Instructions: This quiz has 30 questions. The use of calculators is forbidden. Click on the box with the right answer. To initialise the quiz you must click on "BEGIN QUIZ." When you finish the quiz you click on "END QUIZ" in order to see your score.

Begin Quiz Answer each of the following.

1. $A \in \mathbf{M}_{3\times 2}(\mathbb{R}), B \in \mathbf{M}_{2\times 3}(\mathbb{R})$, and $C \in \mathbf{M}_{2\times 2}(\mathbb{R})$. Which of the following matrix operations is impossible to carry out?

- ☐ ACB
- ☐ $BA + C$
- ☐ $A + B^T$
- ☐ $A^T C - B^T$

2. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Which of the following statements is (are) true about A?

$\mathbf{I}$: $A^n = 3^{n-1}A$ for positive integer n;
$\mathbf{II}$: A is invertible;
$\mathbf{III}$: if $AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, then $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

- ☐ **I** only
- ☐ **I** and **III** only
- ☐ **I** and **II** only
- ☐ **II** and **III** only

3. Which of the following statements is (are) true regarding arbitrary $\mathbf{2}\times\mathbf{2}$ matrices $\boldsymbol{A},\boldsymbol{B}$ with real number entries? Here $\boldsymbol{I}$ is the $\mathbf{2}\times\mathbf{2}$ identity matrix and $\mathbf{0}$ is the $\mathbf{2}\times\mathbf{2}$ zero-matrix.

$$\begin{aligned}&\mathbf{I}: A^2=-I \text{ is always impossible;}\\ &\mathbf{II}: AB=\mathbf{0} \text{ only when } A=\mathbf{0} \text{ or } B=\mathbf{0};\\ &\mathbf{III}: (A+B)^2=A^2+2AB+B^2\end{aligned}$$

☐ **I** only ☐ **I** and **II** only ☐ **I** and **III** only ☐ **III** only ☐ all are false

4. Find t if $\begin{bmatrix}1&1&1\\1&-1&1\\1&1&1\end{bmatrix}\begin{bmatrix}1&2&3\\3&1&2\\2&3&1\end{bmatrix}=\begin{bmatrix}6&6&6\\0&t&2\\6&6&6\end{bmatrix}$.

☐ **0** ☐ **2** ☐ **4** ☐ **6**

5. If $\begin{bmatrix}1&-1&1\\1&1&-1\\-1&1&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}$ then $2x+4y+8z=$

☐ $3\alpha+3\beta+6\gamma$ ☐ $5\alpha+3\beta+8\gamma$ ☐ $2\alpha+4\beta+8\gamma$ ☐ $5\alpha+3\beta+6\gamma$

6. Which of the following statements is true regarding the system of linear equations

$$x + y + z + a - b = 3$$

$$x + 2y + z + 3a - 2b = 5$$

$$y + z + a = 3$$

- ☐ the system possesses the unique solution $(1, 1, 1, 1, 1)$
- ☐ the system has infinitely many solutions and 1 degree of freedom
- ☐ the system has infinitely many solutions and 2 degrees of freedom
- ☐ the system has infinitely many solutions and 3 degrees of freedom
- ☐ the system has no solutions

7. The inverse of the matrix $M = \begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 2 \\ 4 & 3 & 1 \end{bmatrix}$ is the matrix $M^{-1} = \frac{1}{6}\begin{bmatrix} a & 8 & 5 \\ 8 & b & -4 \\ 4 & -2 & -2 \end{bmatrix}$, where $(a, b) =$

☐ $(-7, -10)$ ☐ $(7, 10)$ ☐ $(8, -4)$ ☐ $(1, -3)$

8. For which a is the matrix $\begin{bmatrix} -1 & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{bmatrix}$ singular (non-invertible)?

☐ $a \in \{-3, -1\}$ ☐ $a \in \{3, 1\}$ ☐ $a \in \{-3, 1\}$ ☐ $a \in \{3, -1\}$

Home Page

Title Page

Contents

◀◀ ▶▶

◀ ▶

Go Back

Full Screen

Close

Quit

9. Find t so that $\begin{bmatrix} 1 \\ 1 \\ t \end{bmatrix}$ and $\begin{bmatrix} 2t \\ 1 \\ -1 \end{bmatrix}$ be perpendicular.

☐ $t=0$ ☐ $t=2$ ☐ $t=-1$ ☐ $t=1$

10. Find $(\vec{\mathbf{i}} - \vec{\mathbf{j}} - \vec{\mathbf{k}}) \times (\vec{\mathbf{i}} + \vec{\mathbf{j}} + \vec{\mathbf{k}})$.

☐ $-2\vec{\mathbf{j}} - 2\vec{\mathbf{k}}$ ☐ $-2\vec{\mathbf{j}} + 2\vec{\mathbf{k}}$ ☐ $2\vec{\mathbf{j}} + 2\vec{\mathbf{k}}$ ☐ $2\vec{\mathbf{j}} - 2\vec{\mathbf{k}}$

11. Find the equation of the plane containing the point $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and perpendicular to the line

$$x = 1, \qquad y = -1 + t, \qquad z = 2t.$$

☐ $y + 2z = -3$ ☐ $y + 2z = 3$ ☐ $x + y + 2z = 4$ ☐ $y - 2z = 3$

12. Which of the following sets is (are) (a) vector space(s) of the set of $\mathbf{M}_{2\times 2}(\mathbb{R})$?

- $V_1 = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbf{M}_{2\times 2}(\mathbb{R}) : a + d = 0 \right\}$.
- $V_2 = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbf{M}_{2\times 2}(\mathbb{R}) : ab = 0 \right\}$.
- $V_3 = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbf{M}_{2\times 2}(\mathbb{R}) : a - b + c - d = 0 \right\}$.

☐ V_1 only ☐ V_2 only ☐ V_1 and V_2 only ☐ V_1 and V_3 only ☐ all of V_1, V_2, V_3

13. What is the dimension of the following vector subspace

$$V = \left\{ \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \in \mathbf{M}_{3\times 3}(\mathbb{R}) : a + e + i = 0 = c + e + g \right\}$$

of $\mathbf{M}_{3\times 3}(\mathbb{R})$?

☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 9

14. Which set of vectors spans $\mathbb{R}^3$?

$$B_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}. \qquad B_2 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

$$B_3 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}. \qquad B_4 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

☐ B_4 only
☐ B_1 and B_4 only
☐ B_3 and B_4 only
☐ B_2 and B_3 only
☐ all of B_1, B_2, B_3, B_4

15. Which set of vectors forms a basis for $\mathbb{R}^3$?

$$B_1 = \left\{ \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix} \right\}. \qquad B_2 = \left\{ \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}2\\1\\0\end{bmatrix} \right\}.$$

$$B_3 = \left\{ \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\3\end{bmatrix} \right\}. \qquad B_4 = \left\{ \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}.$$

- ☐ B_4 only
- ☐ B_1 and B_4 only
- ☐ B_3 and B_4 only
- ☐ B_2 and B_3 only
- ☐ all of B_1, B_2, B_3, B_4

16. The outputs $L\begin{bmatrix}x\\y\end{bmatrix}$ for functions $L : \mathbb{R}^2 \to \mathbb{R}^2$ are given below. Which output is not that of a linear transformation?

☐ $\begin{bmatrix}x+y\\y-x\end{bmatrix}$ ☐ $\begin{bmatrix}2x\\y\end{bmatrix}$ ☐ $\begin{bmatrix}x+1\\y-1\end{bmatrix}$ ☐ $\begin{bmatrix}y+2x\\x\end{bmatrix}$

17. Let $L : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation with $L\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}1\\2\\0\end{bmatrix}$ and $L\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix}$. Find $L\begin{bmatrix}5\\4\end{bmatrix}$.

☐ $\begin{bmatrix}5\\4\\0\end{bmatrix}$ ☐ $\begin{bmatrix}5\\7\\-3\end{bmatrix}$ ☐ $\begin{bmatrix}2\\3\\-1\end{bmatrix}$ ☐ $\begin{bmatrix}5\\7\\-5\end{bmatrix}$

18. Let $L : M_{2\times 2}(\mathbb{R}) \to \mathbb{R}$ be a linear transformation with $L\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a + 2b + 3c$. Find $\dim \ker L$.

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

19. Let $L : M_{2\times 2}(\mathbb{R}) \to \mathbb{R}$ be a linear transformation with $L\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a + 2b + 3c$. Find $\dim \operatorname{Im} L$.

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

20. Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $\dim \ker L = 2$. Which of the following is always true?

- I: L is injective.
- II: L is surjective.
- III: $\ker L$ is a plane in $\mathbb{R}^3$.
- IV: $\operatorname{Im} L$ is a line in $\mathbb{R}^3$.

☐ I and II only
☐ III and IV only
☐ I and III only
☐ II and IV only
☐ all of I, II, III, IV

21. Find $\det \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{bmatrix}$.

☐ -1 ☐ 0 ☐ 1 ☐ 2

22. Find $\det \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 1 & 1 \\ -1 & -1 & 0 & 1 & 1 \\ -1 & -1 & -1 & 0 & 1 \\ -1 & -1 & -1 & -1 & 0 \end{bmatrix}$.

☐ -1 ☐ 0 ☐ 1 ☐ 5 ☐ -5

23. A 5×5 matrix A has characteristic polynomial $p(\lambda) = (\lambda - 1)^3(\lambda - 2)(\lambda - 3)$. Find $\det A$.

☐ -2 ☐ 6 ☐ -6 ☐ 0

24. A 4×4 matrix A with $\det A = 2$ undergoes the following successive transformations becoming the matrix B: (i) its first two rows are exchanged, (ii) to the first column the remaining columns are added, (iii) each element of the matrix is doubled. What is $\det B$?

☐ $\frac{1}{32}$ ☐ -32 ☐ 32 ☐ $-\frac{1}{32}$ ☐ 0

25. The 2×2 matrices A and B satisfy $\det 2A = 4$ and $\det B^3 = 27$. What is the determinant of the inverse of their product AB?

☐ $\frac{1}{3}$ ☐ 54 ☐ 6 ☐ 3 ☐ 12

26. What is the characteristic polynomial $p(\lambda)$ of $A = \begin{bmatrix} 1 & -3 \\ 5 & 1 \end{bmatrix}$?

☐ $\lambda^2 + 2\lambda - 16$ ☐ $\lambda^2 + 2\lambda + 16$ ☐ $\lambda^2 - 2\lambda - 16$ ☐ $\lambda^2 - 2\lambda + 16$

27. A 2×2 matrix with real entries has trace -1 and determinant -6. What its characteristic polynomial $p(\lambda)$?

☐ $(\lambda - 2)(\lambda - 3)$ ☐ $(\lambda + 2)(\lambda - 3)$ ☐ $(\lambda - 2)(\lambda + 3)$ ☐ $(\lambda + 2)(\lambda + 3)$

Home Page
Title Page
Contents
Page 9 of 9
Go Back
Full Screen
Close
Quit

Situation: Problems 28 through 30 refer to the 2×2 matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, which has eigenvalues $\lambda = 5$ and $\lambda = -1$.

28. Which of the following are eigenvectors of A?

$$I:\begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad II:\begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad III:\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

☐ I only ☐ II only ☐ I and II only ☐ I and III only ☐ II and III only

29. What is the sum of the eigenvalues of A^2?

☐ 4 ☐ 6 ☐ 26 ☐ 24

30. What is A^n?

☐ $\begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 5^n & 0 \\ 0 & (-1)^n \end{bmatrix}\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 5^n & 0 \\ 0 & (-1)^n \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 5^n & 0 \\ 0 & (-1)^n \end{bmatrix}\begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 5^n & 0 \\ 0 & (-1)^n \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$

End Quiz

Score: Correct

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