1 Warning

Community College of Philadelphia is a firm adherent to the principle of academic freedom. In light of this, faculty are not required to follow a particular approach or a particular textbook for the courses they teach. Most faculty, however, have more or less uniform guidelines for specific courses, and indeed, many use a particular textbook or approach in order to conform to area institutions. Therefore, the sample syllabus found here is not binding to faculty, but represents a synthesis of what most faculty do or aspire to do when they teach a particular course. What follows should not be interpreted as a prescription, but rather, as a means to help the placement of our students in transfer institutions.

2 Catalogue Description

Matrices, determinants, vector spaces, inner product spaces, eigenvalues, eigenvectors, linear transformations and applications. Prerequisites: MATH 171 and MATH 172. (MATH 172 may be taken concurrently).

3 Allotted Time

Math 270 is a four credit course. Thus it meets for $\mathbf{4} \times \mathbf{14} = \mathbf{56}$ hours in a semester, including two hours for a final examination. Instructors usually give three or four exams (generally lasting at least **55** minutes), and a **2**-hour long final exam.

4 Topics Outline

- Binary Operations. Equivalence Relations. Finite and Infinite Fields. Characteristic of a field.
- Matrix Operations. Operations over different fields. Dilatation Matrices. Transvection Matrices. Transposition Matrices. Permutation Matrices. Rank of a Matrix.
- Linear Systems. Linear Systems over different fields. Criteria for existence of solutions. Gauss-Jordan Reduction.
- Inversion of Matrices.
- $\mathbb{R}^3$ and $\mathbb{R}^n$. Vectors in two and three dimensional space. The dot product in $\mathbb{R}^n$. The cross product in $\mathbb{R}^3$. Lines and Planes in $\mathbb{R}^3$. Cauchy-Schwarz-Bunyakovsky Inequality in $\mathbb{R}^3$ and $\mathbb{R}^n$. Projection of a vector.
- Vector Spaces. Examples of finite element and infinite element vector spaces. Examples of Finite Dimensional and Infinite Dimensional Vector Spaces. Spanning Sets. Bases. Steinitz Replacement Theorem. Number of elements of an ordered basis of a finite dimensional vector space over a finite field.
- Linear Mappings. Kernel and Image of a Linear Mapping. The dimension theorem.
- Matrix Representation of Linear Mappings. Geometrical Examples in $\mathbb{R}^2$ and $\mathbb{R}^3$.
- Determinants. Algebraic Properties of Determinants. Geometric Properties of Determinants. Laplace Expansion of a Determinant. Cramer's Rule.
- Eigenvalues and Eigenvectors: Characteristic Polynomials. Similar Matrices. Diagonizability of Matrices. Powers of Square Matrices. Cayley-Hamilton Theorem.
- The Minimal Polynomial.

5 Competencies

1. The Student will demonstrate understanding of finite fields by
   - producing the sum and multiplication table of some finite fields

- solving linear and quadratic equations in finite fields

2. The Student will solve linear systems of equations by,
   (a) applying elementary row operations to the system,
   (b) applying Gauss-Jordan elimination.
   (c) determining when a system possesses no solution, multiple solutions and exactly one solution.

3. The Student will demonstrate proficiency in Matrix Algebra by,
   (a) performing matrix addition, scalar multiplication, and matrix multiplication in both finite and infinite fields
   (b) finding the transpose, the cofactor, and the adjoint matrices of a given matrix,
   (c) finding the inverse of an invertible matrix by either the formula or applying row operations,
   (d) solving linear systems applying matrix algebra.

4. The Student will demonstrate proficiency in the manipulation of determinants by,
   (a) evaluating determinants by both row reduction and cofactor expansion,
   (b) deriving theoretical consequences from various theorems about determinants

5. The Student will demonstrate knowledge of the Euclidean space $\mathbb{R}^{n}$ and its properties by,
   (a) performing basic operations in
   (b) determining if a subset of $\mathbb{R}^{n}$ is a subspace of $\mathbb{R}^{n}$
   (c) recognizing the Euclidean norm, distance, and inner product,
   (d) determining if a set of vectors is linearly independent,
   (e) finding basis for $\mathbb{R}^{n}$
   (f) finding orthonormal basis for $\mathbb{R}^{n}$ applying the Gram-Schmidt process.

6. The Student will recognize linear transformations between $\mathbb{R}^{n}$ and $\mathbb{R}^{m}$ and
   (a) calculate the kernel and the range of a linear transformation,
   (b) find a matrix representation of a given linear transformation,
   (c) describe the geometric properties of basic linear transformations in the plane.

7. The Student will demonstrate knowledge of eigenvalues and eigenvectors by,
   (a) defining eigenvalues eigenvectors of a matrix and a linear transformation,
   (b) . finding eigenvalues and eigenspaces of a matrix,
   (c) recognizing the differences between algebraic multiplicity and geometric multiplicity of an eigenvalue.

8. The Student will demonstrate knowledge of abstract spaces by
   (a) performing vector operations in spaces such as space of continuous functions, space of polynomials functions, and space of matrices,
   (b) finding basis and coordinate vectors in abstract spaces.