Instructions: This quiz has 46 questions. The use of calculators is forbidden. Click on the box with the right answer. To initialise the quiz you must click on "BEGIN QUIZ." When you finish the quiz you click on "END QUIZ" in order to see your score.

Begin Quiz Answer each of the following.

1. Consider the region in the first quadrant bounded by $y = x^3$ and $x = y^3$. Rotate this region about the line $y = -1$. What is the volume of the resulting solid?

☐ $\frac{16}{35}\pi$ ☐ $\frac{51}{35}\pi$ ☐ $\frac{23}{14}\pi$ ☐ $\frac{86}{35}\pi$ ☐ $\frac{31}{10}\pi$ ☐ $\frac{166}{35}\pi$

2. The base of a solid is a circular disk of radius 3. Find the volume of the solid if parallel cross-sections perpendicular to the base are isosceles right triangles with hypotenuse lying along the base. Refer to figure 1

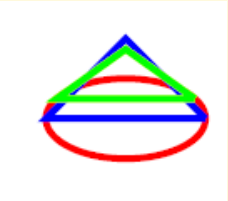

Figure 1: Problem 2.

☐ 36 ☐ 18 ☐ 9 ☐ 1 ☐ $\sqrt{2}$ ☐ π

3. Find the average value of the function

$$f(\theta) = \sec(\theta)\tan(\theta)$$

on the interval $[0, \pi/4]$.

☐ 0 ☐ $\sqrt{2}-1$ ☐ $\frac{2}{\pi}$ ☐ $\frac{1}{2}$ ☐ $\frac{4\sqrt{2}-4}{\pi}$ ☐ diverges

4. Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{e^{e^x}}\right)$.

☐ $e^{\ln x}$ ☐ $e^{e^{e^x}}$ ☐ $e^x e^{e^{e^x}}$ ☐ $e^x e^{e^x} e^{e^{e^x}}$ ☐ $e^{e^x} e^{e^{e^x}}$

5. Let

$$f(x) = \frac{10^x}{10^x + 1}.$$

Find the inverse function f^{-1}.

☐ $f^{-1}(x) = \log_{10}\left(\frac{x}{1-x}\right)$

☐ $f^{-1}(x) = \log_{10}\left(\frac{x}{1+x}\right)$

☐ $f^{-1}(x) = \log_{10}\left(\frac{x-1}{x+1}\right)$

☐ $f^{-1}(x) = \log_{10}\left(\frac{x+1}{x-1}\right)$

☐ $f^{-1}(x) = \log_{10}\left(\frac{x+1}{1-x}\right)$

☐ $f^{-1}(x) = \log_{10}\left(\frac{1-x}{x+1}\right)$

6. Find the limit

$$\lim_{x\to\infty} \arcsin\left(\frac{1+\sqrt{3}\,x^3}{1+2x^3}\right).$$

☐ 0 ☐ $\frac{\pi}{6}$ ☐ $\frac{\pi}{4}$ ☐ $\frac{\pi}{3}$ ☐ $\frac{\pi}{2}$ ☐ 1

7. $\displaystyle\int_2^3 \frac{x^3-x^2+1}{x^2-x}\,\mathrm{d}x =$

☐ $\frac{1}{2}+\ln 2$

☐ $\frac{5}{2}+\ln 3$

☐ $\frac{5}{2}-\ln 3$

☐ $\ln 3-\ln 4$

☐ $\frac{5}{2}+\ln 3-2\ln 2$

☐ $\frac{5}{2}-\ln 3+2\ln 2$

8. $\displaystyle\int \frac{e^{3x}}{1+e^{6x}}\mathrm{d}x =$

☐ $3\arctan e^{3x}+C$ ☐ $\frac{1}{3}\arctan e^{3x}+C$ ☐ $\arctan e^{3x}+C$ ☐ $\arctan e^{x/3}+C$

9. $\displaystyle\int_0^3 \frac{\mathrm{d}x}{(x-1)^3} =$

☐ $\frac{3}{8}$ ☐ $\frac{1}{2}$ ☐ $\frac{9}{4}$ ☐ 0 ☐ $\frac{8}{3}$ ☐ diverges

10. Find the surface area generated by revolving the curve

$$y = \sqrt{1 - x^2}, \qquad 0 \le x \le \frac{1}{2}$$

about the x-axis. Refer to figure 2

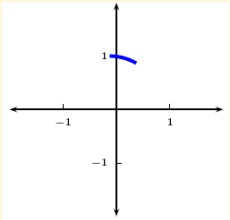

Figure 2: Problem 10.

☐ 2π ☐ π ☐ $\frac{4}{3}\pi$ ☐ $\frac{3}{4}\pi$ ☐ 1 ☐ $\frac{4}{3}$

11. Compute the arc length of the curve $y = 1 + \frac{2}{3}(x-1)^{3/2}$ for $x \in [1, 4]$.

☐ $\frac{14}{3}$ ☐ $\frac{2}{3}\ln 3$ ☐ $\frac{4}{3}\pi$ ☐ 4.75 ☐ 1 ☐ $\frac{1}{2}$

12. Let $\mathscr{C}$ be the curve defined by:

$$x = t - t^2, \qquad y = t^2 + 4, \qquad t \in \mathbb{R}.$$

What is the slope of the line tangent to $\mathscr{C}$ at $(0, 5)$?

☐ -2 ☐ 2 ☐ $\frac{1}{2}$ ☐ $-\frac{1}{2}$ ☐ -1 ☐ 1

13. Find the length of the curve with parametric equations

$$x = \sin t + \cos t \qquad y = \sin t - \cos t$$

for $0 \le t \le 2\pi$.

☐ $\sqrt{2}$ ☐ $2\sqrt{2}$ ☐ $2\sqrt{2}\,\pi$ ☐ 3π ☐ 9

14. Four polar plots appear below.

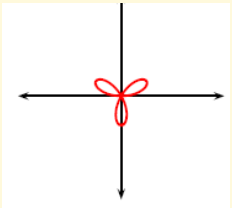

Figure 3: I

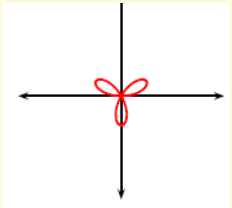

Figure 4: II

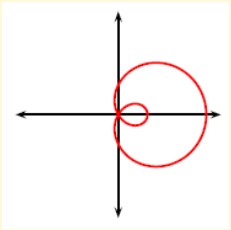

Figure 5: III

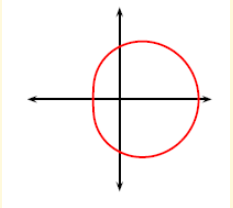

Figure 6: IV

Also, four polar equations are given below

$$\alpha : r = \sin 3\theta; \qquad \beta : r = \cos 3\theta; \qquad \gamma : r = 1 + 2\sin\theta; \qquad \delta : r = 2 + \sin\theta$$

Which choice gives the correct match with the polar equation?

☐ (I, β), (II, α), (III, δ), (IV, γ)
☐ (I, β), (II, α), (III, γ), (IV, δ)
☐ (I, β), (II, γ), (III, α), (IV, δ)

15. Find the area of the region between the two curves with polar equations

$$r = \sqrt{2} \quad \text{and} \quad r = \frac{1}{\sin\theta}, \quad \frac{\pi}{4} \le \theta \le \frac{3}{4}\pi.$$

☐ $\frac{\pi}{2}$ ☐ $\frac{\pi-1}{2}$ ☐ $\frac{\pi-2}{2}$ ☐ $\frac{1}{2}$ ☐ $\ln 2$ ☐ $\ln\frac{\pi}{2}$

16. Find the limit of the sequence

$$\lim_{n\to\infty} n\tan\frac{1}{n}.$$

☐ 2 ☐ 1 ☐ -1 ☐ $\frac{\pi}{2}$ ☐ 0 ☐ divergent

17. A sequence is defined by

$$a_1 = 2; \quad a_2 = 2^{1+1/2}; \quad a_3 = 2^{1+1/2+1/4}; \quad \dots \quad a_n = 2^{1+1/2+1/2^2+\cdots+1/2^{n-1}}.$$

Determine $\lim_{n\to\infty} a_n$.

☐ 0 ☐ 1 ☐ $\sqrt{2}$ ☐ 2 ☐ 4 ☐ $+\infty$

18. Find the interval of convergence of $\sum_{n=2}^{\infty} \frac{1}{\ln n}(x-1)^n$.

☐ $0 < x < 2$ ☐ $-2 \le x < 2$ ☐ $0 \le x \le 2$ ☐ $0 \le x < 1$ ☐ $0 \le x < 2$

19. Consider the following series:

$$I: \sum_{n=1}^{\infty} \frac{1}{n^{5/3}}; \quad II: \sum_{n=1}^{\infty} 5^{n/3}; \quad III: \sum_{n=1}^{\infty} \frac{1}{n^{3/5}}; \quad IV: \sum_{n=1}^{\infty} \frac{1}{5^{n/3}}.$$

Which of the following statements is true?

- ☐ I, II, III, IV all converge
- ☐ I and II converge, but III and IV diverge
- ☐ I and III converge, but II and IV diverge
- ☐ I and IV converge, but II and III diverge
- ☐ I converges, but II, III, and IV diverge
- ☐ all diverge

20. f is a differentiable function and $\int_1^2 xf'(x)\mathrm{d}x = 10;\ f(1) = 4; f(2) = 6$. Find $\int_1^2 f(x)\mathrm{d}x$

☐ 1 ☐ -2 ☐ 5 ☐ 10

21. f is a continuous function such that $f(x) + f(a-x)$ does not vanish on the interval $[0; a]$. Find $\int_0^a \frac{f(x)}{f(x)+f(a-x)}\mathrm{d}x$.

☐ $f(a) - f(0)$ ☐ $af(a)$ ☐ a ☐ $\frac{a}{2}$

22. f is a continuous function such that $f(x)f(a-x) = 1$ and $a > 0$. Find $\int_0^a \frac{1}{f(x)+1}\mathrm{d}x$.

☐ $f(a) - f(0)$ ☐ $af(a)$ ☐ a ☐ $\frac{a}{2}$

23. $\int \sqrt{4-x^2}\ \mathrm{d}x =$

- ☐ $2\arcsin\frac{x}{2}+\frac{x}{2}\sqrt{4-x^2}+C$
- ☐ $\frac{x}{2}\sqrt{4-x^2}+C$
- ☐ $2\arccos\frac{x}{2}+\frac{x}{2}\sqrt{4-x^2}+C$
- ☐ $2\arccos\frac{x}{2}+C$

24. $\int_2^4 \frac{2x+4}{\sqrt{x^2+4x}}\ \mathrm{d}x =$

- ☐ $4\sqrt{2}-2\sqrt{3}$
- ☐ $8\sqrt{2}-4\sqrt{3}$
- ☐ $8\sqrt{2}+4\sqrt{3}$
- ☐ $4\sqrt{2}-8\sqrt{3}$

25. $\int_0^1 \frac{\mathrm{d}x}{x^2-4} =$

- ☐ $\frac{\pi}{2}$
- ☐ $\frac{8}{9}$
- ☐ $-\frac{1}{4}\ln 3$
- ☐ $\ln\frac{3}{4}$

26. $\displaystyle\int \frac{\mathrm{d}x}{x^2+3} =$

- ☐ $\frac{1}{3}\arctan\frac{x}{3}+C$
- ☐ $\frac{1}{\sqrt{3}}\arctan\frac{x}{\sqrt{3}}+C$
- ☐ $\arctan\frac{x}{3}+C$
- ☐ $\arctan\frac{x}{\sqrt{3}}+C$

27. $\displaystyle\int \frac{\mathrm{d}x}{x^2-2x-3} =$

- ☐ $\frac{1}{4}\ln\left|\frac{x-3}{x+1}\right|+C$
- ☐ $\frac{1}{4}\ln\left|\frac{x+1}{x-3}\right|+C$
- ☐ $\frac{1}{4}\ln|(x-3)(x+1)|+C$
- ☐ $\ln|x^2-2x-3|+C$

28. $\displaystyle\int_{-1}^{1} \ln(x+2)\ \mathrm{d}x =$

☐ $3\ln 3$ ☐ $3\ln 3-3$ ☐ $3\ln 3-2$ ☐ $\ln 3$

29. $\displaystyle\int_{0}^{\pi} x\sin 2x\ \mathrm{d}x =$

☐ π ☐ 0 ☐ $-\frac{\pi}{2}$ ☐ $\frac{\pi}{2}$

30. $\displaystyle\int_0^{\pi/2} \frac{\mathrm{d}x}{1+(\tan x)^{\sqrt{2}}} =$

☐ π ☐ 0 ☐ $\frac{\pi}{2}$ ☐ $\frac{\pi}{4}$

31. $\displaystyle\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{2n}$

☐ 1 ☐ e ☐ $e^{1/2}$ ☐ e^2

32. $\displaystyle\lim_{n\to+\infty}\left(1+\frac{1}{2n}\right)^{n}$

☐ 1 ☐ e ☐ $e^{1/2}$ ☐ e^2

33. $\{a_n\}_{n=1}^{\infty}$ is a sequence of non-negative real numbers such that $\displaystyle\sum_{n=1}^{\infty} a_n$ converges. Which of the following series always converges?

☐ $\displaystyle\sum_{n=1}^{\infty} \sin a_n$

☐ $\displaystyle\sum_{n=1}^{\infty} \sqrt{a_n}$

☐ $\displaystyle\sum_{n=1}^{\infty} \ln(2+\tan a_n)$

☐ $\displaystyle\sum_{n=1}^{\infty} (\ln n)a_n$

34. Which of the following series diverges?

- ☐ $\sum_{n=1}^{\infty} \frac{1}{n^{\ln n}}$
- ☐ $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/\ln n}}$
- ☐ $\sum_{n=3}^{\infty} \frac{(\ln \ln n)^2}{n(\ln n)^2}$
- ☐ $\sum_{n=1}^{\infty} \tan\left(\frac{1}{n\sqrt{n}}\right)$

35. $\cos(x+1) =$

- ☐ $\sum_{n=0}^{\infty} (\cos 1)\frac{(-x)^n}{n!}$
- ☐ $\sum_{n=0}^{\infty} (-1)^n \frac{(1-x)^n}{n!}$
- ☐ $\sum_{n=0}^{\infty} (\sin 1)(-1)^n \frac{x^{2n}}{(2n)!} - (\cos 1)(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
- ☐ $\sum_{n=0}^{\infty} (\cos 1)(-1)^n \frac{x^{2n}}{(2n)!} - (\sin 1)(-1)^n \frac{x^{2n+1}}{(2n+1)!}$

36. $\sum_{n=1}^{\infty} \frac{7^n}{8^{n-1}} =$

☐ 7 ☐ $\frac{8}{7}$ ☐ 56 ☐ 49

37. $\sum_{n=1}^{\infty} n \left(\frac{7}{8}\right)^{n-1} =$

- ☐ 8
- ☐ 64
- ☐ 56
- ☐ 49

38. $\sum_{n=0}^{\infty} \frac{1}{(n+1)!} =$

☐ $e-1$ ☐ e ☐ $e+1$ ☐ e^{-1}

39. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{(n+1)!} =$

☐ e^{-1} ☐ $e^{-1}+1$ ☐ $e^{-1}-1$ ☐ $1-e^{-1}$

40. $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n}}{(2n)!} =$

☐ $\frac{\pi}{\pi-1}$ ☐ 0 ☐ 1 ☐ -1

41. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} =$

☐ e ☐ $\ln \frac{1}{2}$ ☐ 1 ☐ $\ln 2$

42. $\sum_{n=2}^{\infty} \frac{1}{n^3 - n} =$

☐ 1 ☐ $\frac{1}{2}$ ☐ $\frac{1}{4}$ ☐ $\frac{1}{6}$

43. $\sum_{n=1}^{\infty} \frac{1}{n^2 + n} =$

☐ 1 ☐ $\frac{1}{2}$ ☐ $\frac{1}{4}$ ☐ $\frac{1}{6}$

44. $\sum_{n=0}^{\infty} \arctan \frac{1}{n^2 + n + 1} =$

☐ π ☐ $\frac{\pi}{4}$ ☐ $\frac{\pi}{2}$ ☐ $\frac{\pi}{6}$

45. What are all the values for which $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n^2}$ converges?

☐ $\{x : 0 \leq x \leq 2\}$
☐ $\{x : 0 \leq x \leq 1\}$
☐ $\{x : -1 \leq x \leq 2\}$
☐ $\{x : -1 \leq x \leq 1\}$

46. Let $\sum_{n=0}^{\infty} a_n x^n = (1-x)^{1/2} + (1+x)^{1/2}$ be the Maclaurin expansion of

$$(1-x)^{1/2} + (1+x)^{1/2}.$$

Find

$$(a_0)^2 + (a_1)^2 + (a_2)^2.$$

☐ $\frac{65}{16}$ ☐ $\frac{63}{16}$ ☐ $\frac{7}{4}$ ☐ $\frac{9}{4}$ ☐ $\frac{81}{16}$ ☐ $\frac{15}{4}$

End Quiz

Score: Correct

Author: David A. SANTOS, May 2006