

1. Find the coefficient of x^2 in the polynomial

$$P(x) = (1 - x)(1 + 2x)(1 - 3x)(1 + 4x) \cdots (1 - 9x)(1 + 10x).$$

Solution: \square *Multiplying every consecutive pair we obtain five quadratic factors:*

$$\begin{aligned} (1 - x)(1 + 2x)(1 - 3x)(1 + 4x) \cdots (1 - 9x)(1 + 10x) &= (1 + x - 2x^2)(1 + x - 12x^2)(1 + x - 30x^2) \\ &\quad \cdot (1 + x - 56x^2)(1 + x - 90x^2) \\ &= \end{aligned}$$

To find the coefficient of x^2 either one takes one x from exactly two of the factors and a 1 from the other three, or one takes the coefficient of x^2 from one of the factors and a 1 from each of the other four factors. Thus the coefficient of x^2 is

$$\binom{5}{2} - 2 - 12 - 30 - 56 - 90 = 10 - 2 - 12 - 30 - 56 - 90 = -180.$$

\square