5}{2} \times 1 = \frac{5}{2} \times \frac{3}{3} = \frac{15}{6}$ b) $\frac{5}{2} = \frac{5}{2} \times 1 = \frac{5}{2} \times \frac{6}{6} = \frac{30}{12}$
2. Find a fraction equivalent to $\frac{3}{5}$ with denominator 100. Show all the steps.
$\frac{3}{5} = \frac{3}{5} \times 1 = \frac{3}{5} \times \frac{20}{20} = \frac{60}{100}$
3. Find a fraction equivalent to $\frac{7}{10}$ with denominator 50. Show all the steps.
$\frac{7}{10} = \frac{7}{10} \times 1 = \frac{7}{10} \times \frac{5}{5} = \frac{35}{50}$

a)	$\frac{3}{7}, \frac{12}{28}$ Th	ey are equivalent since $\frac{3}{7} \times \frac{4}{4} = \frac{12}{28}$
b)	$\frac{2}{3}, \frac{5}{6}$ The	y are not equivalent since $\frac{2}{3} \times \frac{2}{2} = \frac{4}{6} \neq \frac{5}{6}$
c)	$\frac{0}{5}, \frac{0}{3}$	They are equivalent since they both are equivalent to 0
d)	$\frac{7}{7}, \frac{345}{345}$	They are equivalent since they are both equivalent to 1
e)	$\frac{14}{2}, \frac{21}{3}$	They are equivalent since they are both equivalent to 7
f)	$\frac{1}{16}, \frac{1}{32}$	They are not equivalent. If the numerators are the same, then they
		would be equivalent if the denominators are also the same.
g)	$\frac{1}{4}, \frac{2}{8}$	They are equivalent since $\frac{1}{4} \times \frac{2}{2} = \frac{2}{8}$

Lesson 14 Exercises

$1 \frac{9}{3} - \frac{3 \times 3}{3} - \frac{3}{3} \times \frac{3}{3} - \frac{3}{3} \times 1 - \frac{3}{3}$
$\frac{1}{21} - \frac{1}{7 \times 3} - \frac{1}{7} \times \frac{1}{3} - \frac{1}{7} \times \frac{1}{7} - \frac{1}{7}$
2. $\frac{81}{54} = \frac{9 \times 9}{6 \times 9} = \frac{3 \times 3 \times 9}{2 \times 3 \times 9} = \frac{3}{2} \times \frac{3}{3} \times \frac{9}{9} = \frac{3}{2} \times 1 \times 1 = \frac{3}{2} \times 1 = \frac{3}{2}$
3. $\frac{0}{5} = 0$
4. $\frac{88}{4} = \frac{4 \times 22}{4 \times 1} = \frac{4}{4} \times \frac{22}{1} = 1 \times \frac{22}{1} = \frac{22}{1} = 22$
5. $\frac{1230}{1230} = 1$
$6. \ \frac{9240}{220} = \frac{924 \times 10}{22 \times 10} = \frac{462 \times 2 \times 10}{11 \times 2 \times 10} 42 \times \frac{11}{11} \times \frac{2}{2} \times \frac{10}{10} = 42$
7. $\frac{5 \times 45}{45 \times 3} = \frac{45}{45} \times \frac{5}{3} = 1 \times \frac{5}{3} = \frac{5}{3}$
8. $\frac{7 \times 100}{14 \times 10} = \frac{7 \times 10 \times 2 \times 5}{2 \times 7 \times 10} = \frac{7}{7} \times \frac{10}{10} \times \frac{2}{2} \times 5 = 1 \times 1 \times 1 \times 5 = 1 \times 1 \times 5 = 1 \times 5 = 5$
9. $\frac{28}{5 \times 28} = \frac{28 \times 1}{28 \times 5} = \frac{28}{28} \times \frac{1}{5} = 1 \times \frac{1}{5} = \frac{1}{5}$
0.10.05.0.10.05
10. $\frac{9 \times 10 \times 25}{25 \times 9 \times 10} = \frac{9 \times 10 \times 25}{9 \times 10 \times 25} = \frac{9}{9} \times \frac{10}{10} \times \frac{25}{25} = 1 \times 1 \times 1 = 1 \times 1 = 1$

Simplify the fractions. Show all the steps of the simplification.

Lesson 16 Exercises

Compute and simplify

1. $\frac{10}{3} \times \frac{9}{2} = \frac{2 \times 5}{3 \times 1} \times \frac{3 \times 3}{2 \times 1} = \frac{5}{1} \times \frac{3}{1} = \frac{15}{1} = 15$
$16 9 70 \cancel{4} \times 4 \cancel{3} \times 3 \cancel{7} \times \cancel{5} \times 2 4 \times 3 \times 2 24$
$2. \frac{2}{5} \times \frac{7}{7} \times \frac{12}{12} = \frac{7}{\cancel{3} \times 1} \times \frac{7}{\cancel{3} \times 1} \times \frac{7}{\cancel{3} \times \cancel{3}} = \frac{1}{1 \times 1} = \frac{1}{1} = 24$
3. $\frac{100}{9} \left(-\frac{6}{20} \right) = -\frac{1\cancel{3} \times 10}{\cancel{3} \times 3} \left(\frac{\cancel{3} \times \cancel{2}}{\cancel{2} \times 1\cancel{3}} \right) = -\frac{10}{3}$
$2\left(-21\right)\left(-15\right) = 2\left(-21\right)\left(-15\right) = 2\left(-21\right)\left($
$4. -\frac{2}{7} \left(-\frac{21}{10} \right) \left(-\frac{15}{4} \right) = -\frac{2 \times 1}{7 \times 1} \left(\frac{7 \times 5}{2 \times 5} \right) \left(\frac{2 \times 5}{2 \times 2} \right) = -\frac{1 \times 5 \times 5}{1 \times 2 \times 2} = -\frac{9}{4}$
2(-0)(-25)(7+2+7+2-7+2+1)(-7+4)(-2+2+2+2-7)
$5. -\frac{36}{25} \times \frac{90}{7} \times \left(-\frac{35}{120}\right) = \frac{\cancel{2} \times \cancel{3} \times \cancel{2} \times \cancel{3}}{\cancel{3} \times \cancel{5}} \times \frac{\cancel{3} \times \cancel{3} \times \cancel{10}}{\cancel{7} \times \cancel{1}} \times \frac{\cancel{7} \times \cancel{2}}{\cancel{3} \times \cancel{2} \times \cancel{2} \times \cancel{10}} = \frac{\cancel{3} \times \cancel{3} \times \cancel{3}}{\cancel{5} \times \cancel{1}} = \frac{\cancel{27}}{\cancel{5}}$
6. $\frac{5}{8}(-16) = -\frac{5}{8}\left(\frac{16}{1}\right) = -\frac{5}{8}\left(\frac{2\times8}{1}\right) = -\frac{5\times2}{1} = -10$
7. $9 \times \left(-\frac{4}{15}\right) \times \frac{1}{2} = -\frac{\cancel{3} \times \cancel{3}}{1} \times \frac{\cancel{2} \times \cancel{2}}{\cancel{3} \times \cancel{5}} \times \frac{1}{\cancel{2}} = -\frac{\cancel{3} \times \cancel{2} \times \cancel{1}}{1 \times \cancel{5}} = -\frac{6}{5}$
8. $-\left(\frac{1}{3}\right)^3 = -\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = -\frac{1 \times 1 \times 1}{3 \times 3 \times 3} = -\frac{1}{27}$
9. $\left(\frac{2}{5}\right)^2 = \left(\frac{2}{5}\right)\left(\frac{2}{5}\right) = \frac{2 \times 2}{5 \times 5} = \frac{4}{25}$
$10\left(-\frac{1}{2}\right)^4 = -\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{1 \times 1 \times 1 \times 1}{2 \times 2 \times 2 \times 2} = -\frac{1}{16}$

 $11. \left(\frac{3}{5}\right)^2 = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

$$12. - \left(\frac{1}{5}\right)^2 = -\left(\frac{1}{5}\right) \times \left(\frac{1}{5}\right) = -\frac{1}{25}$$

$$13. \left(-\frac{1}{3}\right)^2 = \left(-\frac{1}{3}\right) \times \left(-\frac{1}{3}\right) = \frac{1}{9}$$

$$14. - \left(-\frac{1}{9}\right)^2 = -\left(-\frac{1}{9}\right) \left(-\frac{1}{9}\right) = -\frac{1}{81}$$

$$15. \left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$$

$$16. \left(-\frac{3}{4}\right)^3 = \left(-\frac{3}{4}\right) \times \left(-\frac{3}{4}\right) \times \left(-\frac{3}{4}\right) = -\frac{27}{64}$$

$$17. - \left(-\frac{2}{3}\right)^3 = -\left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) = \frac{8}{27}$$

$$18. \left(-\frac{1}{2}\right)^4 = \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) = \frac{1}{16}$$

Lesson 17

Divide and simply. When simplifying, show all the factors that are canceled.

1. $\frac{1}{5} \div \frac{5}{4} = \frac{1}{5} \times \frac{4}{5} = \frac{1 \times 4}{5 \times 5} = \frac{4}{25}$
Q
2. $\frac{-\frac{8}{25}}{\frac{2}{5}} = -\frac{8}{25} \div \frac{2}{5} = -\frac{8}{25} \times \frac{5}{2} = -\frac{\cancel{2} \times 4}{\cancel{3} \times 5} \times \frac{\cancel{3}}{\cancel{2}} = -\frac{4}{5}$
16(3) 16 2 32
3. $-\frac{10}{9} \div \left(-\frac{5}{2}\right) = \frac{10}{9} \times \frac{2}{3} = \frac{52}{27}$ because both fractions were negative the result is +
40
4. $\frac{-\frac{49}{64}}{-\frac{8}{7}} = \frac{49}{64} \div \frac{8}{7} = \frac{49}{64} \times \frac{7}{8} = \frac{49 \times 7}{64 \times 8} = \frac{343}{512} \qquad \frac{-}{-} = +$ recall that:
5. $-\frac{36}{25} \div 12 = -\frac{36}{25} \div \frac{12}{1} = -\frac{36}{25} \times \frac{1}{12} = -\frac{3 \times 12}{25} \times \frac{1}{12} = -\frac{3 \times 1}{25} = -\frac{3}{25}$
6. $\frac{3}{-\frac{27}{2}} = 3 \div \left(-\frac{27}{2}\right) = \frac{3}{1} \div \left(-\frac{27}{2}\right) = -\frac{3}{1} \times \frac{2}{27} = -\frac{3}{1} \times \frac{2}{3} = -\frac{2}{1 \times 9} = -\frac{2}{9}$
7. $-\frac{1}{3} \div (-3) = \frac{1}{3} \div \frac{3}{1} = \frac{1}{3} \times \frac{1}{3} = \frac{1 \times 1}{3 \times 3} = \frac{1}{9}$ recall that $(-) \div (-) = +$
8. $\frac{\frac{4}{9}}{8} = \frac{4}{9} \div 8 = \frac{4}{9} \div \frac{8}{1} = \frac{4}{9} \times \frac{1}{8} = \frac{\cancel{4}}{9} \times \frac{1}{2 \times \cancel{4}} = \frac{1}{9 \times 2} = \frac{1}{18}$

Lesson 18 Exercises

1. For each pair of fractions find the equivalent fractions having the same denominator.
(a) $\frac{2}{3}; \frac{5}{9}$ 9 is the largest denominator. It can be a common denominator since $\frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$ so $\frac{6}{9}; \frac{5}{9}$ have the same denominator.
(b) $\frac{7}{12}$; $\frac{3}{5}$ 12 is the largest denominator. It can not be a common denominator. Since
$12 \times 5 = 60$, we can consider 60 as a common denominator.
$\frac{7}{12} \times \frac{5}{5} = \frac{35}{60} and \frac{3}{5} \times \frac{12}{12} = \frac{36}{60} \text{; therefore } \frac{35}{60} \text{; and } \frac{35}{60} \text{have the same denominator.}$
c) $\frac{7}{24}$; $\frac{11}{30}$ 30 is the largest denominator. It can not be a common denominator. If we notice that
$30 \times 4 = 120$ and $24 \times 5 = 120$, then we can use 120 as a common denominator.
$\frac{7}{24} \times \frac{5}{5} = \frac{35}{120} and \frac{11}{30} \times \frac{4}{4} = \frac{44}{120} \text{ making } \frac{35}{120}; \frac{44}{120} \text{ the new pair of fractions with the same denominator.}$
(d) $\frac{33}{20}; \frac{65}{56}$ 56 is the largest denominator. It can not be used as a common denominator.
We can use the result of 20×56 as a common denominator (there are other possibilities!).
$\frac{33}{20} \times \frac{56}{56} = \frac{1848}{1120} \text{ and } \frac{65}{56} \times \frac{20}{20} = \frac{1300}{1120} \text{ making } \frac{1848}{1120}; \frac{1300}{1120} \text{ the new pair of fractions}$
with the same denominator

Lesson 18 Exercises

2. Fill in the blank using either '<' or '>' as appropriate. Explain your answer

(a) $\frac{235}{233} > \frac{498}{501}$ we see that $\frac{235}{233} > 1$ and $1 > \frac{498}{501}$ therefore $\frac{235}{233} > \frac{498}{501}$	
(b) $-\frac{143}{4} < 0$ all negative numbers are less than 0.	
(c) $-1 < -\frac{7}{8}$ $\frac{7}{8} < 1$, therefore $-\frac{7}{8} > -1$ another way to reason	
it out is that $-\frac{7}{8}$ is between 0 and -1, so -1 must be smaller.	
(d) $-1230 < \frac{1}{10}$ all negative numbers are less than any positive number.	
(e) $\frac{7}{12} < \frac{3}{5}$ here we need to compare them once their denominators are the same.	
A possible common denominator is the result of 12×5 , that is 60.	
$\frac{7}{12} \times \frac{5}{5} = \frac{35}{60}$ and $\frac{3}{5} \times \frac{12}{12} = \frac{36}{60}$ so based on the new fractions we can determine that the first fraction is less than the second fraction.	
(f) $\frac{9}{24} > \frac{11}{30}$ here again we need to compare them once their denominators are the	
same. One possibility for a common denominator is 120. $\frac{9}{24} \times \frac{5}{5} = \frac{45}{120}$ and $\frac{11}{30} \times \frac{4}{4} = \frac{44}{120}$ so based on	
the new fractions we can tell that the first fraction is greater than the second fraction.	

Lesson 20 Exercises

Compute and simplify. Show all the steps of the computation. When simplifying show the factors that are canceled.

1. $\frac{5}{9} + \frac{4}{9} = \frac{5+4}{9} = \frac{9}{9} = 1$
2. $\frac{5}{9} - \frac{2}{9} = \frac{5-2}{9} = \frac{3}{9} = \frac{3 \times 1}{3 \times 3} = \frac{1}{3}$
3. $\frac{2}{3} + 4 = \frac{2}{3} + \frac{4}{1} = \frac{2}{3} + \frac{4 \times 3}{1 \times 3} = \frac{2}{3} + \frac{12}{3} = \frac{2 + 12}{3} = \frac{14}{3}$
4. $\frac{12}{5} - 1 = \frac{12}{5} - \frac{1}{1} = \frac{12}{5} - \frac{1 \times 5}{1 \times 5} = \frac{12}{5} - \frac{5}{5} = \frac{12 - 5}{5} = \frac{7}{5}$
5. $\frac{9}{5} + \frac{3}{10} = \frac{9 \times 2}{5 \times 2} + \frac{3}{10} = \frac{18}{10} + \frac{3}{10} = \frac{18 + 3}{10} = \frac{21}{10}$
6. $\frac{7}{9} - \frac{11}{6} = \frac{7 \times 2}{9 \times 2} - \frac{11 \times 3}{6 \times 3} = \frac{14}{18} - \frac{33}{18} = \frac{14 - 33}{18} = \frac{-19}{18} = -\frac{19}{18}$
7. $\frac{1}{2} + \frac{5}{3} + \frac{7}{6} = \frac{1 \times 3}{2 \times 3} + \frac{5 \times 2}{3 \times 2} + \frac{7}{6} = \frac{3}{6} + \frac{10}{6} + \frac{7}{6} = \frac{3 + 10 + 7}{6} = \frac{20}{6} = \frac{2 \times 10}{2 \times 3} = \frac{10}{3}$
$8. \frac{2}{5} + 3 + \frac{5}{6} = \frac{2}{5} + \frac{3}{1} + \frac{5}{6} = \frac{2 \times 6}{5 \times 6} + \frac{3 \times 30}{1 \times 30} + \frac{5 \times 5}{6 \times 5} = \frac{12}{30} + \frac{90}{30} + \frac{25}{30} = \frac{12 + 90 + 25}{30} = \frac{127}{30}$
9. $-\frac{5}{9} + \frac{4}{9} = \frac{-5+4}{9} = \frac{-1}{9} = -\frac{1}{9}$ 10. $-\frac{5}{9} - \frac{2}{9} = \frac{-5-2}{9} = \frac{-7}{9} = -\frac{7}{9}$
11. $\frac{2}{3} - 4 = \frac{2}{3} - \frac{4}{1} = \frac{2}{3} - \frac{4 \times 3}{1 \times 3} = \frac{2}{3} - \frac{12}{3} = \frac{2 - 12}{3} = \frac{-10}{3} = -\frac{10}{3}$
$12. \ \frac{12}{5} - 3 = \frac{12}{5} - \frac{3}{1} = \frac{12}{5} - \frac{3 \times 5}{1 \times 5} = \frac{12}{5} - \frac{15}{5} = \frac{12 - 15}{5} = \frac{-3}{5} = -\frac{3}{5}$
$13. \frac{9}{5} + \left(-\frac{3}{10}\right) = \frac{9 \times 2}{5 \times 2} + \left(-\frac{3}{10}\right) = \frac{18}{10} + \left(-\frac{3}{10}\right) = \frac{18}{10} + \left(\frac{-3}{10}\right) = \frac{18 + \left(-3\right)}{10} = \frac{15}{10} = \frac{5 \times 3}{5 \times 2} = \frac{3}{2}$
$14. \frac{7}{9} - \frac{5}{6} = \frac{7 \times 2}{9 \times 2} - \frac{5 \times 3}{6 \times 3} = \frac{14}{18} - \frac{15}{18} = \frac{14 - 15}{18} = \frac{-1}{18} = -\frac{1}{18}$

Lesson 21 - Exercises

Compute and simplify. Show all steps of the computation and when simplifying show the factors.

$1. \frac{1}{2} - \frac{5}{3} - \left(-\frac{7}{6}\right) = \frac{1}{2} - \frac{5}{3} + \frac{7}{6} = \frac{1 \times 3}{2 \times 3} - \frac{5 \times 2}{3 \times 2} + \frac{7}{6} = \frac{3}{6} - \frac{10}{6} + \frac{10}{6$
$2. -2 + \frac{5}{7} - \frac{1}{3} = \frac{-2}{1} + \frac{5}{7} - \frac{1}{3} = \frac{-2 \times 21}{1 \times 21} + \frac{5 \times 3}{7 \times 3} - \frac{1 \times 7}{3 \times 7} = \frac{-42}{21} + \frac{15}{21} - \frac{7}{21} = \frac{-\frac{-27}{-42 + 15} - 7}{21} = \frac{-\frac{-34}{21}}{21} = -\frac{\frac{-34}{21}}{21} = -\frac{34}{21}$
$3. -\frac{2}{9} - \frac{1}{3} - \frac{3}{2} = -\frac{2 \times 2}{9 \times 2} - \frac{1 \times 6}{3 \times 6} - \frac{3 \times 9}{2 \times 9} = -\frac{4}{18} - \frac{6}{18} - \frac{27}{18} = \frac{-4 - 6 - 27}{18} = \frac{-37}{18} = -\frac{37}{18}$
4. $1 + \frac{1}{2} - \frac{3}{2} - \frac{5}{2} = \frac{1}{1} + \frac{1}{2} - \frac{3}{2} - \frac{5}{2} = \frac{1 \times 2}{1 \times 2} + \frac{1}{2} - \frac{3}{2} - \frac{5}{2} = \frac{2}{2} + \frac{1}{2} - \frac{3}{2} - \frac{5}{2} = \frac{2}{2 + 1 - 3 - 5} = \frac{2}{2 + 1 - 3 - 5} = \frac{-5}{2} = -\frac{5}{2}$
5. $\frac{2}{3} - \frac{3}{5} + \frac{1}{2} = \frac{2 \times 10}{3 \times 10} - \frac{3 \times 6}{5 \times 6} + \frac{1 \times 15}{2 \times 15} = \frac{20}{30} - \frac{18}{30} + \frac{15}{30} = \frac{20 - 18 + 15}{30} = \frac{17}{30}$
$\frac{1}{3} + 2 - \frac{1}{2} - \frac{1}{4} = \frac{1}{3} + \frac{2}{1} - \frac{1}{2} - \frac{1}{4} = \frac{1 \times 4}{3 \times 4} + \frac{2 \times 12}{1 \times 12} - \frac{1 \times 6}{2 \times 6} - \frac{1 \times 3}{4 \times 3} = \frac{4}{12} + \frac{24}{12} - \frac{6}{12} - \frac{3}{12} = 6.$ $= \frac{\frac{22}{4 + 24} - 6 - 3}{12} = \frac{19}{12}$
7. $\frac{1}{3} - \frac{1}{3} \times 3 = \frac{1}{3} - \frac{1}{3} \times \frac{1}{1} = \frac{1}{3} - \frac{3}{3} = \frac{1}{3} = \frac{1}{3} = -\frac{2}{3} = -\frac{2}{3}$
8. $\left(\frac{1}{3} - \frac{1}{3}\right) \times 3 = \left(\frac{1-1}{3}\right) \times 3 = \left(\frac{0}{3}\right) \times 3 = 0 \times 3 = 0$

Exercises 21 continued

$$\begin{array}{c} \frac{1}{3}\left(1+\frac{2}{3}\right)+\frac{1}{3}=\frac{1}{3}\left(\frac{1}{1}+\frac{2}{3}\right)+\frac{1}{3}=\frac{1}{3}\left(\frac{1\times3}{1\times3}+\frac{2}{3}\right)+\frac{1}{3}=\frac{1}{3}\left(\frac{3}{3}+\frac{2}{3}\right)+\frac{1}{3}=\frac{1}{3}\left(\frac{3+2}{$$

Lesson 24

Exercises:
1. Write the mixed number as a fraction
a) $3\frac{4}{7} = 3 + \frac{4}{7} = \frac{21}{7} + \frac{4}{7} = \frac{25}{7}$
b) $1\frac{5}{7} = 1 + \frac{5}{7} = \frac{7}{7} + \frac{5}{7} = \frac{12}{7}$
c) $2\frac{3}{5} = 2 + \frac{3}{5} = \frac{10}{5} + \frac{3}{5} = \frac{13}{5}$
d) $5\frac{1}{6} = 5 + \frac{1}{6} = \frac{30}{6} + \frac{1}{6} = \frac{31}{6}$
2. Write the fraction as a mixed number if possible. Then locate it on a number line.
a) $\frac{19}{3} = \frac{18+1}{3} = \frac{18}{3} + \frac{1}{3} = 6 + \frac{1}{3} = 6\frac{1}{3}$. Locate between 6 and 7. Divide the interval
between 6 and 7 into 3 equal parts and mark the 1 st spot
11
b) $\frac{11}{12}$ is not a mixed number since it is between 0 and 1. Notice that the numerator is
smaller than the denominator. Divide the interval between 0 and 1 into 12 equal parts
and mark the 11 th spot.
c) $\frac{14}{5} = \frac{10+4}{5} = \frac{10}{5} + \frac{4}{5} = 2 + \frac{4}{5} = 2\frac{4}{5}$ Locate between 2 and 3. Divide the interval
between 2 and 3 into 5 equal parts and mark the 4 th spot.
2 0
d) $\frac{28}{4} = 7$ Though not a mixed number, plot and label position 7.
e) $\frac{40}{7} = \frac{35+5}{7} = \frac{35}{7} + \frac{5}{7} = 5 + \frac{5}{7} = 5\frac{5}{7}$. Locate between 5 and 6. Divide the interval
between 5 and 6 into 7 equal parts and mark the 5 spot.

Lesson 25 Exercises: Compute. Write your answer both as a fraction (or integer) and a mixed number (if it applies) $3\frac{2}{5} + 7\frac{1}{2} = 3 + \frac{2}{5} + 7 + \frac{1}{2} = (3+7) + (\frac{2}{5} + \frac{1}{2}) = 10 + \frac{6+5}{15} = 10 + \frac{11}{15} = 10\frac{11}{15}$ 1. the answer as a fraction is $:10\frac{11}{15} = 10 + \frac{11}{15} = \frac{150}{15} + \frac{11}{15} = \frac{150 + 11}{15} = \frac{161}{15}$ $1\frac{4}{5}+2\frac{1}{2}=1+\frac{4}{5}+2+\frac{1}{2}=(1+2)+(\frac{4}{5}+\frac{1}{2})=3+\frac{8+5}{10}=3+\frac{13}{10}=3+\frac{10+3}{1$ ^{2.} $3 + \frac{10}{10} + \frac{3}{10} = 3 + 1 + \frac{3}{10} = 4 + \frac{3}{10} = 4\frac{3}{10}$ the answer as a fraction is $:4\frac{3}{10} = 4 + \frac{3}{10} = \frac{40+3}{10} = \frac{43}{10}$ $5\frac{1}{3} - 2\frac{8}{9} = \left(5 + \frac{1}{3}\right) - \left(2 + \frac{8}{9}\right) = \left(\frac{15+1}{3}\right) - \left(\frac{18+8}{9}\right) = \frac{16}{3} - \frac{26}{9} = \frac{48}{9} - \frac{26}{9} = \frac{22}{9}$ 3. now as a mixed number : $\frac{22}{9} = \frac{18+4}{9} = \frac{18}{9} + \frac{4}{9} = 2 + \frac{4}{9} = 2\frac{4}{9}$ $4\frac{2}{7}-3=\left(4+\frac{2}{7}\right)-3=\left(\frac{28+2}{7}\right)-\frac{21}{7}=\frac{30}{7}-\frac{21}{7}=\frac{9}{7}$ 4. now as a mixed number : $\frac{9}{7} = \frac{7+2}{7} = \frac{7}{7} + \frac{2}{7} = 1 + \frac{2}{7} = 1\frac{2}{7}$ 5. $10 \div 2\frac{1}{2} = 10 \div \left(2 + \frac{1}{2}\right) = 10 \div \frac{5}{2} = 10 \times \frac{2}{5} = \frac{10}{1} \times \frac{2}{5} = \frac{2 \times 5}{1} \times \frac{2}{5} = \frac{4}{1} = 4$

Lesson 25 continued Exercises: $4\frac{4}{7}+3=4+\frac{4}{7}+3=4+3+\frac{4}{7}=7+\frac{4}{7}=7\frac{4}{7}$ 6. as a fraction : $7\frac{4}{7} = 7 + \frac{4}{7} = \frac{49}{7} + \frac{4}{7} = \frac{49+4}{7} = \frac{53}{7}$ $3\frac{2}{3} + \frac{3}{7} = 3 + \frac{2}{3} + \frac{3}{7} = 3 + \frac{14}{21} + \frac{9}{21} = 3 + \frac{23}{21} = 3 + \frac{21+2}{21} = 3 + \frac{21}{21} + \frac{2}{21} = 3 + \frac{2}{21} + \frac{2}{21} + \frac{2}{21} = 3 + \frac{2}{21} + \frac{2}{21} = 3 + \frac{2}{21} + \frac{2}{21} = 3 + \frac{2}{21} + \frac{2}{21} + \frac{2}{21} = 3 + \frac{2}{21} + \frac{2}{21} + \frac{2}{21} = 3 + \frac{2}{21} + \frac{2}{21} + \frac{2}{21} = 3 + \frac{2}{21} + \frac{2}{21} + \frac{2}{21} = 3 + \frac{2}{21} +$ 7. $3+1+\frac{2}{21}=4+\frac{2}{21}=4\frac{2}{21} \quad as \ a \ fraction: 4\frac{2}{21}=4+\frac{2}{21}=\frac{84}{21}+\frac{2}{21}=\frac{84+2}{21}=\frac{86}{21}$ $\overline{8. \quad 2\frac{1}{4} \times 3\frac{5}{9}} = \left(2 + \frac{1}{4}\right) \times \left(3 + \frac{5}{9}\right) = \frac{9}{4} \times \frac{32}{9} = \frac{9 \times 4 \times 8}{4 \times 9} = \frac{8}{1} = 8$ 9. $5\frac{1}{5} \div 1\frac{3}{10} = \left(5 + \frac{1}{5}\right) \div \left(1 + \frac{3}{10}\right) = \frac{26}{5} \div \frac{13}{10} = \frac{26}{5} \times \frac{10}{13} = \frac{13 \times 2 \times 5 \times 2}{5 \times 13} = \frac{4}{1} = 4$ $10. \quad 10 \times 7\frac{3}{5} = 10 \times \left(7 + \frac{3}{5}\right) = 10 \times \frac{38}{5} = \frac{10}{1} \times \frac{38}{5} = \frac{2 \times 5 \times 38}{1 \times 5} = \frac{2 \times 38}{1} = \frac{76}{1} = 76$

Lesson 26	
Exercises:	
	1. Find the decimal notation for each of the following fractions
	2391 22.01 since 100 has two zeros, the result needs 2 divits to the right of
	a) $\frac{100}{100}$ = 25.91 since 100 has two zeros, the result needs 2 digits to the right of
	the decimal point.
	91
	b) $\frac{1000}{1000}$ = 0.091 since 1000 has 3 zeros, the result needs 3 digits to the right of the
	decimal point Here we must add a zero to the left of 91 to accomplish the goal and
	write the correct result.
	9 9×2 18
	c) $\frac{1}{5} = \frac{1}{5 \times 2} = \frac{1}{10} = 1.8$ after changing the original denominator to a power of 10,
	$5 - 5 \times 2 - 10$ we see that 10 has one zero so the result needs 1 digit to the right of the decimal
	noint
	point.
	13 13 × 25 325
	d) $\frac{15}{4} = \frac{15 \times 25}{4 \times 25} = \frac{525}{100} = 3.25$ after changing the original denominator to a power
	$4 + 4 \times 25 = 100$
	decimal point
	7 7 × 5 35
	e) $\frac{7}{2} = \frac{7 \times 5}{2 \times 5} = \frac{35}{10} = 3.5$ after changing the original denominator to a power of 10,
	$2 2 \times 3 10$
	we see that 10 has one zero so the result needs 1 digit to the right of the decimal
	point.
	7
	f) $\frac{7}{100} = 0.07$ since 100 has 2 zeros, the result needs 2 digits to the right of the
	decimal point. Here we must add a zero to the left of / to accomplish the goal and
	while the correct result.
	$2 2 \times 4 = 8$
	g) $\frac{2}{25} = \frac{2 \times 4}{25 \times 4} = \frac{6}{100} = 0.08$ after changing the original denominator to a power of
	$25 25 \times 4 100$
	10, we see that 100 has two zeros so the result needs 2 digits to the right of the
	decimal point. Notice that we must add a zero to the left of 8 to accomplish the goal
	and write the correct result.
	501 501 2 1002
	h) $\frac{301}{2} = \frac{301 \times 2}{2} = \frac{1002}{2} = 1.002$ after changing the original denominator to a
	$500 500 \times 2 1000$
	power of 10, we see that 1000 has three zeros so the result needs 3 digits to the right
	of the decimal point.

Lesson 26 continued:
2. Fill in the blank with either '=', '<', or '>' as appropriate.
a) $\frac{501}{500} < 2$ since $\frac{501}{500} = \frac{501 \times 2}{500 \times 2} = \frac{1002}{1000} = 1.002$ and 1.002 is between 1 and 2
b) $5.0000 = 5$ since the 4 zeros to the right of the decimal point in 5.0000 do not change the fact that it is equivalent to 5.
c) $5.000001 > 5$ since 5.000001 is between 5 and 6
d) 9 > 8.9999999999 since 8.999999999 is between 8 and 9

Lesson 27 Exe	preises:
1. Find d	ecimal notation for each of the following numbers
a)	$\frac{139}{8} = 17.375$ Divide 8 into 139 and carefully set the decimal point and add
nee	cessary zeros to complete the division.
b) div	$-\frac{32}{25} = -\frac{32 \times 4}{25 \times 4} = -\frac{128}{100} = -1.28$ this could have also been found using long vision
c)	$\frac{31}{12} = 2.58\overline{3}$ Be sure to carry out the long division to 'see' the repeating 3's
d)	$2\frac{103}{330} = 2.3\overline{12}$ Be sure to carry out the long division far enough to 'see' the beating 12.
e)	$-\frac{5}{16} = -0.3125$
f) ap	$\frac{100}{7} = 14.\overline{285714}$ don't get discouraged here. Keep the division going by adding propriate zeros until you begin to repeat. It is a fairly long problem.
2. Write	each of the following numbers as a fraction.
a) 10	$25.6655 = \frac{256655}{10000}$ notice the 4 numbers after the decimal point and the 4 zeros in 000. Recall that this connection is very important.
b)	$0.0043 = \frac{43}{10000}$ notice the 4 numbers after the decimal point and the 4 zeros in 000. Recall that this connection is very important.
c) Re	$-12.01 = -\frac{1201}{100}$ notice the 2 numbers after the decimal point and the 2 zeros in 100.
d)	$(1.3)^2 = 1.3 \times 1.3 = \frac{13}{10} \times \frac{13}{10} = \frac{13 \times 13}{10 \times 10} = \frac{169}{100}$ notice we did this as we have learned
be	tore. After rewriting the decimals as fractions, perform fraction multiplication.

Lesson 28

Exercises 28.1:
Fill in the blank using either '=', '<', or '>' as appropriate. Explain your answer
1. $\frac{2}{3} = 0.\overline{6}$ changing $\frac{2}{3}$ to a decimal will result in the repeating decimal
2. $2.999 < 3.00$ our decision can be made from the ones digit since 2<3.
3. $9.330 < 9.6$ the comparison can be made from the tenths digit since $3 < 6$
4. $-34.2230 > -34.24$ when comparing the positive decimals, we base our comparison on the hundredths digit and see that 2<4 so with the negative digits we reverse that and make our decision.
5. $0.0001 > -234$ all positive values are greater than any negative value
2250 2250
6. $3.25 = \frac{3250}{1000}$ when changing $\frac{3250}{1000}$ to a decimal we get 3.250 which is equivalent
to 5.25 since the chang o does not after the value.
Exercises 28.2:
Compute
1. $17.3 + 9.77 = 27.07$ be sure to line up the decimal points
2. $12.12 + 7 + 9.9 = 29.02$ be sure to line up the decimal points. Remember that $7 = 7.0$.
3. $221.321 + 45.9 + 99 + 5.35 = 371.571$ be sure to line up the decimal points. Remember that $99 = 99.0$
4. $35.1 - 12.532 = 22.568$ be sure to line up the decimal points and add appropriate zeros in 35.1 (35.100) to accomplish the subtraction correctly.
5. $9 - 7.351 = 1.649$ be sure to line up the decimal points and add appropriate zeros in 9
(9.000) to accomplish the subtraction correctly.
6. $14.12 - 8 = 6.12$ be sure to line up the decimal points and add appropriate zeros in 8 (8.00) to accomplish the subtraction correctly.

Lesson 29
Exercises:
Compute
12.2 - 3.31 = -(2.2 + 3.31) = -5.51
2. $-1.15 + 5.321 = +(5.321 - 1.15) = 4.171$
3. $0.16 - 1.76 = -(1.76 - 0.16) = -1.60 = -1.60$
$4. \ 3.41 - (-1) = 3.41 + 1 = 4.41$
$5 \cdot 1 \cdot 93 - 3 = -(3 - 1 \cdot 93) = -1 \cdot 07$
5. 1.75 5 -(5 1.75) -1.07
65 - (-6.713) = -5 + 6.713 = +(6.713 - 5) = 1.713
7. $8.2 - 9.17 - 3 + 1.1 = -0.97 - 3 + 1.1 = -3.97 + 1.1 = -2.87$ the stages show how we
associate from left to right (one operation at a time) since the problem is all addition an
subtraction.
8 (11) (11) 11 - 11 + 11 11 - 22 11 - 11 once we change the problem
to all addition and subtraction remembering to change all negative followed by negative f
to a positive, we then associate from left to right (one operation at a time) since the
problem is all addition and subtraction.
9. $0.351 - 1.23 + 2 = -0.879 + 2 = 1.121$ the stages show how we associate from left to
right (one operation at a time) since the problem is all addition and subtraction.
$\frac{105.37-(-5.37)-4.65+7-3.3339=-5.37+5.37-4.65+7-3.3339}{-04.65+7-3.22200-2.25-2.22200-0.09200}$
= 0 - 4.65 + 7 - 3.3339 = -4.65 + 7 - 3.3339 = 2.35 - 3.3339 = -0.9839 once we always the problem to all addition and subtraction remembering to always all possible.
followed by negative to a positive, we then associate from left to right (one operation at
a time) since the problem is all addition and subtraction.
11. 8 - 8.001 + 1 - (-0.3) + 1 = 8 - 8.001 + 1 + 0.3 + 1 = -0.001 + 1 + 0.3 + 1
= 0.999 + 0.3 + 1 = 1.299 + 1 = 2.299 once we change the problem to all addition and
subtraction remembering to change all negative followed by negative to a positive, we
then associate from left to right (one operation at a time) since the problem is all
addition and subtraction.

Lesson 30	
Exercises:	
Comp	nte
	1. $3.52 \times (-4.111) = -(3.52 \times 4.111) = -14.47072$ note carefully the number of digits
	after the decimal point (5) represents the total of the number of digits past the decimal point of the two numbers in the product $(2 + 3 = 5)$
	2. $(-2.8) \times (-9.7) = 2.8 \times 9.7 = 27.16$ note carefully the number of digits after the
	decimal point (2) represents the total of the number of digits past the decimal point of the two numbers in the product $(1 + 1 = 2)$
	3. $53.41 \times 1000 = 53410^{\circ}$ here we 'moved' the decimal point 3 spaces to the right of the original location within 53.41 and had to add a zero (0) after the one (1) to accomplish the goal.
	4. $9 \times 0.01 = 0.09$ here we 'moved' the decimal point 2 spaces to the left of the original location (recall that $9 = 9.0$). we had to add a zero (0) to the left of 9 to accomplish the goal.
	5. $3.25 \times 5.5 \times (-2.1) = -(3.25 \times 5.5 \times 2.1) = -(17.875 \times 2.1) = -37.5375$ we first see
	that the result is negative and then the answer will have a total of 4 digits past the decimal point since the sum of the digits past the decimal points of the three numbers in the product is $(2 + 1 + 1 = 4)$
	6. $(-0.1) \times (-0.1) \times (-0.01) = -(0.1 \times 0.1 \times 0.01) = -(0.01 \times 0.01) = -0.0001$ we first
	see that the result is negative and then the answer will have a total of 4 digits past the decimal point since the sum of the digits past the decimal points of the three numbers in the product is $(1 + 1 + 2 = 4)$
	7. $(-6.63) \times 0.01 \times (-2.2) \times 1000 = 6.63 \times 0.01 \times 2.2 \times 1000$ = $0.0663 \times 2200 = 145.8600 = 145.86$ is positive. We can use associative property (since it is all multiplication) with the first two numbers and the last two numbers of the problem to simplify our work since we see
	multiplication using powers of 10, both a negative power $(10^{-2} = 0.01)$ and a positive
	power $(10^3 = 1000)$. Our result gets simplified since the two zeros after the 6 do not
	add any additional value and hence can be eliminated.

Lesson 30 Exercises continued

8. $(-5.5)^3 = (-5.5) \times \overline{(-5.5)} \times (-5.5) = -(5.5 \times 5.5 \times 5.5) = -(30.25 \times 5.5) = -166.375$
 once we determine that the result will be negative, we multiply using the associative property from left to right noting that three (3) digits will be after the decimal point in the product.
9. $-(-2.11)^3 = -[(-2.11) \times (-2.11) \times (-2.11)] = -[-(2.11 \times 2.11 \times 2.11)]$
$= 2.11 \times 2.11 \times 2.11 = 4.4521 \times 2.11 = 9.393931$
once we determine that the result will be positive, we multiply using the associative property from left to right noting that six (6) digits will be after the decimal point in the product.
*
10. $-(-33.1)^2 = -[(-33.1) \times (-33.1)] = -[33.1 \times 33.1] = -1095.61$
once we determine that the result will be negative, we then note that two digits will be after the decimal point in the product.
11. $(0.1)^5 = 0.1 \times 0.1 \times 0.1 \times 0.1 \times 0.1 = 0.00001$
here we can see that we will simply be multiplying by 1 with itself, noting that the result will have 5 digits after the decimal point in the product ending in 1, so the rest must be zeros.

Lesson 31
Exercises:
Compute:
1. $40.1951 \div (-11) = -(40.1951 \div 11) = -3.6541$ be sure to notice that the result is
negative
$2 -5.932 \div (-12) = 5.932 \div 12 = 0.494\overline{3}$ once we determine the result is positive
continue the division until you see the repeating 3's and then record the result correctly
continue the division until you see the repeating 5 s and then record the result concerny.
$3 - 893 16 \div 200 = -(893 16 \div 200) = -44658$ be sure to notice that the result is
5.095.10.200 = (095.10.200) = 1.1000 be such to indice that the result is
negative and then add the necessary zeros to conclude the division.
$A = (4.78000) \cdot 1000 = (4.78000 \cdot 1000) = 0.00478000$
$4. \ (-4./8999) \div 1000 = -(4./8999 \div 1000) = -0.004/8999$
once we determine that the result is negative, the division can be done by moving or
shifting the decimal point three (3) spaces to the left, adding the appropriate zeros. This
is because we are dividing by a power of 10, in this case $10^3 = 1000$.
5. $0.1 \div 200 = 0.0005$ be sure to set the decimal point and add the necessary zeros to
complete the work using the long division procedure.
6. $-69.797 \div 13 = -(69.797 \div 13) = -5.369$
7. $0.35888 \div (-9) = -(0.35888 \div 9) = -0.039875$ once we determine the result is
negative note carefully where to place the zeros in the quotient and continue to do the
division until you see the repeating 5's and then record the result correctly

Lesson 32	
Evaraisas.	
Excretises.	
Compute	
1.	$0.9435 \div 0.3 = \frac{0.9435}{0.3} = \frac{0.9435 \times 10}{0.3 \times 10} = \frac{9.435}{3} = 9.435 \div 3 = 3.145$
2	$28.875 \div (-1.25) = -(28.875 \div 1.25) = -\left(\frac{28.875}{1.25}\right) = -\left(\frac{28.875 \times 100}{1.25 \times 100}\right)$
2.	$= -\left(\frac{2887.5}{125}\right) = -(2887.5 \div 125) = -23.1$
3	$(-3274.5) \div 1.11 = -(3274.5 \div 1.11) = -\left(\frac{3274.5}{1.11}\right) = -\left(\frac{3257.5 \times 100}{1.11 \times 100}\right)$
5.	$= -\left(\frac{327450}{111}\right) = -2950$
4.	$(-5.392) \div (-0.02) = 5.392 \div 0.02 = \frac{5.392}{0.02} = \frac{5.392 \times 100}{0.02 \times 100}$ 539.2
	$\frac{1}{2} = 539.2 \div 2 = 269.6$
5.	$85938 \div 1.2 = \frac{85938}{1.2} = \frac{85938 \times 10}{1.2 \times 10} = \frac{859380}{12} = 859380 \div 12 = 71615$
6.	$(-10.25) \div (-2.5) = 10.25 \div 2.5 = \frac{10.25}{2.5} = \frac{10.25 \times 10}{2.5 \times 10} = \frac{102.5}{25} = 102.5 \div 25 = 4.1$
	$63.212 \div (-0.01) = -(63.212 \div 0.01) = -\left(\frac{63.212}{0.01}\right)$
7.	(63.212×100) (6321.2)
	$= -\left(\frac{1}{0.01 \times 100}\right) = -\left(\frac{1}{1}\right) = -6321.2$
Q	$(-9.21) \div (0.001) = -(9.21 \div 0.001) = -\left(\frac{9.21}{0.001}\right)$
0.	$= -\left(\frac{9.21 \times 1000}{0.001 \times 1000}\right) = -\left(\frac{9210}{1}\right) = -9210$

Lesson 32 Exercises Continued

9.
$$(-538) \div (-0.1) = 538 \div 0.1 = \frac{538}{0.1} = \frac{538 \times 10}{0.1 \times 10} = \frac{5380}{1} = 5380$$

10
$$0.02 \div 0.1 = \frac{0.02}{0.1} = \frac{0.02 \times 10}{0.1 \times 10} = \frac{0.2}{1} = 0.2$$

$$35.8722 \div (-0.001) = -(35.8722 \div 0.001) = -\left(\frac{35.8722}{0.001}\right)$$

11.
$$= -\left(\frac{35.8722 \times 1000}{0.001 \times 1000}\right) = -\left(\frac{35872.2}{1}\right) = -35872.2$$

Lesson 33 Exerc	cises
1	$-1+3\times(-0,1) = -1+(-0,3) = -1=0,3=-(1+0,3) = -1,3$
1.	$-1+5\times(-0.1) = -1+(-0.5) = -1-0.5 = -(1+0.5) = -1.5$
2.	$(-1+3)\times(-0.1)=2\times(-0.1)=-(2\times0.1)=-0.2$
3.	$1.12 \times (0.3)^2 = 1.12 \times 0.09 = 0.1008$
4.	$10 - 5 \times 2.2 - (0.2)^3 \times 10^3 = 10 - 5 \times 2.2 - 0.008 \times 1000 = 10 - 11 - 8 = -1 - 8 = -9$
no ex We	te the work and the order. We first went thru the problem and performed all the ponentiations. Then we went thru again and performed all the multiplications. Lastly e went thru and associated all the addition and subtraction from left to right.
5.	$(10-5) \times 2.2 - (-0.2)^3 \times 10^2 = 5 \times 2.2 - (-0.2)^3 \times 10^2 = 5 \times 2.2 - (-0.008) \times 100$ = 5 × 2 2 + 0.008 × 100 = 11 + 0.8 = 11.8
no Th all su	the work and the order. We first went thru the problem and worked on the group. Then we performed all the exponentiations. Then we went thru again and performed the multiplications. Lastly we went thru and associated all the addition and btraction from left to right.
	$-3 + 10(6 - 7.28) - 3 \div 0.3 \times 10 = -3 + 10(-1.28) - 3 \div 0.3 \times 10$
6.	$= -3 + (-12.8) - 3 \div 0.3 \times 10 = -3 + (-12.8) - 3 \div 0.3 \times 10$
	$= -3 - 12.8 - 10 \times 10 = -3 - 12.8 - 100 = -(3 + 12.8 + 100) = -115.8$
no Th di we an	te the work and the order. We first went thru the problem and worked on the group. Then we went thru again, several times, and performed all the multiplications and visions from left to right and rewrote a $+(-)$ as a - Lastly we went thru and saw we are combining three negative quantities which is a negative sum, which we wrote out, and then did the calculation.
7.	$\frac{(0.4)^3 \times (-0.1)^3 \times 10}{(0.4)^2 \times 10^2} = \frac{0.4 \times 0.4 \times 0.4 \times (-0.1)^3 \times 10}{0.4 \times 0.4 \times 10 \times 10} = \frac{0.4 \times (-0.1)^3}{10}$ $\frac{0.4 \times (-0.001)}{10} = \frac{-(0.4 \times 0.001)}{10} = \frac{-0.0004}{10} = -0.00004$
no the At	the here that the first thing was to try and simplify the expression since we noticed that e bases for the exponentials were the same in the numerator and the denominator. fter writing out the factors and producing several ones (1) in the problem by dividing $\begin{pmatrix} N \\ \end{pmatrix}$
a c nu di le:	quantity by itself $\left(\frac{N}{N} = 1\right)$, we determined the value of the exponential in the imerator and then multiplied by 0.4 to determine the value of the numerator and lastly, vided by 10 which moved the decimal point in the numerator value one space to the ft and our result was negative since we were dividing a negative by a positive.

Lesson 33 Exercises continued			
$(2.2 - 3.3)(2.5 - 2.50)$ $(-1.1) \times 0$ 0			
8. $\frac{1}{2.567} = \frac{1}{2.567} = \frac{1}{2.567} = 0$			
several things to note. The group () followed by the group () or ()() implied			
multiplication. When evaluating the groups as the first step, it was noted that $2.5 = 2.50$			
so their difference is 0. This means the numerator would have a value of 0 (any			
quantity times 0 equals 0) and when the denominator remained a nonzero value, the			
result was zero since zero divided by a nonzero number is always zero.			
$2 - 0.1 \times 3$ $2 - 0.3$ 1.7 1.7×10 17 1.7×26 $0.47\overline{2}$			
9. $\frac{1}{4 - 0.1 \times 4} = \frac{1}{4 - 0.4} = \frac{1}{3.6} = \frac{1}{3.6 \times 10} = \frac{1}{3.6} = \frac{1}{3.6$			
here we worked on the numerator and the denominator until they became single values			
and then did the division.			
$2(-2.11+1.1) \times 4 - 2 \times (-1.01) \times 4 - (2 \times 1.01) \times 4 - 1 \times 4 - 4$			
10.			
after performing the work in the group and rewriting all the factors clearly and the			
multiplications, we can see some ones (1's) within the problem as the factors divide			
themselves. In this case we have a negative one (-1) in the numerator and the			
conclusion is for the result to be -4.			

Exercises: 1. A store offers a 15% discount for an item. The original price was \$32. What is the price after the discount? Solution: If there is a 15% discount then the customer will pay 85% of the cost (100 - 15=85). That means we want to take 85% of \$32. This is a multiplication problem: 0.85 × 32 = 27.20. Thus the item cost \$27.50 after the 15% discount. 2. 35% of what quantity is equal to 14. Solution: 0.35 times some quantity will equal 14. This means that 14 divided by 0.35 will give us the value of the quantity. Hence: $14 \div 0.35 = 1400 \div 35 = 40$. Therefore the quantity has a value of 40. 3. The price of one gallon of gas increase from \$2.50 to \$2.55. What percent was the increase? Solution: The increase was \$0.05. To determine the percent of increase we compare
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the increase of 0.05 to the original price of \$2.50. That would give us a fraction of
0.05 If we compute the indicated division we will see
$\frac{1}{2.50}$. If we compute the indicated division we will see:
0.05 0.5
$\frac{1}{2.50} = \frac{1}{25} = 0.5 \div 25 = 0.02 = \frac{1}{100}$ once we know that we can conclude that 2% is the
resulting increase since percent (%) is "per 100" and is the numerator value of any
fraction with a denominator of 100. The percent of increase is 2%.

Ν	lath016	Exam1	CEMEC	Page 713
Ti	me: 40 minutes		No calculators	
1. (a)	Compute. Use the '=' 4^3	sign.		
b)	100×29			
c)	1 ⁵³			
d)	0 ²⁴			
e)	8^0			

- **f)** 10×104
- 2. Fill in the blank using either the '=' or \neq as appropriate. Show your work.
 - a) 8×3 6×4
 - b) 2^4 4^2
 - c) 25^1 1^{25}
 - d) 0^3 0^0

3. Determine, without calculations whether the following are **true or false**. Justify your answer. (for example, state if the order of operations, Commutative or Associative property are applicable)

- a) 8552 + 367 = 367 + 8552
- **b)** $(9 \times 71) + 3 = 9 \times (71 + 3)$
- c) $45 + 37 \times 19 = 45 + (37 \times 19)$

- 4. Circle the larger in each pair of numbers. Show your work.
- a) 6^1 or 1^6
- b) 0^8 or 8^0
- 5. Compute. Use the '=' sign. Show every step clearly.
- a) $15^0 + 4^3$
- b) $10 \times (6 2 + 9)$
- c) $9 2 \times 4$
- **d)** $5 \times (15 7)^0$
- e) $5 + 3 \times 4 3$

f) $2 + 10^3$

g) $4 \times 10^2 + 5 \times 10^2$

h) $14^{0} + 3 \times 1^{5}$

i) $(3+8) \times (9-7)$

j) $5 + 2(9 - 6)^2$

Math 016Test2Name_____

No Calculator, no notes and Show your work!!!

1. Find the opposite of each of the following integers. (3pts)

- a. -6 opposite _____
- b. 7 opposite _____
- c. -(-7) opposite_____

2. Compute if possible or write the expression is undefined. Use the "=" correctly. Show your steps if necessary. Pay close attention to the operation and the signs of the numbers. (20 pts)

a. -2×10 b. $-4 \times (-3)$ c. $-4 \times 2 \times (-3)$ d. $\frac{-2}{0}$ e. $\frac{0}{-3}$ f. $-18 \div 6$ g. $\frac{-24}{-12}$ h. $+(-8) \div (-2)$ i $\frac{16}{-4}$ j. $-4 \times 2 \times (-1) \times (-2)$ 3. Compute. Show correct use of "=" sign between stages (steps) of work (28 pts)

- a. -4 + 0
- b. -100 + 100
- c. 0 + (-8)
- d. -6 + (-2)
- e. -14 + 7
- f. 7 + (-11)
- g. -7 + (-11)
- $h. \quad -22 \ + \ 28$
- i. -6 3
- j. -14 (-2)
- k. -5 (+2)
- l. 6-9
- m. 8 (-9)
- n. -5 4
- 4. Compute and show your work. Show correct use of = sigh between stages (steps) of work (8pts)
 - a. -2⁴
 - b. (-2)⁴
 - c. $(-1)^{17} + (-1)^{18}$
 - d. $(-3)^3 + (-2)^0$

5. Fill in the blank using either , <, >, =, as appropriate (5 pts)

a. $-11 _ 5$ b. $-7 _ -9$ c. $-(+9) _ -9$ d. $-8 _ -(-8)$ e. $\frac{-14}{7} _ \frac{14}{-7}$

6. Determine <u>without calculating</u> which of the following are true or false and circle accordingly. <u>If true</u> state property of addition or multiplication that makes statement true. (5 pts)

a.	-879 + 156 = 156 + (-879)	Т	F			
b.	-197 × (-204) = -204 × 197	Т	F			
c.	[-62 + (-37)] + (-19) = -62 + [-37]	+ (-19))]	Т	F	

7. Compute showing one step for each operation \cdot Make sure that you use the "=" sign correctly. Be sure to follow the order of operations very carefully. (15 pts)

a. $-4 + 5 \times (-2)$

b..
$$(9-2)(7-11) \div 2(3+4)$$

c. .
$$[(-3)^2 + 9] \div [9 \times (-2)]$$

d.
$$(-5^2 - 1^2 - 2^3)(-3 - 7)$$

e. $[5-5(4-2\times3)] + [6-(-8 \div 2)]$

- 8. Read each of the following very carefully and determine if the statement is true or false. Circle your choice. (10 points)
 - a) T or F : -7 is a natural number.
 - b) T or F : The difference of two natural numbers is always a natural number.
 - c) T or F : When you add two integers of the same sign you sometimes subtract their values.
 - d) T or F : The quotient of two integers is always an integer
 - e) T or F : The product of five integers is sometimes negative.
 - f) T or F : The opposite of a negative integer is a positive integer
 - g) T or F : The opposite of the opposite of negative 6 is 6
 - h) T or F : The product of a positive integer and a negative integer is sometimes negative.
 - i) T or F : The sum of a positive and a negative integer is always positive.
 - j) T or F : -x, where x is some number, is always a negative number
- 9. Plot and label the following on their respective number line. Be sure to also indicate the position of 0 (6 points)
 - a) Plot and label: -5

b) Plot and label: -(- 14)
Math 016

Exam 3

CEMEC

1. Fill in the blank using either =, <,or > as appropriate.



e. $\frac{-5}{-6}$ $\frac{-6}{5}$

2. Compute. Use the = sign. If the resulting fraction is an integer, indicate it.

- **a.** $\frac{5}{3} \times \frac{2}{7}$
- **b.** 5 $x \frac{1}{9}$
- **c.** $\frac{2}{3} \times 6$

d.
$$\frac{4}{11} \times 0$$

e.
$$\frac{7}{6} \times 1$$

3. Find 2 fractions equivalent to $\frac{3}{2}$. Show all the steps.

4. Find a fraction equivalent to $\frac{9}{10}$ with denominator 50. Show all the steps.

- 5. Simplify the fractions. Show all the steps of the simplification.
- **a.** $\frac{12}{21}$
- **b.** $\frac{81}{63}$
- **c.** $\frac{5 \ x \ 23}{23 \ x \ 4}$
- **d.** $\frac{7 \ x \ 12 \ x \ 27}{27 \ x \ 7 \ x \ 12}$

6. Compute and simplify. Show all of the factors that can be cancelled.

- **a.** $\frac{8}{3} \times \frac{9}{4}$ **b.** $\frac{3}{5} \times \frac{5}{7}$ **c.** $\frac{12}{6} \times \frac{8}{2} \times \frac{35}{7}$
- **d.** $\frac{2}{3}$ (-15)
- **e.** 4 $x \left(-\frac{10}{15}\right) x \frac{5}{2}$
- $\mathbf{f.} \quad -\left(-\frac{1}{2}\right)^3$
- **g.** $\left(\frac{3}{5}\right)^2$
- **h.** $-\left(-\frac{2}{3}\right)^4$
- i. $\left(-\frac{3}{5}\right)^3$
- **j.** $-\left(\frac{1}{7}\right)^2$
- 7. Divide and simplify. When simplifying show all of the factors that are cancelled.



8. Determine and state which of the following pair of fractions are equivalent. Show all of your work.

a. $\frac{4}{7}, \frac{16}{28}$

- **b.** $\frac{3}{5}, \frac{7}{10}$
- **c.** $\frac{0}{2}$, $\frac{0}{6}$

d. $\frac{9}{9}$, $\frac{157}{157}$

e. $\frac{18}{3}$, $\frac{24}{4}$

Math 016 Exam 4

1.	$\frac{5}{12} + \frac{25}{12}$
2.	$\frac{7}{10} + 2$
3.	$\frac{1}{8} + \frac{3}{5} + \frac{51}{40}$
4.	$\frac{5}{7} + 3 + \frac{1}{3}$
5.	$\frac{1}{2} + \frac{1}{3} + \frac{1}{7}$
6.	$-\frac{3}{11}+\frac{25}{11}$
7.	$-\frac{17}{20}-\frac{7}{20}$
8.	$4 - \frac{21}{5}$
9.	$\frac{11}{3} - 3 - \frac{3}{5}$
10.	$-\frac{4}{7}+\frac{4}{5}-\frac{18}{35}$
11.	$\frac{4}{5} - \left(-\frac{3}{4}\right) - 3$
12.	$4\left(-\frac{5}{12}\right)$
13.	$4 - \frac{5}{12}$

14.
$$\frac{1}{4} - \frac{1}{4} \times 4$$

15. $\left(\frac{1}{4} - \frac{1}{4}\right) \times 4$
16. $\left(2 - \frac{1}{7}\right) \times 2 - 4$
17. $1 + \left(\frac{1}{3}\right)^2 + \frac{1}{5}$
18. $2 - \left(\frac{\frac{1}{3}}{\frac{1}{3} - \frac{1}{6}}\right) \times \frac{1}{2}$

Questions 19 and 20. Write the mixed number as a fraction.

19.
$$9\frac{1}{5}$$
 20. $3\frac{1}{11}$

Questions 21 and 22. Write the fraction as a mixed number. Then, locate it on a number line.

Questions 23 through 26. Compute. Give your answer both as a fraction (or an integer) and as a mixed number, if it applies.



CEMEC 016

Name:

Find decimal notation for each of the following fractions. Make sure your answer is either a terminating or a repeating decimal. (13 pts)

1.	$\frac{403}{100}$	
2.	$\frac{17}{20}$	
3.	$\frac{9}{4}$	
4.	$\frac{31}{12}$	

Write each of the following numbers as fractions: (9 pts)

 $5. \ 0.074$

6. 5.03

7. $(0.2)^2$

Fill in the blank using either '=', '<' or '>' as appropriate (6 pts)

- 8. 2.131____2.082
- 9. -0.075____-0.32
- 10. $\frac{403}{1000}$ _____ 0.403

Compute the following. Show your work. (15 pts)

11. 56.8-9.26

12. 4.69 – (-12.7)

13. 0.24 – 1

14. -1.04-3.27

15. 2.87 + 4 + 0.36

Multiply or divide. Show your work. (18 pts)

16. $0.55 \div 11$

17. 14.6×100

18. $-5.12 \times (-4.2)$

19. $15.623 \times 7.26 \times 0 \times (-0.4)$

20. $8.02 \div (-0.001)$

21. $15.036 \div (-0.3)$

Evaluate the powers. Show all work. (6 pts)

22. $(0.1)^4$

23. $-(-0.2)^3$

Compute. Show all of your steps. (33 pts)

24. $2 + 4 \times 1.08$

25. $(6.45 - 9.5)^0 - 1.6$

26. $9.9 \div 0.3^2$

27. $-3 \times 0.02 - 5 \times 0.1$

28. $-4 - 3(5 - 6.6)^2$

29. -1-2(-0.1)-0.11

30. (-1-2)(-0.1-0.01)